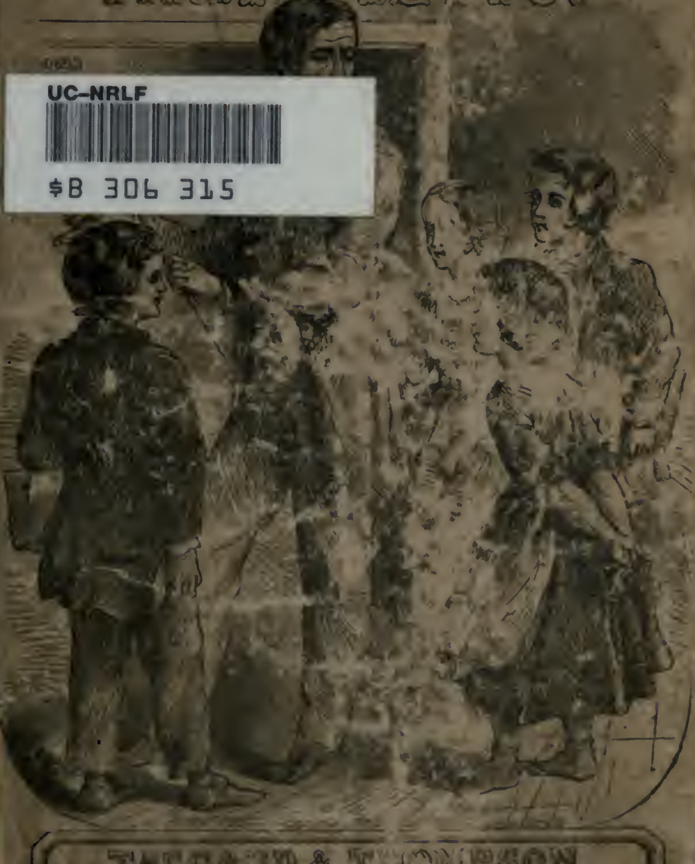
EATON'S COMMON SCHOOL ARITHMETIC.



TAGGARD & THOMPSON.
BOSTON.

THE

COMMON SCHOOL ARITHMETIC;

COMBINING

ANALYSIS AND SYNTHESIS;

ADAPTED TO

THE BEST MODE OF INSTRUCTION IN THE ELEMENTS OF WRITTEN ARITHMETIC.

BY

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BOSTON:
TAGGARD AND THOMPSON,
29 CORNHILL.
SAN FRANCISCO: H. H. BANCROFT & COMPANY.
1864.

ANDOVER:
ELECTROTYPED AND PRINTED
BY W. F. DRAPER.

PREFACE.

THERE is a large class of pupils whose limited time renders it impossible for them to pursue an extended mathematical course. The author, in accordance with his original intention to prepare a series of text-books in Arithmetic, has now endeavored to adapt this work to the wants of this class of pupils.

With this purpose in view, the simple, elementary, practical principles of the science are more fully presented than in his larger work, while the more intricate and less important parts have been treated more briefly or entirely omitted. A corresponding change in the character of the examples has also been made.

As in the larger work, so here, constant attention has been paid to the brevity, simplicity, perspicuity, and accuracy of expression; and no effort has been spared in the endeavor to render the mechanical execution appropriate and attractive.

Definitions, tables, and explanations of signs have been distributed through the book where their aid is needed, to enable the pupil to learn them more readily than when they are presented collectively.

Nearly all the examples have been prepared for this book, and are different from those of the larger work; still, to secure uniformity of language (a matter of great importance, as every experienced teacher knows), the leading examples in the several subjects, the definitions and rules, with few exceptions, have been intentionally retained with but little modification.

Articles on United States Money, Percentage, Stocks, Custom-House Business, and Exchange have been prepared for this book; and all the principles requisite for a practical business life have been presented in a simple, intelligible, attractive manner, and with sufficient minuteness and fullness and a due regard to logical arrangement.

Brief, suggestive questions have been placed at the bottom of the page, designed in no way to interfere with the free, original questioning which every *teacher* will adopt for himself, but merely to aid the young and inexperienced pupil in fixing his attention upon the more important parts of the subject.

Here, as in the larger work, some of the answers to examples have been given to inspire confidence in the learner, and others are omitted to secure the discipline resulting from proving the operations, a discipline and a benefit which the pupil should not forego nor the teacher neglect.

Fully appreciating the favor which has been bestowed on his other works, the author sends this forth, hoping it may commend itself to the approval of committees and teachers, and that it may be found adapted to contribute in some measure to the happiness and improvement of the class of pupils for whom it is designed.

A Key, containing the Answers not given in this book, is published for the use of Teachers.

PHILLIPS ACADEMY, ANDOVER,
April 19, 1862.

CONTENTS.

SIMPLE NUMBERS.

REDUCTION OF COMPOUND NUMBERS.

GENERAL PRINCIPLES.

COMMON FRACTIONS.

1*

DECIMAL FRACTIONS.

UNITED STATES MONEY.

COMPOUND NUMBERS.

PERCENTAGE.

MISCELLANEOUS.

ARITHMETIC.

ARTICLE 1. ARITHMETIC is the *science* of numbers, and the *art* of computation.

A NUMBER is a *unit* or a *collection of units*, a unit being *one*, i. e. *a single thing of any kind;* thus, in the number *six* the unit is *one;* in *ten days* the unit is *one day.*

2. All numbers are *concrete* or *abstract.*

A CONCRETE NUMBER is a number that is applied to a particular object; as six books, ten men, four days.

An ABSTRACT NUMBER is a number that is *not* applied to any particular object; as six, ten, seventeen.

3. Arithmetic employs six different operations, viz. *Notation, Numeration, Addition, Subtraction, Multiplication,* and *Division.*

NOTATION AND NUMERATION.

4. NOTATION is the art of expressing numbers and their relations to each other by means of *figures and other symbols.*

5. NUMERATION is the art of reading numbers which have been expressed by figures.

ART. 1. What is Arithmetic? What is a Number? A Unit? 2. What is a Concrete Number? An Abstract Number? 3. How many operations in Arithmetic? What are they? 4. What is Notation? 5. Numeration?

6. Two methods of notation are in common use: the *Arabic* and the *Roman.*

7. The ARABIC NOTATION, or that brought into Europe by the Arabs, employs *ten figures* to express numbers, viz.:

0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Naught,	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine.

These figures are called *digits*, from the Latin *digitus, a finger;* a term probably applied to figures from the custom of *counting upon the fingers.*

8. The first Arabic figure, 0, is called *a cipher, naught,* or *zero,* and, standing alone, it signifies *nothing.*

Each of the remaining nine figures represents the number placed under it, and for convenience in distinguishing them from 0, they are called *significant figures.*

9. No number greater than *nine* can be expressed by a single Arabic figure, but by repeating the figures, and arranging them differently, all numbers may be represented.

Ten is expressed by writing the figure 1 at the left of the cipher; thus, 10. In like manner, twenty, thirty, forty, etc., are expressed by placing 2, 3, 4, etc., at the left of 0; thus,

20,	30,	40,	50,	60,	70,	80,	90.
Twenty,	Thirty,	Forty,	Fifty,	Sixty,	Seventy,	Eighty,	Ninety.

10. The numbers from 10 to 20 are expressed by placing the figure 1 at the left of each of the significant figures; thus,

11,	12,	13,	14,	15,	16,	17,	etc.
Eleven,	Twelve,	Thirteen,	Fourteen,	Fifteen,	Sixteen,	Seventeen,	etc.

In a similar manner all the numbers, up to one hundred, may be written; thus,

21,	36,	66,	98,	etc.
Twenty-one,	Thirty-six,	Sixty-six,	Ninety-eight,	etc.

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6. How many methods of Notation? What? **7.** How many figures in the Arabic Notation? What called? Why? **8.** What is the first figure, 0, called? The others? Why? **9.** The largest number expressed by one figure? Ten, how expressed? Twenty? **10.** Numbers from ten to twenty, how expressed?

11. One hundred is expressed by placing the figure 1 at the left of *two ciphers;* thus 100. In like manner two hundred, three hundred, etc., are written; thus,

200,	300,	600,	800,	etc.
Two hundred,	Three hundred,	Six hundred,	Eight hundred,	etc.

12. The other numbers, up to one thousand, may be expressed by putting a significant figure in the place of one or each of the ciphers in the above numbers; thus,

Two hundred and three, expressed in figures, is 203,
Six hundred and eighty, expressed in figures, is 680,
Nine hundred and ninety-eight, expressed in figures, is 998.

13. The PLACE of a figure is the *position* it occupies with reference to other figures; thus, in 436, the 6, counting from the *right*, is in the *first* place, 3 is in the *second* place, and 4 in the *third* place.

The figure in the *first place* represents *simple units*, or units of the *first order;* the *second* figure represents *tens*, or units of the *second order;* the *third*, *hundreds*, or units of the *third order;* the *fourth*, *thousands*, or units of the *fourth order*, etc.; thus, in the number 3576, the 6 is 6 units of the *first* order; the 7 tens is 7 units of the *second* order; the 5 hundreds is 5 units of the *third* order, etc.

14. From the foregoing it will be seen that each *significant* figure has *two values*; one of which is *constant* (i. e. *always the same*), the other *variable;* thus, in each of the numbers 2, 20, and 200, the left hand figure is *two;* but in the *first* it is two *units;* in the *second*, two *tens;* and in the *third*, two *hundreds.*

The former of these values is the *inherent* or *simple* value, and the latter is the *local* or *place* value.

15. *It is also evident that the value of a figure is made ten fold by removing it one place toward the left; a hundred fold by removing it two places, etc.;* i. e. ten units of the first order

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11. One hundred, how expressed? Two hundred? **12.** Other numbe s, how expressed? **13.** What is the *place* of a figure? What does the figure in the first place represent? Second place? Third? **14.** How many, and what values has a figure? **15.** How does moving a figure towards the left affect its value?

make one ten, ten tens make one hundred, ten hundreds make one thousand, and, in short, *ten units of any order make one unit of the next higher order.*

16. The *cipher,* when used with other figures, fills a place that would otherwise be vacant; thus, in 206 the cipher occupies the place of *tens,* because there are *no tens expressed* in the given number.

17. The figures of large numbers, for convenience in reading, are often separated by commas into periods or groups.

There are two methods of *numerating:* the FRENCH and the ENGLISH. By the French method a period consists of *three* figures; by the English, of *six.* The French method is most convenient, and principally used in this country.

18. By the FRENCH METHOD OF NUMERATION the first or right-hand period contains units, tens, and hundreds, and is called the *period of units;* the second period contains thousands, tens of thousands, and hundreds of thousands, and is called the *period of thousands;* etc., as in the following

FRENCH NUMERATION TABLE.

Tens of Quintillions.	Quintillions,	Hundreds of Quadrillions,	Tens of Quadrillions,	Quadrillions,	Hundreds of Trillions,	Tens of Trillions,	Trillions,	Hundreds of Billions,	Tens of Billions,	Billions,	Hundreds of Millions,	Tens of Millions,	Millions,	Hundreds of Thousands,	Tens of Thousands,	Thousands,	Hundreds,	Tens,	Units,
2	8,	7	6	9,	5	4	0,	7	0	6,	4	7	6,	0	0	1,	8	4	3.
7th period, Quintillions.		6th period, Quadrillions,			5th period, Trillions,			4th period, Billions,			3d period, Millions,			2d period, Thousands,			1st period, Units,		

16. For what is the cipher used? 17. How many methods of numerating? What are they? Which is generally used in this country? 18. Name the different periods in the French Numeration Table. Repeat the table.

19. The value of the figures in this table, expressed in words, is twenty-eight quintillion, seven hundred and sixty-nine quadrillion, five hundred and forty trillion, seven hundred and six billion, four hundred and seventy-six million, one thousand, eight hundred and forty-three.

NOTE. The READING of a number consists of two distinct processes: First, reading the *order of the places*, beginning at the right hand; thus, units, tens, hundreds, etc., as in the *Numeration Table;* and, second, reading the *value* of the figures, beginning at the left, as above. To distinguish these processes, the first may be called *numerating*, and the second *reading*, the number.

20. The table can be extended to any number of places, adopting a new name for each succeeding period. The periods above quintillions are sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, etc.

21. To numerate and read a number according to the French method:

RULE. 1. *Beginning at the right, numerate and point off the number into periods of* THREE *figures each.*

2. *Beginning at the left, read each period separately, giving the name of each period except that of units.*

EXERCISES IN NUMERATION BY THE FRENCH METHOD.

22. Let the learner read the following numbers:

1.	24	11.	7,435,720,597
2.	357	12.	74,690,007,467
3.	4,649	13.	297,999,399,089
4.	95,679	14.	6,137,731,975,468
5.	549,517	15.	45,719,456,972,145
6.	5,745,328	16.	457,749,136,958,083
7.	52,073,712	17.	3,125,945,654,315,756
8.	243,967,184	18.	57,963,568,194,437,973
9.	4,674,925,178	19.	367,942,143,866,145,316
10.	43,404,876,347	20.	3,593,047,671,350,486,950

19. What is the value of the number expressed in the table? Reading a number consists of how many processes? What are they? **20.** What are the names of periods above Quintillions? **21.** Rule for numerating and reading a number by the French method?

23. To write numbers by the French method:

RULE. 1. *Beginning at the left, write the figures belonging to the highest period.*

2. *Write the figures of each successive period in their order, filling each vacant place with a cipher.*

EXERCISES IN FRENCH NOTATION AND NUMERATION.

24. Let the learner write the following numbers in figures, and read them by the French method:

1. Two units of the third order and five of the first. Ans. 205.

NOTE. Since no figure of the second order is given, a *cipher* is written in the second place.

2. Six units of the fourth order, three of the second, and eight of the first. Ans. 6,038.

3. One unit of the seventh order, three of the sixth, seven of the third, and two of the second. Ans. 1,300,720.

4. Five units of the fifth order and three of the fourth.

5. Six units of the fourth order and one of the third.

6. Two units of the eighth order and three of the sixth.

7. Nine units of the ninth order, six of the fifth, one of the second, and three of the first.

25. Express the following numbers in figures by the French notation:

1. Three hundred and fifty-six. Ans. 356.
2. Six hundred and fifty-three. Ans. 653.
3. Five hundred and sixty-three. Ans. 563.
4. Three hundred and sixty-five.
5. Six hundred and fifty-one.
6. One thousand, six hundred and fifty-one. Ans. 1,651.
7. Forty-two thousand, five hundred and fifty-four.
8. Eight hundred sixteen thousand, and two hundred.
9. Six million, one hundred four thousand, two hundred and seventy-six. Ans. 6,104,276.

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23. Rule for writing numbers by the French method?

10. Three hundred six thousand, five hundred and two.

11. Nine hundred forty-six million, five hundred fourteen thousand, nine hundred and twenty-five.

12. Six billion, fifteen million, seven thousand, and four hundred. Ans. 6,015,007,400.

13. Five million, six hundred fifty-one thousand, four hundred and six.

14. Seventy-four million.

15. Sixty-three million, fourteen thousand, and seven hundred.

26. By the ENGLISH METHOD OF NUMERATION, the first period contains units, tens, hundreds, thousands, tens of thousands, and hundreds of thousands, and is called the *period of units;* the second period contains millions, tens of millions, hundreds of millions, thousands of millions, tens of thousands of millions, and hundreds of thousands of millions, and is called the *period of millions ;* etc., as in the following

ENGLISH NUMERATION TABLE.

Tens of Trillions.	Trillions,	Hundreds of Thousands of Billions,	Tens of Thousands of Billions,	Thousands of Billions,	Hundreds of Billions,	Tens of Billions,	Billions,	Hundreds of Thousands of Millions,	Tens of Thousands of Millions,	Thousands of Millions,	Hundreds of Millions,	Tens of Millions,	Millions,	Hundreds of Thousands,	Tens of Thousands,	Thousands,	Hundreds,	Tens,	Units,
2	8,	7	6	9	5	4	0,	7	0	6	4	7	6,	0	0	1	8	4	3.
4th period, Trillions.		3d period, Billions,						2d period, Millions,						1st period, Units,					

26. By the English numeration what figures are in the first period? Second period? Third? Repeat the table.

27. The value of the figures in this table, is twenty-eight trillion, seven hundred sixty-nine thousand five hundred and forty billion, seven hundred six thousand four hundred and seventy-six million, one thousand eight hundred and forty-three.

28. The *names* of the figures *and their values are the same* in the two tables for the first *nine* places from the right, after which they are *alike in value but different in name.* A trillion by the English method is much more than by the French.

29. To numerate and read a number according to the English method:

RULE. 1. *Beginning at the right, numerate and point off the number into periods of* SIX *figures each.*

2. *Beginning at the left, read each period separately, giving the name of each period except that of units.*

EXERCISES IN NUMERATION BY THE ENGLISH METHOD.

30. Read the following numbers:

1.	684	4.	87,658765,647596
2.	853697	5.	95467,694164,745689
3.	7,474569	6.	47,678600,709050,359691

31. To write numbers by the English method:

RULE. 1. *Beginning at the left, write the figures belonging to the highest period.*

2. *Write the figures of each successive period in their order, filling each vacant place with a cipher.*

EXERCISES IN ENGLISH NOTATION AND NUMERATION.

32. Write the following, and read by the English method:

1. Five units of the eighth order, six of the seventh, two of the fourth, and one of the third. Ans. 56,002100.

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27. What number is expressed by the table? **28.** Are the *names* of figures alike in the French and English tables? Their *values*, alike or unlike? **29.** Rule for numerating and reading a number by the English method? **31.** Rule for writing a number by the English method?

2. Nine units of the fourteenth order, two of the twelfth, three of the eleventh, six of the eighth, nine of the sixth, two of the fifth, and three of the fourth. Ans. 90,230060,923000.

3. Two units of the ninth order, six of the sixth, one of the fifth, two of the third, seven of the second, and five of the first.

33. Express the following numbers by the English Notation:

1. Seventy-two million, six hundred thirteen thousand four hundred and forty-six. Ans. 72,613446.

2. Five hundred seventeen billion, three hundred twenty-two thousand one hundred fourteen million, eight hundred forty-one thousand nine hundred and sixty-nine.

3. Two hundred and ten billion, and six thousand.

NOTE. These and other exercises will be varied and extended by the teacher as circumstances may dictate.

34. The ROMAN NOTATION, or that used by the ancient Romans, employs *seven capital letters* to express numbers, viz.:

I,	V,	X,	L,	C,	D,	M.
One,	Five,	Ten,	Fifty,	One hundred,	Five hundred,	One thousand.

All other numbers may be expressed by combining and repeating these letters,

35. The Roman Notation is based on the following principles:

1st. When two or more letters of equal value are united, or when a letter of less value *follows* one of greater, the *sum* of their values is indicated; thus, XXX stands for 30, LXV for 65, CC for 200, MDCLXVII for 1667.

2d. When a letter of less value is placed *before* one of greater, the *difference* of their values is indicated; as, IX stands for 9, XL for 40, XC for 90.

3d. When a letter of less value stands *between* two of greater value, the less is to be *taken from the sum of the other two;* as, XIV stands for 14, XIX for 19, CXL for 140.

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34. How many and what characters are employed in the Roman Notation? What is the value of each? **35.** What is the first *principle* in Roman Notation? Second? Third?

4th. A letter with a line over it represents a number one thousand times as great as the same letter without the line; thus X stands for *ten*, but $\overline{X}$ stands for *one thousand times ten*, i. e. *ten thousand;* M stands for *one thousand*, but $\overline{M}$ for *one thousand times one thousand.*

TABLE OF ROMAN NUMERALS.

I	1	XVI	16	CCCC	400
II	2	XVII	17	D	500
III	3	XVIII	18	DC	600
IV	4	XIX	19	DCCCC	900
V	5	XX	20	M	1000
VI	6	XXI	21	MD	1500
VII	7	XXIV	24	MDC	1600
VIII	8	XXV	25	MDCLXV	1665
IX	9	XXIX	29	MDCCXLIX	1749
X	10	XXX	30	MDCCCXVI	1816
XI	11	XL	40	MDCCCLXII	1862
XII	12	L	50	$\overline{V}$	5000
XIII	13	LX	60	$\overline{L}$	50000
XIV	14	XC	90	$\overline{C}$	100000
XV	15	C	100	$\overline{M}$	1000000

EXERCISES IN ROMAN NOTATION.

36. Express the following numbers by letters:

1. Twelve. Ans. XII.
2. Eighteen. Ans. XVIII.
3. Twenty-nine.
4. Ninety-nine.
5. Two hundred and eighty-four.
6. One thousand four hundred and forty-six.
7. One thousand six hundred and forty-four.
8. The present year, A. D. ——.

NOTE. The Roman notation is very inconvenient for Arithmetical operations, and the Roman numerals are now seldom used, except for numbering the pages of a preface, the divisions of a discourse, and the sections, chapters, and other divisions of a book.

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35. What is the fourth principle in Roman Notation? **36.** Are Roman numerals much used in arithmetical operations? Why? For what are they used?

37. Besides the Arabic and the Roman figures, there are various marks used to indicate that certain operations are to be performed, such, e. g., as the *sign of addition*, +; the *sign of subtraction*, —; etc. These signs will be given, and their uses explained when their aid is needed.

ADDITION.

38. ADDITION is the putting together of two or more numbers of the same kind, to find their *sum* or *amount*.

The *sum* or *amount* of two or more numbers is a number which contains the same number of units as the two or more numbers put together; thus, 7 is the sum of 3 and 4, because there are just as many units in 7 as in 3 and 4 put together; for a like reason 11 *days* is the sum of 2 days, 4 days, and 5 days.

Ex. 1. James has 4 marbles, John has 5, and Henry has 3; how many marbles have they all?

To solve this example, add the numbers 4, 5, and 3: thus, 4 and 5 are 9, and 3 are 12; therefore James, John, and Henry have 12 marbles, Ans.

2. How many are 3 and 6? 6 and 3? 2 and 5 and 7?

39. A SIGN is a mark which indicates an operation to be performed, or which is used to shorten some expression.

40. The *sign of dollars* is written thus, $; e. g. $2 represents *two dollars;* $10, *ten dollars,* etc.

41. The *sign of equality*, =, signifies that the quantities between which it stands are equal to each other; thus, $1 = 100 cents, i. e. one dollar equals one hundred cents.

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37. What characters are used in Arithmetic besides the Arabic and Roman figures? For what?

38. What is Addition? Sum or amount? 39. A sign? 40. Make the sign of dollars on the black-board. 41. Make the sign of equality. What does it mean?

42. The *sign of addition*, $+$, called *plus*, denotes that the quantities between which it stands are to be added together; thus, $3+2=5$, i. e. three plus two equals five, or three and two are five.

43. Three *dots*, thus, $\therefore$, are the symbol for *therefore*, *hence*, or *consequently*; thus, $2+3=5$, and $3+2=5$, $\therefore 2+3=3+2$, i. e. *therefore* the sum of 2 and 3 is equal to the sum of 3 and 2.

Ex. 3. William paid \$4 for a pair of skates, \$3 for a sled, and \$1 for a knife; what did he pay for all?

$\$4+\$3+\$1=\$8$, Ans.

4. What is the sum of $\$6+\$3$? $\$5+\$2+\$8$?

5. What is the sum of $4+6+2+3$? $3+5+8+2$?

44. To add when the numbers are large and the amount of each column is less than 10.

6. A manufacturer sold 125 yards of cloth to one merchant, 342 to another, and 231 to another; how many yards did he sell in all? Ans. 698.

OPERATION.

	1 2 5
	3 4 2
	2 3 1
Sum,	6 9 8

Having arranged the numbers so that units stand under units, tens under tens, etc., add the units; thus, 1 and 2 are 3, and 5 are 8, and set the result under the column of units. Then add the tens; thus, 3 and 4 are 7, and 2 are 9, set down the result, and so proceed till all the columns are added.

Ex. 7.	8.	9.	10.
4 2 5	1 2 7	1 0 6	6 2 0 4
1 4 3	3 4 1	3 4 1	2 4 1 3
2 3 1	2 1 0	1 2 1	1 2 3 1
Sum, 7 9 9	6 7 8	5 6 8	9 8 4 8

11.	12.	13.	14.
2 0 0 0	1 1 2 1	1 1 2 0 0	1 0 0 0
2 3 4 5	5 1 2 7	2 5 4 1 3	2 7 4 3
1 4 2 3	2 3 4 0	3 2 1 4 2	3 1 5 4
3 2 3 1	1 4 0 0	2 1 0 3 4	1 0 0 1

42. Make the sign of addition. **43.** Sign for *therefore*. **44.** How are numbers arranged for addition? Which column is added first? Its sum, where placed?

15. What is the sum of 1243, 2112, and 1313? Ans. 4668.

16. What is the sum of 2013, 1421, 2132, and 1231?

17. A gentleman paid $125 for a horse, $231 for a chaise, and $32 for a harness; what did he pay for all? Ans. $388.

45. To add when the amount of any column is 10 or more.

18. Add together 27, 93, and 145. Ans. 265.

OPERATION.

27
93
145
Ans. 265

Having arranged the numbers, add the column of units; thus, 5 and 3 are 8, and 7 are 15 units (= 1 ten and 5 units). The 5 units are placed under the column of units, and the 1 ten is added to the column of tens; thus, 1 and 4 are 5, and 9 are 14, and 2 are 16 tens (= 1 hundred and 6 tens). The 6 tens are set under the tens, and the 1 hundred is added to the 1 hundred in the third column, making 2 hundreds to be set under the third column.

19.	20.	21.	22.
276	748	4681	36487
483	249	7362	10462
874	838	8428	38420
Ans. 1633	1835	20471	85369

23.	24.	25.	26.
417	246	3874	34827
819	385	1920	5148
234	274	4208	97604
846	961	3186	27
721	249	8004	86129
Ans. 3037			

27.	28.	29.	30.
46723	4628	327	3
5432	94342	56948	784
46	4	4876	98643

31. Add 3467, 82, 946, 13845, and 426. Ans. 18766.

32. Add 64287, 342, 8694, 32, and 46872.

33. Add 3462, 8, 97, 4682, 3800, and 47289.

34. Add 384, 16942, 34, 87, 6294, and 3274.

46. The examples already given embrace all the principles in addition. Hence, to add numbers,

RULE. *Write the numbers in order, units under units, tens under tens, etc. Draw a line beneath, add together the figures in the units' column, and, if the sum be less then ten, set it under that column; but, if the sum be ten or more, write the units as before, and add the tens to the next column. Thus procced till all the columns are added.*

47. PROOF. *The usual mode of proof is to begin at the top and add downward. If the work is right, the two sums will be alike.*

NOTE 1. By this process, we *combine* the figures *differently*, and hence shall *probably* detect any mistake which may have been made in adding *upward*.

ILLUSTRATION.

	Ex. 35.
	37684
	48297
	68746
	94852
Sum,	249579
Proof,	249579

In adding *upward* we say, 2 and 6 are 8, and 7 are 15, and 4 are 19, etc.; but in adding *downward*, we say, 4 and 7 are 11, and 6 are 17, and 2 are 19, etc., thus obtaining the *same result*, but by *different combinations*.

If we do not obtain the same result by the two methods, one operation or the other is wrong, perhaps both, and the work must be *carefully* performed again.

NOTE 2. *In adding it is not desirable to name the figures that we add; thus, in example* 35, *instead of saying* 2 *and* 6 *are* 8, *and* 7 *are* 15, *and* 4 *are* 19, *it is shorter, and therefore better, to say* 2, 8, 15, 19; *setting down the* 9, *say* 1, 6, 10, 19, 27, *etc.*

36. What is the sum of 8432, 42698, 34, 1892, 70068, 5142, and 68742? Ans. 197008.

37. What is the sum of 2468, 13579, 276, and 42?

38. What is the sum of 3406, 872, 6541, 2, and 17?

39. What is the sum of 3910, 4, 876, 27, and 89462?

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46. If the amount of any column is ten or more, where is the right-hand figure of the amount written? What is done with the left-hand figure? Repeat the rule for Addition. **47.** How is Addition proved? Why not add *upward a second time?* Is it desirable to name the figures as we add them?

	Ex. 40.	41.	42.	43.
	51000	20404	21153	31201
	11608	44346	25000	22222
	38020	93040	15000	66666
	49132	90000	55555	55555
	12883	95000	54445	33333
Sum,	162643			
Proof,	162643			

44. How many are 876 + 9287 + 69842 + 7700? Ans. 87705.

45. How many are 36904 + 216 + 8942 + 47?

46. How many are 18 + 4 + 76984 + 327 + 14?

47. 846 + 972 + 84 + 300 = how many? Ans. 2202.

48. 2468 + 9867 + 37428 + 278 = how many?

49. 3004 + 6094 + 87642 + 36 = ? Ans. 96776.

50. 2468 + 13579 + 100 + 6042 + 187 + 19 = ?

51. Add four hundred and sixty-two; three thousand two hundred and fourteen; seventy-nine thousand six hundred and fifty-nine; and two hundred and eighty-four. Ans. 83619.

52. Add four hundred and fifty-six; eight thousand, four hundred and seventy-two; fifteen thousand, seven hundred and twenty-one; forty-three million, seven hundred and thirty-three thousand, eight hundred and fifty-nine; and ten.

53. The population of England in 1851 was 16921888; of Scotland, 2888742; of Wales, 1005721; of Ireland, 6515794. What was the population of Great Britain and Ireland?

54. England and Wales contain about 55100 square miles; Scotland 29600; and Ireland, 32000; what is the area of the British Islands? Ans. 116700 square miles.

55. By the census of 1860, the number of inhabitants of Maine, was 628276; of New Hampshire, 326072; of Vermont, 315116; of Massachusetts, 1231065; of Rhode Island, 174621; of Connecticut, 460151; what was the population of New England? Ans. 3135301.

56. The area of Maine is 35000 square miles; N. H., 8030; Vt., 8000; Mass., 7250; R. I., 1200; Ct., 4750. What is the area of New England?

57. In 1850 the population of Maine was 583169; of New Hampshire, 317976; of Vermont, 314120; of Massachusetts, 994514; of Rhode Island, 147545; of Connecticut, 370792; what was the population of these six States in 1850?

58. A merchant, commencing business, had in cash, \$4376; goods worth \$3780; bank stock worth \$2700; and other property valued at \$5496. In a year he gained \$2475; what was he worth at the end of the year?

59. In one year a farmer sold a pair of oxen for \$125, two cows for \$75, three swine for \$96, twenty sheep for \$120, and a horse for \$156; what did he receive for all?

60. On Monday, a merchant sold goods for \$357, on Tuesday, for \$463, on Wednesday, for \$279, on Thursday, for \$318, on Friday, for \$687, and on Saturday for \$348; what was the value of the goods sold during the week?

61. In 1850 the population of New York was 515547; of Philadelphia, 340045; of Baltimore, 169054; of Boston, 136881; of New Orleans, 116375; and of Cincinnati, 115436; what was the number of inhabitants in these six cities in 1850?

62. In the middle of the nineteenth century the population of London was about 2363141; of Paris, 1053897; of Constantinople, 786990; of St. Petersburg, 478437; of Vienna, 477846; of Berlin, 441931; and of Naples, 416475; what was the population of these seven cities?

63. In 1850 the population of the United States was about 23191876; of Great Britain and Ireland, 27332145; of France, 35783170; of Russia, 62088000; and of Austria, 36514397; what was the population of these five countries?

64. The population of North America is about 39257819; of South America, 18373188; of Europe, 265368216; of Asia, 630671661; of Africa, 61688779, and of Oceanica, 23444082; what is about the population of the globe? Ans. 1038803745.

65. The cost of the American army for five successive years, commencing in 1812, was \$12187046, \$19906362, \$20608366, \$15394700, and \$16475412; what was the cost for five years?

66. B owes to C \$150, to D \$4682, to E \$267, to F \$54, and to G \$1353; how much does he owe?

67.	68.	69.	70.
95690	19998	59059	28738
58689	58596	79819	52903
19821	01298	18582	75755
55555	41239	93977	27579
12677	93333	50504	11111
24764	47804	56667	88888
24914	87046	84769	76554
25900	98764	25251	32690
24878	58698	24274	12465
19864	95490	55628	54000
27414	98695	72869	22878
29925	96564	27121	40502
27208	90825	46862	28276
16502	92672	62128	27262
21778	92267	74279	61625
25427	76152	24725	52465
24521	97267	76592	27248
47214	73017	15172	47510

71. In January there are 31 days, in February 28, in March 31, in April 30, in May 31, in June 30, in July 31, in August 31, in September 30, in October 31, in November 30, and in December 31; how many days are there in a year?

72. A gardener has 3476 apple trees, 8476 pear trees, 5684 peach trees, 1845 plum trees, 4680 quince trees, and 9487 ornamental trees; how many trees are there in his nursery?

73. The first of three numbers is 4768, the second is 8942, and the third is as much as the other two; what is the sum of the three numbers?

74. I have $376 in one bank, $4678 in another, and in another as much as in both of these; how much money have I in the three banks?

75. An army consists of 276450 infantry, 14875 cavalry, 27846 artillery men, and 127462 riflemen; what is the number of men in the army?

76. A carpenter engaged to build 4 houses, the first for $3462, the second for $6875, the third for $8963, and the fourth for $12462; what shall he receive for the four houses?

SUBTRACTION.

48. SUBTRACTION is taking a *less number* from a *greater* number of the same kind, to find their *difference.*

The greater number is called the MINUEND; the less number is called the SUBTRAHEND; and the result is called the DIFFERENCE or REMAINDER.

Ex. 1. Arthur had 7 apples, but he has given 4 of them to Mary; how many apples has he now?

Ans. 3; because 4 apples taken from 7 apples leave 3 apples.

2. John having 17 marbles, lost 7 of them; how many had he left?

49. The *sign of subtraction*, —, called *minus*, signifies that the number after it is to be taken from the number before it; thus, $7 - 4 = 3$, i. e. seven minus four, or seven diminished by four, equals three.

3. How many are $10 - 6$? Ans. 4.

4. How many are $12 - 8$? $12 - 4$? $16 - 6$?

NOTE. When the numbers are small, the subtraction is readily performed *in the mind;* but when they are large, the work is more easily done *by writing the figures*, as in the following examples.

50. To subtract when no figure in the subtrahend is greater than the corresponding figure in the minuend.

5. From 796 take 582.

OPERATION.	
Minuend,	7 9 6
Subtrahend,	5 8 2
Remainder,	2 1 4

This example is solved by taking the 2 units from 6 units, 8 tens from 9 tens, and 5 hundreds from 7 hundreds, giving 214 for the remainder.

	6.	7.	8.	9.
Minuend,	4 6 9	5 6 4 2	9 8 7 4	8 0 7 2
Subtrahend,	3 2 7	4 1 3 0	3 6 2 3	3 0 5 1
Remainder,	1 4 2	1 5 1 2	6 2 5 1	5 0 2 1

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48. What is Subtraction? Minuend? Subtrahend? Remainder? **49.** Make the *sign* of subtraction. Its meaning? How do we subtract when the numbers are small? How when they are large?

	10.	11.	12.	13.
From	2741	5462	6408	8420
Take	1301	1350	3207	3110
Ans.	1440			

14. A farmer bought a farm for $4875 and sold it again for $3463; how much did he lose by the transactions? Ans. $1412.

15. By the census of 1860, the population of Maine was 628276, and that of New Hampshire was 326072; how many more people were there in Maine than in New Hampshire?

16. If I borrow $4687 and afterwards pay $2423, how much do I still owe?

51. To subtract when any figure in the minuend is less than the corresponding figure in the subtrahend.

17. From 483 take 257.

OPERATION.

Minuend,	483
Subtrahend,	257
Remainder,	226

There are two methods of explaining this operation:

1st. As we cannot take 7 units from 3 units, *one* of the 8 *tens* is put with the 3 units, making 13 units, and then, 7 units from 13 units leave 6 units. Now as *one* of the 8 *tens* has been put with the 3 *units*, only 7 tens remain in the minuend, and 5 tens from 7 tens leave two tens, and, finally, 2 hundreds from 4 hundreds leave 2 hundreds; ∴ the entire remainder is 226.

2d. Instead of *taking away* 1 of the 8 tens in the minuend, we may *add* 1 ten to the 5 tens in the subtrahend, and then take the *sum* (6 tens) from the 8 tens, since the result is 2 tens by either process.

The second mode depends on the principle, *that, if two numbers are equally increased, the difference between them remains unchanged;* thus, the difference between 9 and 4 is 5, and, if 10 is added to both 9 and 4, making 19 and 14, the difference still is 5. Now, in solving Ex. 17 by the second method, we add 10 *units* to the *minuend* and 1 *ten (the same as* 10 *units)* to the *subtrahend*, and ∴ find the *same remainder* as by the first method.

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51. How many methods of subtracting when a figure of the minuend is less than the one under it? What is the first method? Second? The second depends on what principle? By the second method, *is the same number added to minuend and subtrahend? How?*

52. The preceding examples illustrate all the principles in subtraction. Hence, to perform subtraction,

RULE. 1. *Write the less number under the greater, units, under units, tens under tens, etc., and draw a line beneath.*

2. *Beginning at the right hand, take each figure of the subtrahend from the figure above it, and set the remainder under the line.*

3. *If any figure in the subtrahend is greater than the figure above it, add* TEN *to the upper figure and take the lower figure from the* SUM; *set down the remainder and, considering the next figure in the minuend* ONE LESS, *or the next figure in the subtrahend* ONE GREATER, *proceed as before.*

53. PROOF. *Add the subtrahend and the remainder together, and the sum should be the minuend.*

NOTE 1. This proof rests upon the self-evident truth, that *the whole of a thing is equal to the sum of all its parts;* thus, the *minuend* is separated into the *two parts, subtrahend and remainder; hence* the *sum* of those parts *must be* the *minuend.*

	Ex. 18.
Minuend,	68745
Subtrahend,	26854
Remainder,	41891
Proof,	68745

As the *sum* of the subtrahend and remainder is the minuend, the work is supposed to be right.

	19.	20.	21.
Minuend,	9875	532769	5784268
Subtrahend,	265	278493	3296416
Remainder,	9610	254276	
Proof,	9875	532769	

	22.	23.	24.
From	468724	5406872	9846237
Take	259782	2304798	9468714
Ans.	208942		

52. The rule for Subtraction? 53. How is Subtraction proved? On what principle does this proof rest?

	25.	26.	27.
From	243769	8876048	7777777
Take	243627	2960040	5666669

Ex. 28.

	(5)	(9)	(12)
Minuend,	6	0	2
Subtrahend,	4	3	8
Remainder,	1	6	4

Here we cannot take 8 from 2, nor can we borrow from the tens' place, as that place is occupied by 0; but we can borrow *one* of the 6 *hundreds* and separate the one hundred into 9 *tens* and 10 *units*; then, putting the 9 tens in the place of tens and adding the 10 units to the 2 units, we can subtract 8 from 12, 3 from 9, and 4 from 5.

NOTE 2. This process will probably be more readily *understood* by the young learner than the second method given in the *rule*, though the latter, being thought more convenient, is usually adopted.

	29.	30.	31.
From	8702	4003	870000
Take	2465	1876	324872

32. From 804 take 567. Ans. 237.
33. From 4687 take 2398.
34. From 87062 take 36981.
35. Subtract 2437 from 8064. Ans. 5627.
36. Subtract 160874 from 4769872.
37. Subtract 3768942 from 7000000.
38. Take 87406 from 95472. Ans. 8066.
39. Take 2704698 from 8749206.
40. How many are 3642 less 1468? Ans. 2174.
41. How many are 87649 less 24065?
42. 8749 — 3684 = how many? Ans. 5065.
43. 7248 — 2943 = how many?
44. The difference between two numbers is 365 and the greater number is 876; what is the less? Ans. 511.
45. What number added to 3876 will give 7469?
46. What number taken from 8742 leaves 3748?

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53. What is there peculiar in Ex. 28? Explain the process.

47. The sum of two numbers is 8629, and the less of the two numbers is 2689; what is the greater? Ans. 5940.

48. The sum of two numbers is 8426, and the greater is 7162; what is the less?

49. From fourteen million, eight hundred and sixty-two thousand, three hundred and twenty-five, take six million, six hundred and eighty-six thousand, two hundred and fourteen.

Ans. 8176111.

50. From seven hundred and thirty-three thousand, six hundred and fifty-four, take two hundred and twenty-seven thousand, five hundred and fifteen.

51. How many years from the discovery of America by Columbus in 1492 to the birth of Washington in 1732?

52. How many years have elapsed since the discovery of America in 1492?

53. By the census of 1860, the number of inhabitants in Massachusetts was 1231065, and the number in Vermont was 315116; how many more in Massachusetts than in Vermont?

54. The population of the United States was 23191876 in 1850, and 17063353 in 1840; what was the increase in ten years?

55. The area of New England is 64230 square miles and the area of Maine is 35000 square miles; what is the area of the other five New England States?

56. About 56619608 bushels of corn were raised in Ohio in 1850, and 73436690 bushels in 1853; what was the increase?

57. Bought a paper mill for $15475, and sold it for $17925; what did I gain?

58. How many are 876942 — 468279?

59. How many are 742006 — 387429?

60. How many are 820654 — 260408?

61. Washington was born in 1732 and died in 1799; at what age did he die?

62. A merchant sold goods to the amount of $4276, and thereby gained $1142; what did the goods cost him?

63. A farm was sold for $3462, which was $876 more than it cost; what did it cost?

64. The distance from the earth to the sun is about 95000000 miles; the distance to the moon is about 240000 miles. How much farther to the sun than to the moon?

65. Methuselah died at the age of 969 years, and Washington at 67; what was the difference of their ages?

66. Mr. Hale, owing a debt of $3762, paid $2486; how much remained unpaid?

Examples in Addition and Subtraction.

1. From the sum of 76 and 92 take 14. Ans. 154.

2. From the sum of the three numbers, 876, 493, and 916, take the sum of 842 and 397. Ans. 1046.

3. I owe 3 notes, whose sum is $600. One of these notes is for $150, another for $200; for what is the third one?

4. My real estate is valued at $4500 and my personal property at $2596. I owe to A $600, to B $1358, and to C $318; what am I worth? Ans. $4820.

5. Bought a barrel of flour for $9, four yards of cloth for $2, and 8 pounds of sugar for $1. In payment I gave a ten and a five dollar bill; what change shall the merchant return to me?

6. Mr. Fox, owning 3762 acres of land, gave 563 acres to his oldest son, and 672 acres to his youngest son; how many acres had he remaining?

7. The area of Maine is 35000 square miles; N. H., 8030; Vt., 8000; Mass., 7250; R. I., 1200; Ct., 4750. Which is the greater, Maine or the rest of N. E.? How much?

8. Gave my note for $3465. Paid $1300 at one time, and $575 at another; how much do I still owe? Ans. $1590.

9. Mr. T., opening an account at the Andover Bank, deposited $187 on Monday, $362 on Tuesday, $580 on Thursday, and $675 on Friday. On Tuesday he withdrew $67, on Wednesday $213, on Friday $350, and on Saturday $125; how much remained on deposit at the close of the week? Ans. $1049.

10. A traveler who was 875 miles from home, traveled toward home 144 miles on Monday, 127 miles on Tuesday, 156 miles on Wednesday, and 157 miles on Thursday; how far from home was he on Friday morning?

11. From the discovery of America by Columbus in 1492, to the settlement of Jamestown in 1607, was 115 years, from the settlement of Jamestown to the Declaration of Independence in 1776, was 169 years, and from the Declaration of Independence to the present time (1862) is 86 years. Methuselah died at the age of 969 years; how much longer did he live than from the discovery of America to the year 1862?

12. Four men, A, B, C, and D, commencing business together, furnished money as follows: A, \$2475; B, \$3475; C, \$2850; and D, \$4500. At the end of a year they closed business, having lost \$3225; how much money had they to divide between them?

MULTIPLICATION.

54. Multiplication is a short method of *adding equal numbers;* that is, *multiplication* is a short method of *finding the sum of the repetitions* of a number.

Or, Multiplication is a short method of finding how many units there are in any number of times a given number.

The Multiplicand is the number to be repeated.

The Multiplier is the number which shows how many times the multiplicand is to be taken.

The Product is the *sum* of the repetitions, or the *result* of the multiplication.

The *Multiplicand* and *Multiplier* are called Factors.

Ex. 1. There are 7 days in 1 week; how many days in 4 weeks?

This example may be solved by addition; thus, $7+7+7+7=28$; or more briefly, by multiplication; thus, 4 times 7 are 28, Ans.

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54. What is Multiplication? Another definition? What is the Multiplicand? Multiplier? Product? What are the Multiplicand and Multiplier called?

55. The pupil, before advancing further, should learn the following

MULTIPLICATION TABLE.

Once		Twice		Three times		Four times		Five times		Six times	
1 is	1	1 are	2	1 are	3	1 are	4	1 are	5	1 are	6
2	2	2	4	2	6	2	8	2	10	2	12
3	3	3	6	3	9	3	12	3	15	3	18
4	4	4	8	4	12	4	16	4	20	4	24
5	5	5	10	5	15	5	20	5	25	5	30
6	6	6	12	6	18	6	24	6	30	6	36
7	7	7	14	7	21	7	28	7	35	7	42
8	8	8	16	8	24	8	32	8	40	8	48
9	9	9	18	9	27	9	36	9	45	9	54
10	10	10	20	10	30	10	40	10	50	10	60
11	11	11	22	11	33	11	44	11	55	11	66
12	12	12	24	12	36	12	48	12	60	12	72

Seven times		Eight times		Nine times		Ten times		Eleven times		Twelve times	
1 are	7	1 are	8	1 are	9	1 are	10	1 are	11	1 are	12
2	14	2	16	2	18	2	20	2	22	2	24
3	21	3	24	3	27	3	30	3	33	3	36
4	28	4	32	4	36	4	40	4	44	4	48
5	35	5	40	5	45	5	50	5	55	5	60
6	42	6	48	6	54	6	60	6	66	6	72
7	49	7	56	7	63	7	70	7	77	7	84
8	56	8	64	8	72	8	80	8	88	8	96
9	63	9	72	9	81	9	90	9	99	9	108
10	70	10	80	10	90	10	100	10	110	10	120
11	77	11	88	11	99	11	110	11	121	11	132
12	84	12	96	12	108	12	120	12	132	12	144

Ex. 2. How many are 8 times 3? 3 times 8? 6 times 4? 4 times 6? 7 times 7? 5 times 9?

3. How many are 9 times 7? 9 times 11? 8 times 6? 6 times 12? 12 times 6? 9 times 8?

4. If I deposit $10 a month in a savings bank, how many dollars shall I deposit in 4 months? In 7 months? In 5 months? In 12 months?

5. When wood is worth \$6 a cord, what shall I pay for 3 cords? 5 cords? 8 cords? 11 cords?

6. In one year there are 12 months, how many months in 2 years? 4 years? 7 years? 12 years?

7. If I study 11 hours in a day, how many hours shall I study in 3 days? 5 days? 8 days? 11 days?

56. To multiply by a single figure.

8. In one bushel are 32 quarts; how many quarts in 6 bushels?

BY ADDITION.	BY MULTIPLICATION.
3 2	3 2
3 2	6
3 2	Product, 1 9 2
3 2	
3 2	
3 2	
Sum, 1 9 2	

In 6 bushels there are, evidently, 6 times as many quarts as in 1 bushel, and the number of quarts in 6 bushels may be obtained by *adding*, as in the margin; or, more briefly, by *multiplying;* thus, 6 times 2 units are 12 units = 1 ten and 2 units; write the 2 units in units' place, and then say 6 times 3 tens are 18 tens, which, increased by the 1 ten previously obtained, make 19 tens = 1 hundred and 9 tens, and these, written in the place of hundreds and tens respectively, give the true product. Hence,

RULE. *Write the multiplier under the multiplicand, and draw a line beneath; multiply the units of the multiplicand, set the units of the product under the multiplier, and add the tens, if any, to the product of the tens, and so proceed.*

	9.	10.	11.
Multiplicand,	4 2 7	1 3 4 7	1 0 6 4
Multiplier,	2	5	8
Product,	8 5 4	6 7 3 5	8 5 1 2

12.	13.	14.	15.
8 4 2 3	5 4 3 6	2 6 4 9 3	7 6 4 8 9
7	9	3	4
5 8 9 6 1			

56. Which figure of the Multiplicand is multiplied first? Where are the units of the product written? What is done with the tens? Repeat the rule.

16.	17.	18.
36042	4787243	3424270
6	9	7
216252		23969890

57. To multiply by two or more figures.

19. How many quarts in 46 bushels?

OPERATION.

Multiplicand,	32
Multiplier,	46
	192
	128
Product,	1472

First multiply by 6, as though 6 were the only figure in the multiplier; then multiply by 4, and set the first figure of this product in the place of *tens;* for multiplying by the 4 *tens* is the same as multiplying by 40, and 40 times 2 units are 80 *units* = 8 *tens;* i. e. the product of *units* by *tens* is *tens.* Having multiplied by each figure in the multiplier, the *sum* of the *partial products* will be the *true product.*

NOTE. So much of the product as is obtained by multiplying the whole multiplicand by *one figure* of the multiplier is called *a partial product;* thus, in the 19th example, 192 is the first partial product and 128 *tens* is the second.

58. Similar reasoning applies however many figures there may be in the multiplier. Hence,

RULE. 1. *Set the multiplier under the multiplicand and draw a line beneath.*

2. *Beginning at the right hand of the multiplicand, multiply the multiplicand by each figure in the multiplier, setting the first figure of each partial product directly under the figure of the multiplier which produces it.*

3. *The* SUM *of these partial products will be the true product.*

59. PROOF. *Multiply the multiplier by the multiplicand, and, if correct, the result will be like the first product.*

NOTE. This proof rests on the principle, that the *order* of the factors is immaterial; thus, $3\times4=4\times3$; $5\times2\times7=2\times7\times5$.

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57. Which figure of the multiplier is first employed? Where is the first figure of each partial product written? What *is* a partial product? 58. Rule for multiplying by two or more figures? 59. Proof? Principle?

Ex. 20. Multiply 5236 by 2413.

	OPERATION.		PROOF.
Multiplicand,	5236		2413
Multiplier,	2413		5236
	15708		14478
	5236		7239
	20944		4826
	10472		12065
Product,	12634468	=	12634468

	21.	22.
Multiplicand,	2640873	12474893
Multiplier,	4622	79

23.	24.	25.	26.
34678	54327	8645	3579
54	324	463	246

27. Multiply 4276 by 356. Ans. 1522256.
28. Multiply 5462 by 248.
29. Multiply 4628 by 336.
30. Multiply 3874 by 846.

60. The *sign of multiplication*, $\times$, signifies that the two numbers between which it stands are to be multiplied together; thus, $6 \times 5 = 30$, i. e. six multiplied by five equals thirty; or, more familiarly, six times five are thirty.

31. How many are 726×27? Ans. 19602.
32. How many are 4628×554? Ans. 2563912.
33. $3648 \times 36 =$ how many? Ans. 131328.
34. $4275 \times 54 =$ how many? Ans. 230850.
35. $3759 \times 8463 = ?$ Ans. 31812417.

36. $53642 \times 63 = ?$	41. $37642 \times 57 = ?$
37. $4620 \times 524 = ?$	42. $37942 \times 386 = ?$
38. $8726 \times 463 = ?$	43. $27403 \times 584 = ?$
39. $7692 \times 356 = ?$	44. $36008 \times 412 = ?$
40. $2146 \times 179 = ?$	45. $81650 \times 789 = ?$

60. Sign of multiplication, what does it signify?

46. If 37 men do a piece of work in 23 days, in how many days will 1 man do the same work?

47. What is the value of 37 acres of land, at $43 per acre?

48. If a horse can travel 41 miles per day, how far can he travel in 17 days?

49. How many yards of cloth in 29 pieces, if each piece contains 31 yards?

61. To multiply by a composite number.

A COMPOSITE NUMBER is the product of two or more numbers; thus 15 is a composite number, whose factors are 3 and 5; and 12 is a composite number, whose factors are 2 and 6, or 3 and 4, or 2, 2, and 3.

It will be observed that a composite number *may have* several sets of factors.

50. If 35 men have $37 each, how many dollars have they all?

OPERATION.

$35 = 5 \times 7.$

Multiplicand,	$37
1st Factor of Multiplier,	5
	$185
2d Factor of Multiplier,	7
Product,	$1295

The 35 men may be separated into 7 groups of 5 men each. Now 1 group of 5 men will have 5 times $37 = $185, and if 1 group has $185, evidently 7 groups will have 7 times $185 = $1295, Ans.; i. e. 7 times 5 times a number are 35 times that number.

51. Multiply 367 by 168. Ans. 61656.

	FIRST OPERATION. $168 = 8 \times 7 \times 3.$		SECOND OPERATION. $168 = 4 \times 7 \times 6.$
Multiplicand,	367		367
First Factor of Multiplier,	8		4
	2936		1468
Second Factor of Multiplier,	7		7
	20552		10276
Third Factor of Multiplier,	3		6
Product,	61656	=	61656

61. What is a composite number? May a composite number have more than one set of factors?

Several other sets of factors of 168 may be used, and give the same product. Every similar example may be solved in like manner. Hence,

RULE. *Multiply the multiplicand by one of the factors of the multiplier, and that product by another factor, and so on until all the factors in the set have been taken; the last product will be the true product.*

52. Multiply 743 by 42, i. e. by 7 and 6. Ans. 31206.
53. Multiply 3467 by 56.
54. $839 \times 54 =$ how many? Ans. 45306.
55. $7869 \times 72 = ?$
56. $469876 \times 81 = ?$
57. $478969 \times 1728 = ?$ Ans. 827658432.
58. $5387469 \times 96 = ?$
59. $987462 \times 49 = ?$

62. To multiply by 10, 100, 1000, or 1 with any number of ciphers annexed:

RULE. *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the number so formed will be the product.*

NOTE. The reason of the rule is obvious. Annexing a cipher removes each figure in the multiplicand one place toward the left, and thus its value is made ten fold (Art. 15).

60. Multiply 74 by 10. Ans. 740.
61. Multiply 869 by 10000. Ans. 8690000.
62. Multiply 4698 by 1000.
63. $76984 \times 100000 = ?$ Ans. 7698400000.
64. $59874 \times 1000000000 = ?$

63. To multiply by 20, 50, 500, 25000, or any similar number:

RULE. *Multiply by the significant figures, and to the product annex as many ciphers as there are ciphers at the right of the significant figures of the multiplier.*

---

61. Rule for multiplying by a composite number? **62.** How is a number multipled by 10? By 100? Why? **63.** How is a number multiplied by 20? Why?

65. Multiply 756 by 30. Ans. 22680.

OPERATION.

```
  756
   30
-----
22680
```

This is upon the principle of Art. 61. The factors of 30 are 3 and 10. Having multiplied by 3, the product is multipled by 10 by annexing 0 (Art. 62).

66. Multiply 743 by 3500.

OPERATION.

```
          743
            7
         ----
         5201
          500
      -------
Product, 2600500
```

The factors of 3500 are 7, 5, and 100, ∴ multiply first by 7, then by 5, then annex two ciphers.

67. Multiply 84693 by 480000. Ans. 40652640000.

68. $8769432 \times 7200000 = ?$

69. $94684235 \times 49000000 = ?$

64. To multiply when there are ciphers at the right of both multiplicand and multiplier:

RULE. *Multiply the significant figures of the multiplicand by those of the multiplier, and then annex as many ciphers to the product as there are ciphers at the right of both factors.*

70. Multiply 8000 by 900.

OPERATION.

```
        8000
         900
     -------
Ans. 7200000
```

The factors of 8000 are 8 and 1000, and those of 900 are 9 and 100. Now, as it is immaterial in what order the factors are taken (Art. 59, Note), first multiply 8 by 9, then multiply this product by 1000 (Art. 62), and this product by 100.

71. Multiply 730000 by 2900.

OPERATION.

```
            730000
              2900
         ---------
            657
           146
         ---------
Product, 2117000000
```

---

64. Rule when there are ciphers at the right of both factors? The reason?

72. Multiply 840 by 2700000. Ans. 2268000000.

73. $7693000 \times 569000 = ?$

65. To multiply when there are ciphers *between* the significant figures of the multiplier:

RULE. *Multiply only by the significant figures of the multiplier, taking care to set the first figure of each partial product directly under the figure of the multiplier which gives that product.*

74. Multiply 5723 by 2004.

OPERATION.

```
           5723
           2004
          -----
          22892
       11446
       --------
Product, 11468892
```

This is only carrying out the principle (in addition) of setting units under units, tens under tens, etc. The 2 of the multiplier is 2000, and 2000 times 3 are 6000, ∴ the 6 of the partial product should be written in the thousands' place, i. e. directly under the 2 of the multiplier.

75. Multiply 3724 by 4008. Ans. 14925792.

76. $698427 \times 420006 = ?$

77. $7800076900 \times 2008040000 = ?$

66. To multiply by 9, 99, or any number of 9's:

RULE. *Annex as many 0's to the multiplicand as there are 9's in the multiplier, and from the number so formed subtract the multiplicand; the remainder will be the product sought.*

78. Multiply 234 by 99.

OPERATION.

```
23400 = 100 times the multiplicand.
  234 =   1 time the multiplicand.
-----   ---
23166 =  99 times the multiplicand, Ans.
```

79. Multiply 3746 by 999. Ans. 3742254.

80. Multiply 427 by 9999.

65. Rule for multiplying when there are ciphers *between* the significant figures of the multiplier? The reason? 66. To multiply by 9? By 99? Rule? Reason?

67. To multiply by 13, 14, 15, 16, 17, etc.:

RULE. *Multiply by the right-hand figure of the multiplier, set the product under the multiplicand,* ONE PLACE FURTHER TO THE RIGHT, *and add.*

81. Multiply 426 by 17.

OPERATION.

```
 4 2 6
 2 9 8 2
 -------
 7 2 4 2, Ans.
```

The 2982 is 7 times 426, and the 426, *standing one place further to the left,* is 10 times 426 (Art. 15), ∴ their sum is 17 times 426.

82. Multiply 342 by 18. By 14. By 16.

In a similar manner multiply by 102, 1005, 10009, etc.

83. Multiply 2463 by 102.

OPERATION.

```
2 4 6 3     = 100 times 2463.
    4 9 2 6 =   2   "     "
----------    ---
2 5 1 2 2 6 = 102   "     "   Ans.
```

84. Multiply 3248 by 104. By 1004. By 1008.

68. To multiply by 21, 31, etc.:

RULE. *Multiply by the left-hand figure of the multiplier, set the product under the multiplicand,* ONE PLACE FURTHER TO THE LEFT, *and add.*

85. Multiply 324 by 21.

SHORT METHOD.

```
  3 2 4
6 4 8
-------
6 8 0 4, Ans.
```

COMMON METHOD.

```
  3 2 4
    2 1
  -----
  3 2 4
6 4 8
-------
6 8 0 4, Ans.
```

86. Multiply 34264 by 81. By 41. By 61.

In like manner multiply by 201, 301, 6001, etc.

87. Multiply 4237 by 501. Ans. 2122737.

88. Multiply 34265 by 801. By 4001. By 30001.

---

67. To multiply by 13? By 15? By 102? By 1005? Reason? 68. To multiply by 21? By 31? By 501? Reason? Why better than the common method?

Miscellaneous Examples in Multiplication.

1. What cost 11 pounds of beef at 9 cents per pound?
Ans. 99 cents.

2. What cost 98 tons of hay at $15 per ton? Ans. $1470.

3. In one hogshead of wine are 63 gallons; how many gallons in 75 hogsheads?

4. In a certain house are 75 rooms, in each room four windows, and in each window 12 panes of glass; how many panes of glass in the house?

5. The earth, in its annual revolution, moves 19 miles in a second; how far will it move in an hour, there being 60 seconds in a minute, and 60 minutes in an hour?

6. Light moves 192000 miles in a second; how far will it move in an hour?

7. How many yards of cloth in 10 bales, each bale containing 25 pieces, and each piece 24 yards?

8. If 12 men do a piece of work in 7 days, in how many days can 1 man do 5 times as much work?

9. Multiply forty-three million, seven hundred and four thousand, eight hundred and sixteen, by forty-two thousand and eight.

10. A man bought 24 city lots at $365 each; what did they all cost him?

11. Multiplicand = 4632; multiplier = 4008; product = ?

12. Multiplier = 3333; multiplicand = 4444; product = ?

Examples in the Foregoing Principles.

1. Two men start from the same place, and travel in the same direction, one at the rate of 56 miles and the other 75 miles per day, how far apart are they at the end of 43 days?

2. Had the men named in Ex. 1 traveled in opposite directions, how far apart would they have been in 56 days?

3. Bought 58 tons of hay for $600 and sold it for $12 per ton; did I gain or lose? How much?

4. Bought 25 horses for $125 each, and 14 pairs of oxen at $87 a pair; what did I pay for all?

5. Bought 56 barrels of flour at \$9 per barrel, and in pay for it gave 48 cords of wood at \$6 per cord, and the rest in money; how much money did I pay?

6. Paid \$7 each for 63 sheep, and sold the flock for \$425; did I gain or lose? How much?

7. A farmer sold 56 bushels of wheat at \$2 per bushel, for which he received 40 yards of cloth at \$2 per yard, and the balance in money; how much money did he receive?

8. A merchant bought 846 barrels of flour for \$7191; he sold 526 barrels at \$9 per barrel, and the remainder at \$8 per barrel; did he gain or lose? How much? Ans. Gained \$103.

9. A man's income is \$1575 a year, and his expenses are \$3 a day; what does he save in a year of 365 days? Ans. \$480.

10. Bought 18 tons of iron at \$39 a ton, and 27 tons at \$41; what shall I gain by selling the whole at \$43 a ton?

11. A drover bought a herd of 33 oxen, paying as many dollars for each ox as there were oxen in the herd. He paid \$500 in money, and gave his note for the balance; what was the size of the note?

12. How many are $8 + 2 \times 7 - 3 \times 5$? Ans. 7.

13. How many are $9 \times 7 + 3 \times 5 - 12$? Ans. 66.

14. How many are $48 - 3 \times 6 - 4$? Ans. 26.

15. The factors of one number are 20, 14, and 23, and of another 16, 8, and 7; what is the difference of the two numbers? Ans. 5544.

16. The President of the United States receives a salary of \$25000 a year; what will he save in a year of 365 days, if his expenses are \$50 a day?

17. A man having a journey of 313 miles to perform in 6 days, travels 54 miles a day for 5 days; how far must he go on the sixth day?

18. A many sold three farms; for the first he received \$3475, for the second, \$925 less than for the first, and for the third, he received twice as much as for the other two; how much did he receive for the three farms?

19. What shall I pay for 25 horses, at \$75 each, and 12 oxen, at \$54 each?

20. If a teacher receives a salary of $800 a year, and pays $210 a year for board, $75 for clothing, $50 for books, and $100 for other expenses, how much will he save in 3 years?

DIVISION.

69. DIVISION is the process of finding how many times one number is contained in another.

The DIVIDEND is the number *to be divided.*

The DIVISOR is the number *by which to divide.*

The QUOTIENT is the *number of times* the dividend contains the divisor.

If the dividend does not contain the divisor an *exact number of times*, the part of the dividend which is left is called the REMAINDER.

NOTE. The remainder is *always* of the *same kind as the dividend, because it is a part* of the dividend.

Ex. 1. How many oranges, at 4 cents each, can be bought for 12 cents?

Ans. As many oranges as there are times 4 cents in 12 cents; 4 cents are contained in 12 cents, 3 times; $\therefore$ 3 oranges, at 4 cents each, can be bought for 12 cents.

2. How many apples, at 2 cents each, can be bought for 10 cents?

Ans. As many as there are times 2 cents in 10 cents, or as there are times 2 in 10, viz. 5.

70. The *sign of division*, $\div$, indicates that the number before it is to be divided by the number after it; thus, $8 \div 2 = 4$, i. e. 8 divided by 2 equals 4, or 2 in 8, 4 times.

3. How many are $6 \div 2$? Ans. 2 in 6, 3 times.

---

69. What is Division? What the Dividend? Divisor? Quotient? Remainder? Of what *kind* is the remainder? **70.** The sign of Division, what does it indicate?

In the same manner, let the pupil explain and recite the following

DIVISION TABLE.

1 ÷ 1 = 1	2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1
2 ÷ 1 = 2	4 ÷ 2 = 2	6 ÷ 3 = 2	8 ÷ 4 = 2
3 ÷ 1 = 3	6 ÷ 2 = 3	9 ÷ 3 = 3	12 ÷ 4 = 3
4 ÷ 1 = 4	8 ÷ 2 = 4	12 ÷ 3 = 4	16 ÷ 4 = 4
5 ÷ 1 = 5	10 ÷ 2 = 5	15 ÷ 3 = 5	20 ÷ 4 = 5
6 ÷ 1 = 6	12 ÷ 2 = 6	18 ÷ 3 = 6	24 ÷ 4 = 6
7 ÷ 1 = 7	14 ÷ 2 = 7	21 ÷ 3 = 7	28 ÷ 4 = 7
8 ÷ 1 = 8	16 ÷ 2 = 8	24 ÷ 3 = 8	32 ÷ 4 = 8
9 ÷ 1 = 9	18 ÷ 2 = 9	27 ÷ 3 = 9	36 ÷ 4 = 9
5 ÷ 5 = 1	6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1
10 ÷ 5 = 2	12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2
15 ÷ 5 = 3	18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3
20 ÷ 5 = 4	24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4
25 ÷ 5 = 5	30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5
30 ÷ 5 = 6	36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6
35 ÷ 5 = 7	42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7
40 ÷ 5 = 8	48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8
45 ÷ 5 = 9	54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9
9 ÷ 9 = 1	10 ÷ 10 = 1	11 ÷ 11 = 1	12 ÷ 12 = 1
18 ÷ 9 = 2	20 ÷ 10 = 2	22 ÷ 11 = 2	24 ÷ 12 = 2
27 ÷ 9 = 3	30 ÷ 10 = 3	33 ÷ 11 = 3	36 ÷ 12 = 3
36 ÷ 9 = 4	40 ÷ 10 = 4	44 ÷ 11 = 4	48 ÷ 12 = 4
45 ÷ 9 = 5	50 ÷ 10 = 5	55 ÷ 11 = 5	60 ÷ 12 = 5
54 ÷ 9 = 6	60 ÷ 10 = 6	66 ÷ 11 = 6	72 ÷ 12 = 6
63 ÷ 9 = 7	70 ÷ 10 = 7	77 ÷ 11 = 7	84 ÷ 12 = 7
72 ÷ 9 = 8	80 ÷ 10 = 8	88 ÷ 11 = 8	96 ÷ 12 = 8
81 ÷ 9 = 9	90 ÷ 10 = 9	99 ÷ 11 = 9	108 ÷ 12 = 9

Ex. 4. 32 are how many times 4? 8? 2? 16?
5. 48 are how many times 4? 6? 12? 8? 3? 16?
6. 36 are how many times 12? 6? 9? 3? 4? 2?
7. 40 are how many times 8? 4? 2? 10? 5? 20?

71. Division is also indicated by the *colon;* thus, $8:2=4$.

Also by writing the divisor before the dividend, with a curved line between; thus, $2)\underline{8\ 4\ 6}$, or thus, $2)\ 8\ 4\ 6\ ($, the quotient to be placed *under* or *at the right* of the dividend, and separated from it by a line.

Also by writing the divisor under the dividend, with a line between; thus, $\frac{6}{2}=3$; i. e. 6 divided by 2 equals 3; or, more familiarly, 2 in 6, 3 times.

Ex. 8. How many are $\frac{8}{2}$? Ans. 2 in 8, 4 times.

The fourth mode of indicating division gives the the following compact and convenient

DIVISION TABLE.

$\frac{1}{1}=1$	$\frac{2}{2}=1$	$\frac{3}{3}=1$	$\frac{4}{4}=1$	$\frac{5}{5}=1$	$\frac{6}{6}=1$
$\frac{2}{1}=2$	$\frac{4}{2}=2$	$\frac{6}{3}=2$	$\frac{8}{4}=2$	$\frac{10}{5}=2$	$\frac{12}{6}=2$
$\frac{3}{1}=3$	$\frac{6}{2}=3$	$\frac{9}{3}=3$	$\frac{12}{4}=3$	$\frac{15}{5}=3$	$\frac{18}{6}=3$
$\frac{4}{1}=4$	$\frac{8}{2}=4$	$\frac{12}{3}=4$	$\frac{16}{4}=4$	$\frac{20}{5}=4$	$\frac{24}{6}=4$
$\frac{5}{1}=5$	$\frac{10}{2}=5$	$\frac{15}{3}=5$	$\frac{20}{4}=5$	$\frac{25}{5}=5$	$\frac{30}{6}=5$
$\frac{6}{1}=6$	$\frac{12}{2}=6$	$\frac{18}{3}=6$	$\frac{24}{4}=6$	$\frac{30}{5}=6$	$\frac{36}{6}=6$
$\frac{7}{1}=7$	$\frac{14}{2}=7$	$\frac{21}{3}=7$	$\frac{28}{4}=7$	$\frac{35}{5}=7$	$\frac{42}{6}=7$
$\frac{8}{1}=8$	$\frac{16}{2}=8$	$\frac{24}{3}=8$	$\frac{32}{4}=8$	$\frac{40}{5}=8$	$\frac{48}{6}=8$
$\frac{9}{1}=9$	$\frac{18}{2}=9$	$\frac{27}{3}=9$	$\frac{36}{4}=9$	$\frac{45}{5}=9$	$\frac{54}{6}=9$
$\frac{7}{7}=1$	$\frac{8}{8}=1$	$\frac{9}{9}=1$	$\frac{10}{10}=1$	$\frac{11}{11}=1$	$\frac{12}{12}=1$
$\frac{14}{7}=2$	$\frac{16}{8}=2$	$\frac{18}{9}=2$	$\frac{20}{10}=2$	$\frac{22}{11}=2$	$\frac{24}{12}=2$
$\frac{21}{7}=3$	$\frac{24}{8}=3$	$\frac{27}{9}=3$	$\frac{30}{10}=3$	$\frac{33}{11}=3$	$\frac{36}{12}=3$
$\frac{28}{7}=4$	$\frac{32}{8}=4$	$\frac{36}{9}=4$	$\frac{40}{10}=4$	$\frac{44}{11}=4$	$\frac{48}{12}=4$
$\frac{35}{7}=5$	$\frac{40}{8}=5$	$\frac{45}{9}=5$	$\frac{50}{10}=5$	$\frac{55}{11}=5$	$\frac{60}{12}=5$
$\frac{42}{7}=6$	$\frac{48}{8}=6$	$\frac{54}{9}=6$	$\frac{60}{10}=6$	$\frac{66}{11}=6$	$\frac{72}{12}=6$
$\frac{49}{7}=7$	$\frac{56}{8}=7$	$\frac{63}{9}=7$	$\frac{70}{10}=7$	$\frac{77}{11}=7$	$\frac{84}{12}=7$
$\frac{56}{7}=8$	$\frac{64}{8}=8$	$\frac{72}{9}=8$	$\frac{80}{10}=8$	$\frac{88}{11}=8$	$\frac{96}{12}=8$
$\frac{63}{7}=9$	$\frac{72}{8}=9$	$\frac{81}{9}=9$	$\frac{90}{10}=9$	$\frac{99}{11}=9$	$\frac{108}{12}=9$

71. Second sign of Division, what is it? Third mode of indicating Division, what is it? Where is the quotient to be written? Fourth method, what? How are the dividend and divisor written in the second Division Table?

Ex. 9. How many are $24 \div 6$, or $\frac{24}{6}$? Ans. 4.
10. How many are $48 \div 8$, or $\frac{48}{8}$?
11. How many are $66 \div 11$, or $\frac{66}{11}$?
12. How many are $84 \div 12$, or $\frac{84}{12}$?
13. How many are $63 \div 9$, or $\frac{63}{9}$? Ans. 7.
14. How many are $48 \div 6$, or $\frac{48}{6}$?
15. How many are $77 \div 11$, or $\frac{77}{11}$?
16. How many are $72 \div 8$, or $\frac{72}{8}$?
17. How many are $96 \div 12$, or $\frac{96}{12}$? Ans. 8.
18. How many are $88 \div 8$, or $\frac{88}{8}$?
19. How many are $72 \div 12$, or $\frac{72}{12}$?

72. When the dividend is large the division may be performed in two ways, as follows:

20. Divide 1384 by 4.

FIRST OPERATION.

```
4)1384(346
  12
  --
   18
   16
   --
    24
    24
    --
     0
```

Having written the divisor and dividend as in the margin, we first inquire how many times 4 is contained in 13, (the fewest figures at the left of the dividend that will contain the divisor,) and find the quotient to be 3, which we set at the right of the dividend. We then multiply the divisor by the quotient, 3, and set the product, 12, under the 13 of the dividend, and subtract it therefrom. To the remainder, 1, we annex 8, the next figure of the dividend, and then inquire how many times the divisor is contained in 18, the second partial dividend; the result, 4, we set as the second figure of the quotient, and then multiply, subtract, annex, etc., as before, until all the figures of the dividend have been taken.

Since the 13 of the dividend is *hundreds*, the 3 of the quotient is *also hundreds;* since the 18 is *tens*, the 4 is *also tens;* and, *universally, any quotient figure is of the same order as the right-hand figure of the dividend taken to obtain that quotient figure.*

---

72. How many ways to perform Division? Of what order is any quotient figure?

The foregoing operation is called *Long Division*, but the work may be much shortened by *carrying the process in the mind, instead of writing it;* thus, having written the divisor and dividend as before, say, 4 in 13, 3 times and 1 remainder; set the quotient, 3, under the 3 of the dividend, and then, *imagining* the remainder, 1, placed *before* the 8, say, 4 in 18, 4 times and 2 remainder; set down the 4 as the second figure of the quotient, and imagine the 2 set before the next figure, and so proceed.

SECOND OPERATION.

Divisor, 4) 1 3 8 4 Dividend.
Quotient, 3 4 6

This operation is called *Short Division*, which is usually adopted when the divisor is so small that the process may be readily carried in the mind. Hence,

73. To perform Short Division:

RULE. *Divide the left-hand figure or figures of the dividend, (the fewest figures in the dividend that will contain the divisor,) and set the quotient under the right-hand figure taken in the dividend; if anything remains, prefix it* MENTALLY *to the next figure in the dividend, and divide the number thus formed as before, and so proceed till all the figures of the dividend have been employed.*

Ex. 21. Divide 24864 by 8.

OPERATION.

Divisor, 8) 2 4 8 6 4 Dividend.
Quotient, 3 1 0 8

22. Divide 3246 by 2. Ans. 1623.
23. Divide 1326 by 3. Ans. 442.
24. Divide 72345 by 5. Ans. 14469.
25. Divide 3283 by 7. Ans. 469.
26. Divide 59684 by 4. Ans. 14921.
27. Divide 69545 by 5. Ans. 13909.
28. Divide 36945 by 9. Ans. 4105.
29. Divide 27512 by 8. Ans. 3439.

---

72. What is the first method of Division called? What the Second? When is Short Division employed? **73.** Rule for Short Division?

30.	31.
Divisor, $8\,)\,\underline{7\,6\,4\,1\,2\,8}$ Dividend	$3\,)\,\underline{2\,1\,3\,6\,4\,2}$
Quotient, $9\,5\,5\,1\,6$	$7\,1\,2\,1\,4$

32.	33.	34.
$6\,)\,\underline{3\,2\,4\,9\,6}$	$2\,)\,\underline{1\,4\,8\,6\,5\,9\,3\,2}$	$9\,)\,\underline{4\,5\,8\,2\,8\,9\,2\,7}$

74. When there is *no remainder*, as in the first thirty-four examples, the division is *complete.* The dividend is then said to be *divisible* by the divisor, and the divisor is called an *exact divisor.*

When there *is a remainder*, as in Ex. 35, the division is *incomplete*, and the dividend is said to be *indivisible* by the divisor.

35. Divide 2781 by 8.

OPERATION.

Divisor, $8\,)\,\underline{2\,7\,8\,1}$ Dividend.

Quotient, $3\,4\,7 \ldots 5$ Remainder.

	Quotients,	Rem.
36. Divide 3654 by 4.	913,	2.
37. Divide 72584 by 5.	14516,	4.
38. $86471 \div 3 =$ how many?	28823,	2.
39. $40505 \div 7 = ?$	5786,	3.

40. $476589 \div 9 = ?$

41. $987654 \div 12 = ?$

42. $334523 \div 11 = ?$

43. In one week there are 7 days; how many weeks in 255 days? Ans. 36 weeks, Rem. 3 days.

44. How many barrels of flour, at $6 a barrel, can be bought for $750?

45. If 6 shillings make a dollar, how many dollars are there in 2736 shillings?

46. If 4 weeks make a month, how many months are there in 624 weeks?

---

74. When is the division complete? When is one number *divisible* by another? What is an exact divisor? When is one number *indivisible* by another?

75. When the divisor is *large*, the operation is usually performed by *Long Division*, as follows:

Ex. 47. Divide 2875 by 23.

OPERATION.

```
23)2875(125
   23
   ---
    57
    46
    ---
    115
    115
    ---
      0
```

This operation is like the first operation in Ex. 20. The partial dividends are 28, 57, and 115; the successive quotient figures are 1, 2, and 5, and these quotient figures multiplied into the divisor, give 23, 46, and 115 for the successive products or subtrahends, and the last product, 115, taken from the last dividend, 115, leaves no remainder;

∴ 125 is the true quotient. Hence,

76. To perform Long Division:

RULE. 1. *Write the divisor and dividend as in short division, and draw a curved line at the right of the dividend.*

2. *Divide the smallest number of figures in the left of the dividend that will contain the divisor, and set the result as the first figure of the quotient at the right of the dividend.*

3. *Multiply the divisor by the quotient figure, and set the product under that part of the dividend taken.*

4. *Subtract the product from the figures over it, and to the remainder annex the next figure of the dividend for a new partial dividend.*

5. *Divide, and proceed as before, until the whole dividend has been divided.*

NOTE 1. It will be seen that the process of dividing consists of four distinct steps, viz.: first, to seek a quotient figure; second, multiply; third, subtract; and, fourth, form a new partial dividend by annexing the next figure of the dividend to the remainder.

NOTE 2. If any partial dividend will not contain the divisor, 0 must be placed in the quotient, and another figure annexed to the partial dividend.

NOTE 3. If the product of the divisor multiplied by the quotient figure

---

75. When is Long Division employed? Explain Ex. 47. 76. Give the rule for Long Division. How many steps in dividing? What are they? Repeat Note 2. Note 3. Note 4.

is greater than the partial dividend, the quotient figure is too large, and must be diminished.

NOTE 4. If the remainder equals or exceeds the divisor, the quotient is too small, and must be increased.

77. *Division is the reverse of multiplication.* In multiplication the two factors are given, and the product is required; in division the product and one factor are given, and the other factor is required. The dividend is the product, and the divisor and quotient are the factors; thus,

IN MULTIPLICATION.		IN DIVISION.						
Factors,	Product.		Dividend,		Divisor,		Quotient.
$5 \times 4 = 20$			20	$\div$	5	$=$	4
		Or,	20	$\div$	4	$=$	5

Hence the

78. PROOF. *Multiply the divisor by the quotient, and to the product add the remainder; the* SUM *should be the dividend.*

48. Divide 2537 by 53.

OPERATION.

```
53)2537(47
   212
   ---
    417
    371
    ---
     46
```

PROOF.

```
  53 Divisor.
  47 Quotient.
 ---
 371
212
  46 Remainder.
----
2537 Dividend.
```

49.

```
21)864(41
   84
   --
    24
    21
    --
     3
```

50.

```
87)3659(42
   348
   ---
    179
    174
    ---
      5
```

51. A flock of 1728 sheep were divided equally in 9 different pastures, how many sheep were there in each pasture?

---

77. What is said of Division and Multiplication? In Multiplication what is given? What required? In Division what is given? Required? 78. How is Division proved?

	Quotients.	Rem.
52. Divide 46782 by 31.	1509,	3.
53. Divide 47086 by 18.	2615,	16.
54. Divide 468074 by 46.	10175,	24.
55. Divide 340068 by 67.	5075,	43.
56. $869432 \div 83 =$ how many?	10475,	7.
57. $937048 \div 99 =$ how many?	9465,	13.
58. $876543 \div 78 =$ how many?	11237,	57.
59. $276984 \div 254 = ?$	1090,	124.

60. $376958 \div 349 = ?$

61. $876598 \div 427 = ?$

62. $469873 \div 789 = ?$

63. $804068 \div 803 = ?$

64. $896842 \div 548 = ?$

65. $569432 \div 45 = ?$

66. $98647324 \div 4893 = ?$

67. $698742346525 \div 6995 = ?$

68. Divide four hundred eighteen thousand, six hundred and forty-eight, by twenty-four. Ans., Quo. 17443, Rem. 16.

69. Divide two hundred one thousand, five hundred and ninety-five acres of land, into twenty-three equal parts.

70. A railroad that cost $3576500 was divided into 7153 equal shares; what was the cost of each share?

71. A farmer raised 2001 bushels of wheat on 87 acres of land; how many bushels did he raise per acre?

72. In how many days will a ship sail 3456 miles, if it sails 144 miles per day?

73. A farmer raised 4088 bushels of corn, his crop averaging 56 bushels per acre; how many acres did he plant?

74. A drover paid $2175 for 29 oxen; how many dollars did he pay for each ox?

75. The product of two numbers is 10707, and one of the numbers is 129; what is the other number?

76. The earth, in its revolution round the sun, moves about 1641600 miles in one day; how far does it move in one second, there being 86400 seconds in a day?

77. Divide $1064 equally among 8 men. Ans. $133.

79. To divide by a composite number.

Ex. 78. Divide $1855 equally among 35 men.

OPERATION.

$35 = 7 \times 5.$

1st Factor,	7)	$ 1 8 5 5	Dividend.
2d Factor,	5)	$ 2 6 5	1st Quotient,
		$ 5 3	True Quotient.

The 35 men may be separated into 7 groups of 5 men each. Then dividing by 7 gives $265 for each group, and dividing the $265 by 5 gives $53 for each man.

NOTE. When a composite number is made up of different sets of factors, as in Ex. 79, it is immaterial which set is taken. It is also immaterial in what order the factors are taken.

79. Divide 10656 by 288.

$288 = 4 \times 6 \times 12 = 6 \times 6 \times 8 = 8 \times 3 \times 12$, etc.

FIRST OPERATION.		SECOND OPERATION.
4) 1 0 6 5 6		6) 1 0 6 5 6
6) 2 6 6 4		6) 1 7 7 6
1 2) 4 4 4		8) 2 9 6
3 7	=	3 7

From these examples we have the following

RULE. *Divide the dividend by one factor of the divisor, and the quotient so obtained by another factor, and so on till all the factors of the set have been used. The last quotient will be the true quotient.*

80. Divide 1551 by 33.
81. Divide 31794 by 42.
82. Divide 47936 by 56.
83. Divide 24840 by 72.
84. Divide 7665 by 105.
85. Divide 1064 by 56.
86. Divide 1984 by 64.
87. Divide 3321 by 81.
88. Divide 187236 by 252.
89. Divide 1255872 by 192.
90. Divide 1365 by 105.
91. Divide 5355 by 315.
92. Divide 6699 by 231.
93. Divide 3822 by 294.
94. Divide 8568 by 504.
95. Divide 7245 by 315.

---

79. Rule for dividing by a Composite Number? Is it material which factor of the divisor is used first?

80. In dividing by the factors of the divisor, there may be a *remainder* after either or each of the divisions.

Should the learner find a difficulty in determining the *true remainder*, he has but to remember that it is *always* of the same kind as the dividend (Art. 69, Note).

96. Divide 86 by 21.

OPERATION.

$$\begin{array}{r|l} 7 & 86 \\ \hline 3 & 12 \ldots 2 \text{ Rem.} \\ \hline \text{Quotient,} & 4 \end{array}$$

In this example, as 86 is the true dividend, 2 is the true remainder.

97. Divide 92 by 28.

OPERATION.

$$\begin{array}{r|l} 4 & 92 \\ \hline 7 & 23 \\ \hline \text{Quotient,} & 3 \ldots 2 \text{ Rem.} \end{array}$$

In this example, as 23 is only one fourth of the true dividend, so the remainder, 2, is only one fourth of the true remainder; $\therefore 2 \times 4 = 8$, true remainder.

98. Divide 527 by 42.

OPERATION.

$$\begin{array}{r|l} 6 & 527 \\ \hline 7 & 87 \ldots 5 \text{ Rem.} \\ \hline \text{Quotient,} & 12 \ldots 3 \text{ Rem.} \end{array}$$

By the explanation of examples 96 and 97, we see that 5 is one part of the true remainder, and that 3, the second remainder, multiplied by 6, the first divisor, is the other part; i. e. $5 + 3 \times 6 = 23$, is the true remainder. The same species of reasoning applies when there are more than two divisors. Hence,

To obtain the true remainder when division is performed by using the factors of the divisor:

RULE. *Multiply each remainder, except that left by the first division, by the continued product of the divisors preceding that which gave the remainders severally, and the sum of the products, together with the remainder left by the first division, will be the true remainder.*

NOTE 1. When there are but two divisors and two remainders, the rule

---

80. Rule for finding the true remainder when the factors of the divisor are used separately? The reason? What is meant by a *continued* product?

only requires the addition of the *first remainder*, to the *product* of the *first divisor and second remainder.*

NOTE 2. When *three* or more factors are multiplied together, the product is called a *continued* product.

99. Divide 1834 by 35. Ans. Quo. 52, Rem. 14.

OPERATION.

$35 = 5 \times 7.$

$$
\begin{array}{l}
5\,)\,\underline{1\,8\,3\,4} \\
\;\;7\,)\,\underline{3\,6\,6} \ldots 4,\text{ 1st Rem.} \\
\text{Quo., } \;5\,2 \ldots 2,\text{ 2d Rem.}
\end{array}
$$

TRUE REMAINDER.

$$
\begin{array}{rl}
4 = & \text{1st Rem.} \\
2 \times 5 = \underline{1\,0} = & \text{2d Rem.} \times \text{1st Div} \\
1\,4 = & \text{True Rem.}
\end{array}
$$

100. Divide 18328 by 385.

OPERATION.

$385 = 5 \times 7 \times 11.$

$$
\begin{array}{l}
5\,)\,\underline{1\,8\,3\,2\,8} \\
\;\;7\,)\,\underline{3\,6\,6\,5} \ldots 3,\text{ 1st Rem.} \\
\;\;1\,1\,)\,\underline{5\,2\,3} \ldots 4,\text{ 2d Rem.} \\
\text{Quo., } \;\;4\,7 \ldots 6,\text{ 3d Rem.}
\end{array}
$$

TRUE REMAINDER.

$$
\begin{array}{rrl}
 & 3 = & \text{1st Rem.} \\
4 \times 5 = & 2\,0 = & \text{1st Prod.} \\
6 \times 7 \times 5 = & \underline{2\,1\,0} = & \text{2d Prod.} \\
 & 2\,3\,3 = & \text{True Rem.}
\end{array}
$$

101. Divide 5273 by 42.

$42 = 6 \times 7.$ Ans. 125 and 23 Rem.

102. Divide 46987 by 504, using the factors of the divisor.

Ans. 93 and 115 Rem.

103. $437298 \div 54 = ?$

104. $216349 \div 64 = ?$

105. $2411 \div 72 = ?$

106. $36067 \div 45 = ?$

107. $65947 \div 25 = ?$

108. $6842 \div 49 = ?$

109. $7829 \div 35 = ?$

110. $3748 \div 42 = ?$

111. $4629 \div 81 = ?$

112. $3643 \div 48 = ?$

81. To divide by 10, 100, 1000, etc.:

RULE. *Cut off, by a point, as many figures from the right hand of the dividend as there are ciphers in the divisor. The figures at the left of the point are the quotient, and those at the right are the remainder.*

113. Divide 756 by 10. Ans. 75.6, i. e. 75 Quo., 6 Rem.

---

81. Rule for dividing by 10? By 100?

NOTE. The reason of the rule is obvious. By taking away the right-hand figure, each of the other figures is brought one place nearer to units, and its value is only one tenth as great as before (Art. 15), and ∴ the whole is divided by 10. For like reasons, cutting off *two* figures divides by 100; cutting off *three* figures divides by 1000, etc.

114. Divide 402763 by 10.

115. Divide 76943 by 100. Ans. 769 and 43 Rem.

116. Divide 98765423 by 100000.

Ans. 987 and 65423 Rem.

117. Divide 3078654321 by 100000000.

82. To divide by 20, 50, 700, or any similar number:

RULE. *Cut off as many figures from the right of the dividend as there are ciphers at the right of the significant figures of the divisor, and then divide the remaining figures of the dividend by the significant figures of the divisor.*

NOTE 1. This is on the principle of dividing by the factors of the divisor; ∴ the true remainder will be found by the rule in Art. 80.

118. Divide 74689 by 8000. Ans. 9 and 2689 Rem.

OPERATION.

8) 74.689

Quotient, 9 . . . 2 Rem.

We divide by 1000 by cutting off 689, which gives 74 for a quotient and 689 for a remainder; then divide 74 by 8, and obtain the quotient, 9, and remainder, 2. This remainder, 2, is 2000, which, increased by 689, gives 2689 for the true remainder (Art. 80).

NOTE 2. It will be observed that the true remainder, in all examples like the 118th, is obtained by *annexing* the 1st to the 2d remainder.

119. Divide 67475 by 2400.

120. Divide 74689 by 4200. Ans. 17 and 3289 Rem.

121. Divide 276987 by 3300.

122. 769842 ÷ 45000 = ? Ans. 17 and 4842 Rem.

123. 9999999 ÷ 33300 = ?

124. 80407080 ÷ 40000 = ?

125. 987654321 ÷ 90900 = ?

---

81. Reason of rule for dividing by 10? **82.** Rule for dividing by 20? By 500? Reason? How is the true remainder found?

General Principles of Division.

83. The value of a quotient depends upon the *relative* values of the divisor and dividend, and not upon their *absolute* values, as will be seen by the following propositions.

(a) *If the divisor remains unaltered, multiplying the dividend by any number is, in effect, multiplying the quotient by the same number;* thus,

$$\begin{array}{rcrcr} 15 & \div & 3 & = & 5 \\ \underline{4} & & & & \underline{4} \\ 60 & \div & 3 & = & 20; \end{array}$$

i. e. multiplying the dividend by 4 multiplies the quotient by 4.

(b) *Dividing the dividend by any number is dividing the quotient by the same number;* thus,

$$\begin{array}{rrcrcl} & 24 & \div & 2 & = & 12 \\ 3) & \underline{24} & & & & \\ & 8 & \div & 2 & = & 4 = 12 \div 3; \end{array}$$

i. e. dividing the dividend by 3 divides the quotient by 3.

(c) *Multiplying the divisor divides the quotient;* thus,

$$\begin{array}{rcrcl} 30 & \div & 2 & = & 15 \\ & & \underline{3} & & \\ 30 & \div & 6 & = & 5 = 15 \div 3; \end{array}$$

i. e. multiplying the divisor by 3 divides the quotient by 3.

(d) *Dividing the divisor multiplies the quotient;* thus,

$$\begin{array}{rcrcl} 40 & \div & 10 & = & 4 \\ & & 5)\,\underline{10} & & \\ 40 & \div & 2 & = & 20 = 4 \times 5; \end{array}$$

i. e. dividing the divisor by 5 multiplies the quotient by 5.

---

83. Does the size of the quotient depend upon the absolute size of divisor and dividend? Upon what does it depend? What is the first proposition? Second? Third? Fourth?

(e) It follows, from (a) and (b), *that the greater the dividend, the greater is the quotient; and the less the dividend, the less the quotient.*

(f) Also, from (c) and (d), *that the greater the divisor, the less is the quotient; and the less the divisor, the greater the quotient.*

84. From the illustrations in Art. 83 we see that any change in the *dividend* causes a SIMILAR change in the quotient, and that any change in the *divisor* causes an OPPOSITE change in the quotient. Hence,

(a) *Multiplying both dividend and divisor by the same number does not affect the quotient;* thus,

$$\begin{array}{r@{\;\div\;}l}
12 & 3 = 4 \\
\underline{2} & \underline{2} \\
24 & 6 = 4, \text{ Quotient unchanged.}
\end{array}$$

(b) *Dividing both dividend and divisor by the same number does not affect the quotient;* thus,

$$\begin{array}{r@{\;\div\;}l}
20 & 10 = 2 \\
5\,)\,\underline{20} & 5\,)\,\underline{10} \\
4 & 2 = 2, \text{ Quotient unchanged.}
\end{array}$$

(c) It follows from (a) and (b), *that the operations of multiplying and dividing by the same number cancel* (i. e. *destroy*) *each other;* e. g.,

If a number be multiplied by any number, and the product be divided by the multiplier, the quotient will be the multiplicand; thus,

$8 \times 7 = 56$, and $56 \div 7 = 8$, the multiplicand.

Also, if a number be divided by any number, and the quotient be multiplied by the divisor, the product will be the dividend; thus,

$15 \div 3 = 5$, and $5 \times 3 = 15$, the dividend.

---

83. What follows from (a) and (b)? From (c) and (d)? **84.** Any change in the dividend, how does it affect the quotient? Any change in the divisor, how? First inference? Second? Third? Illustrate.

85. These general principles may be more briefly stated as follows:

1st. *Multiplying the dividend multiplies the quotient; and dividing the dividend divides the quotient* (Art. 83, a and b).

2d. *Multiplying the divisor divides the quotient; and dividing the divisor multiplies the quotient* (Art. 83, c and d).

3d. *Multiplying both dividend and divisor by the same number; or dividing both by the same number does not affect the quotient* (Art. 84, a and b).

Examples in the Foregoing Principles.

1. How many bushels of corn at $1 per bushel must be given for 6 barrels of flour at $7 per barrel?

2. How many barrels of apples at $2 per barrel must be given for 8 cords of wood at $6 per cord?

3. A speculator bought 640 acres of land at $3 per acre, and sold the whole for $3200; how much did he gain by the transactions? How much per acre?

4. Bought 320 acres of land for $1760, and 320 acres more at $7 per acre, and sold the whole at $6 per acre; did I gain or lose? How much? Ans. Lost $160.

5. The expenses of a boy at school for a year are $126 for board, $24 for tuition, $15 for books, $35 for clothes, $10 for railroad and coach fare, and $9 for other purposes; what will be the expenses of 250 boys at the same rate?

6. If 3 men build 24 rods of wall in 4 days, in how many days will 5 men build 70 rods? Ans. 7.

7. The product of 4 factors is 1155; three of the factors are 3, 5, and 7; what is the fourth? Ans. 11.

8. How many miles per hour must a ship sail to cross the Atlantic, 2880 miles, in 12 days of 24 hours each?

9. The first of 3 numbers is 6, the second is 5 times the first, and the third is 4 times the sum of the other two; what is the difference between the first and third?

---

85. A more brief statement of these principles: First? Second? Third?

10. Sold two cows at $30 apiece, 3 tons of hay at $20 per ton, 50 bushels of corn for $50, and 10 cords of wood at $7 per cord, and received in payment $200 in money, a plow worth $15, 50 pounds of sugar worth $5, and the balance in broadcloth at $4 yer yard; how many yards did I receive? Ans. 5.

11. In how many days of 24 hours each will a ship cross the Atlantic, 2880 miles, if she sails 10 miles per hour?

12. If I receive $60 and spend $40, per month, in how many years of 12 months each shall I save $2160? Ans. 9.

13. What is the value of 27 hogsheads of molasses at $32 per hogshead?

14. What is the value of 87 yards of cloth at $4 per yard?

15. Bought 87 acres of land at $50 per acre, and paid $3150 in cash, and the balance in labor at $240 a year; how many years of labor did it take?

16. Bought 42 yards of cloth at 15 cents per yard, and paid for it in corn at 90 cents per bushel; how many bushels did it take?

17. If I take 13729 from the sum of 8762 and 14967, divide the remainder by 50, and multiply the quotient by 19, what is the product? Ans. 3800.

REDUCTION.

86. All numbers are *simple* or *compound.*

A SIMPLE NUMBER consists of but *one kind* or *denomination;* as 2, $4, 8 books, 5 men, 6 days, 10 miles.

A COMPOUND NUMBER is composed of *two or more denominations;* as 4 days and 7 hours; 3 bushels, 2 pecks, and 5 quarts; 5 rods, 4 feet, and 6 inches.

All abstract numbers (Art. 2) are simple.

---

86. What is a Simple Number? A Compound Number? An Abstrac Number, is it simple or compound?

A concrete number, whether simple or compound, is often called a *Denominate Number.*

NOTE 1. All operations in the preceding pages are upon simple numbers.

NOTE 2. The several parts of a compound number, though of *different denominations*, are yet of the *same general nature;* thus, 2 weeks, 3 days, and 6 hours are SIMILAR *quantities, and constitute a compound number;* but 2 weeks, 3 miles, and 6 quarts are UNLIKE IN THEIR NATURE, *and do* NOT *constitute a compound number.*

87. REDUCTION is changing a number of one denomination to one of another denomination, *without changing its value.*

It is of two kinds, viz. *Reduction Descending* and *Reduction Ascending.*

REDUCTION DESCENDING consists in changing a number from a *higher* to a *lower* denomination.

REDUCTION ASCENDING is changing a number from a *lower* to a *higher* denomination.

ENGLISH MONEY.

88. ENGLISH MONEY is the Currency of Great Britain.

TABLE.

4 Farthings (far. or qr.)	make	1 Penny,	marked	d.
12 Pence	"	1 Shilling,	"	s.
20 Shillings	"	1 Pound,	"	£

£		s.		d.		qr.
				1	=	4
		1	=	12	=	48
1	=	20	=	240	=	960

89. REDUCTION DESCENDING is performed by *multiplication;* thus, to reduce 15£ to shillings, we multiply 15 by 20, because there will be 20 times as many shillings as pounds. So to reduce 15£ and 12s. to shillings, we multiply 15 by 20, and to the product add the 12s.

---

86. A Concrete Number, what is it called? **87.** What is Reduction? How many kinds of Reduction? What are they called? What is Reduction Descending? Reduction Ascending? **88.** What is English Money? Repeat the table. **89.** How is Reduction Descending performed?

In a similar manner all such examples are reduced. Hence,

90. To reduce the higher denominations of a compound number to a lower denomination:

RULE. *Multiply the highest denomination given by the number it takes of the next lower denomination to make one of this higher, and to the product add the number of the lower denomination; multiply this sum by the number it takes of the* NEXT *lower denomination to make one of* THIS; *add as before, and so proceed till the number is brought to the denomination required.*

Ex. 1. Reduce 11£ 17s. 9d. 3qr. to farthings.

OPERATION.

£	s.	d.	qr.
11	17	9	3
20			
237s.			
12			
2853d.			
4			
11415qr., Ans.			

Eleven pounds = 220s., and the 17s. added make 237s. = 2844d., and the 9d. added give 2853d. = 11412qr., which, increased by the 3qr., give 11415 qr., the answer.

2. Reduce 6£ 18s. 4d. 1qr. to farthings. Ans. 6641qr.
3. Reduce 7£ 9s. 3qr. to farthings. Ans. 7155qr.

NOTE. Since there are no pence in the 3d example, there is nothing to add to the product obtained by multiplying by 12.

4. Reduce 27£ 15s. 6d. 2qr. to farthings.
5. Reduce 32£ 8d. 3qr. to farthings.

91. REDUCTION ASCENDING is performed by *division;* thus, to reduce 4299 farthings to pence, we divide the 4299 by 4, because there will be only one fourth as many pence as farthings. Performing the division we obtain 1074d. and a remainder of 3qr. If we wish to reduce the 1074d. to shillings, we divide by 12, because there will be only one twelfth as many shillings as pence, and obtain 89s. and a remainder of 6d. Again,

---

90. Repeat the rule. Explain the process in Ex. 1. How are the 237 shillings obtained? How the 2853 pence? The 11415 farthings? 91. How is Reduction Ascending performed?

the 89s. may be reduced to pounds, by dividing by 20, giving 4£ and a remainder of 9s. Thus we find that 4299qr. are equal to 4£ 9s. 6d. 3qr.

Like reasoning applies to all similar examples. Hence,

92. To reduce a number of a lower denomination to numbers of higher denominations:

RULE. *Divide the given number by the number it takes of that denomination to make one of the next higher; divide the quotient by the number it takes of* THAT *denomination to make one of the* NEXT *higher, and so proceed till the number is brought to the denomination required. The last quotient, together with the several remainders* (Art. 69, Note), *will be the answer.*

93. Reduction Ascending and Reduction Descending *prove* each other.

Ex. 1. Reduce 11415 farthings to pence, shillings, and pounds.

OPERATION.

4) 1 1 4 1 5 qr.
1 2) 2 8 5 3 d. + 3qr.
2 0) 2 3 7 s. + 9d.
1 1 £ + 17s.

First divide by 4 to reduce the farthings to pence; then divide by 12 to reduce pence to shillings; then by 20 to reduce shillings to pounds, and thus obtain 11£ 17s. 9d. 3qr., Ans.

2. Reduce 17229qr. to pence, shillings, and pounds.
Ans. 17£ 18s. 11d. 1qr.

3. Reduce 6874d. to shillings and pounds.
Ans. 28£ 12s. 10d.

NOTE 1. Since Ex. 3rd is given in pence instead of farthings, the first divisor is 12 rather than 4.

4. Reduce 84697qr. to higher denominations.
5. Reduce 124683qr. to higher denominations.
6. Reduce 347624qr. to pence, shillings, and pounds.
7. Reduce 3746d. to shillings and pounds.
8. Reduce 8793s. to pounds.

---

92. Repeat the rule. Explain the process in Ex. 1. How are the 3qr. obtained? How the 9d.? The 17s.? The 11£? **93.** What is the *Proof* in Reduction?

NOTE 2. The numbers employed in the reduction of a compound number are called a *Scale*. The scale is a *descending* scale for *Reduction Descending* and an *ascending* scale for *Reduction Ascending;* thus, in English money the *descending* scale is 20, 12, and 4, and the *ascending* scale is 4, 12, and 20. The *descending* scale consists of the numbers at the left hand of the table, taken in order from the *bottom* to the *top* of the table, and the *ascending* scale consists of the *same numbers taken in the reversed order*, i. e. from the *top* to the *bottom* of the table. In like manner the scale is found in the other tables.

TROY WEIGHT.

94. TROY WEIGHT is used in weighing gold, silver, and precious stones.

TABLE.

24 Grains (gr.)	make	1 Pennyweight,	dwt.
20 Pennyweights	"	1 Ounce,	oz.
12 Ounces	"	1 Pound,	lb.

				dwt.		gr.
		oz.		1	=	24
lb.		1	=	20	=	480
1	=	12	=	240	=	5760

Ex. 1. How many grains in 7lb. 11oz. 14dwt. 18gr.?

OPERATION.

```
  7 lb. 11oz. 14dwt. 18gr.
 12
 ---
 95 oz.
  20
 -----
 1914 dwt.
   24
 -----
 7674
3828
-----
45954 gr., Ans.
```

Ex. 2. Reduce 45954gr. to pounds, ounces, etc.

OPERATION.

```
24 ) 45954 gr.
    20 ) 1914 dwt. + 18 gr.
      12 ) 95 oz.  + 14 dwt.
            7 lb.  + 11 oz.
```

Ans. 7lb. 11oz. 14dwt. 18gr.

NOTE 1. In solving Ex. 1, the several numbers of the lower denomina

---

93. What is a scale? A descending scale? An ascending scale? What are the scales for English money? Where are these scales found? Taken in what order? **94.** For what is Troy Weight used? Repeat the table. Descending scale? Ascending?

tions are added *mentally*, and only the *results are written*; thus, 12 times 7 are 84, and the 11oz. added give 95oz. Then multiplying the 95oz. by 20, and adding the 14dwt., we have 1914dwt. Finally, in multiplying the 1914 dwt. by 24, first multiply by 4, adding in the 18gr., and then multiplying by 2, and adding the results we have 45954gr. for the answer.

NOTE 2. In reducing Ex. 2, if any divisor is so large that the work is not easily done by Short Division, the numbers may be taken upon the slate and the work done by Long Division, setting down only the results.

3. How many grains in 16lb. 8oz. 19dwt.? Ans. 96456gr.
4. Reduce 38695gr. to pounds, etc.
Ans. 6lb. 8oz. 12dwt. 7gr.
5. Reduce 87942gr. to pounds, ounces, etc.
6. Reduce 15lb. 8oz. 6dwt. 15gr. to grains.
7. How many spoons, each weighing 2oz. 8dwt. 20gr., can be made from 2lb. 5oz. 6dwt. of silver? Ans. 12.
8. A jeweller made 8oz. 16dwt. of gold into rings which weighed 3dwt. 16gr. each; how many rings did he make?

APOTHECARIES' WEIGHT.

95. APOTHECARIES' WEIGHT is used in *mixing* or *compounding* medicines; but medicines are *bought* and *sold* by *Avoirdupois Weight.*

TABLE.

20 Grains (gr.)	make	1 Scruple,	sc. or ℈
3 Scruples	"	1 Dram,	dr. or ʒ
8 Drams	"	1 Ounce,	oz. or ℥
12 Ounces	"	1 Pound,	lb. or ℔

						sc.		gr.
				dr.		1	=	20
		oz.		1	=	3	=	60
lb.		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

NOTE 1. The pound, ounce, and grain, in Apothecaries' and Troy Weight are equal, but the ounce is differently subdivided.

---

94. In solving Ex. 1, what is done with the numbers of the lower denominations? In Ex. 2, how is the work done? 95. For what is Apothecaries' Weight used? Repeat the table. Descending scale? Ascending? What denominations of Apothecaries' Weight are like those of Troy Weight?

Ex. 1. How many scruples in 4lb 8℥ 5ʒ 2℈?

OPERATION.

4 lb 8℥ 5ʒ 2℈
12
―――
56℥
8
―――
453ʒ
3
―――
1361℈, Ans.

Ex. 2. In 1361℈ how many pounds, ounces, etc.?

OPERATION.

3) 1361℈
8) 453ʒ + 2℈
12) 56℥ + 5ʒ
4 lb + 8℥

Ans. 4lb 8℥ 5ʒ 2℈.

3. Reduce 6oz. 3dr. 1sc. 19gr. to grains. Ans. 3099gr.

4. Reduce 15984 grains to pounds, ounces, etc.
Ans. 2lb. 9oz. 2dr. 1sc. 4gr.

5. Reduce 876943 grains to higher denominations.

6. Reduce 27lb. 8oz. 7dr. 2sc. 15gr. to grains.

7. How many pounds, ounces, etc., of medicine will an apothecary use in preparing 974 prescriptions of 15 grains each?
Ans. 2lb. 6oz. 3dr. 1sc. 10gr.

AVOIRDUPOIS WEIGHT.

96. Avoirdupois Weight is used for weighing the coarser articles of merchandise, such as hay, cotton, tea, sugar, copper, iron, etc.

TABLE.

16	Drams (dr.)	make	1 Ounce,	oz.
16	Ounces	"	1 Pound,	lb.
25	Pounds	"	1 Quarter,	qr.
4	Quarters	"	1 Hundred Weight,	cwt.
20	Hundred Weight	"	1 Ton,	t.

t.		cwt.		qr.		lb.		oz.		dr.
								1	=	16
						1	=	16	=	256
				1	=	25	=	400	=	6400
		1	=	4	=	100	=	1600	=	25600
1	=	20	=	80	=	2000	=	32000	=	512000

---

96. For what is Avoirdupois Weight used? Table? Scale?

NOTE 1. It was the custom formerly to consider 28lb. a quarter, 112lb. a hundred weight, and 2240lb. a ton; but now the *usual* practice is in accordance with the table.

These different tons are distinguished as the *long* or *gross* ton = 2240lb. and the *short* or *net* ton = 2000lb.

The *gross* ton is still used in the wholesale coal trade, also in estimating goods at the U. S. custom-houses, etc.

NOTE 2. A pound in Avoirdupois Weight is equal to 7000 grains in Apothecaries' or Troy Weight.

Ex. 1. Reduce 6t. 15cwt. 3qr. 20lb. to pounds.

OPERATION.

6t. 15cwt. 3qr. 20lb.
20
135cwt.
4
543qr.
25
2735
1086
13595lb., Ans.

Ex. 2. In 13595lb. how many tons, etc.?

OPERATION.

25) 13595lb.
4) 543qr. + 20lb.
20) 135cwt. + 3qr.
6t. + 15cwt.

Ans. 6t. 15cwt. 3qr. 20lb.

3. Reduce 3t. 6cwt. 2qr. 5lb. 6oz. 10dr. to drams.
Ans. 1703786dr.

4. Reduce 3642897 drams to higher denominations.
Ans. 7t. 2cwt. 1qr. 5lb. 1oz. 1dr.

5. Reduce 37t. 19cwt. 3qr. to pounds.

6. Reduce 17796lb. to higher denominations.

7. Reduce 3t. 19cwt. 3qr. 24lb. 15oz. 15dr. to drams.

8. Reduce 1742684 drams to higher denominations.

9. In 10t. 1cwt. 2qr. 10lb., *net weight*, how many *gross tons*?

10. If a horse eats 22lb. of hay in one day, how many tons will he eat in 365 days? Ans. 4t. 0cwt. 1qr. 5lb.

11. If a blacksmith uses 23lb. 8oz. of iron daily, how many tons will he use in 313 days?

---

96. How many pounds now make a ton? How many formerly? What are the different tons called? For what is the long ton now used? One pound Avoirdupois equals how many grains Troy?

CLOTH MEASURE.

97. Cloth Measure is used in measuring cloths, ribbons, braids, etc.

TABLE.

$2\frac{1}{4}$ Inches (in.)	make	1 Nail,	na.
4 Nails	"	1 Quarter,	qr.
4 Quarters	"	1 Yard,	yd.

yd.		qr.		na.		in.
				1	=	$2\frac{1}{4}$
		1	=	4	=	9
1	=	4	=	16	=	36

Note. Expressions like $\frac{1}{4}$, $\frac{2}{3}$, etc., are called *fractions.* $\frac{1}{4}$ = *one fourth*; $\frac{2}{3}$ = *two thirds*; $2\frac{1}{4}$ = *two and one fourth.* The principles of fractions will be discussed in another place.

Ex. 1. Reduce 15yd. 3qr. 2na. to nails.

OPERATION.

$$\begin{array}{l} 15\text{yd. } 3\text{qr. } 2\text{na.} \\ \underline{\quad 4} \\ 63\text{qr.} \\ \underline{\quad 4} \\ 254\text{na., Ans.} \end{array}$$

Ex. 2. In 254 nails how many yards, quarters, and nails?

OPERATION.

$$\begin{array}{l} 4\,)\,\underline{254}\text{na.} \\ \quad 4\,)\,\underline{63}\text{qr.} + 2\text{na.} \\ \qquad 15\text{yd.} + 3\text{qr.} \end{array}$$

Ans. 15yd. 3qr. 2na.

3. In 27yd. 2qr. 3na. how many nails? Ans. 443.

4. In 873 nails how many yards, etc.? Ans. 54yd. 2qr. 1na.

5. How many dresses may be made from 167yd. 3qr. of silk if each dress requires 15yd. 1qr.? Ans. 11.

6. If 2yd. 3qr. of ribbon are used in trimming one bonnet, how many yards will be used in trimming 5 bonnets?

7. Reduce 43yd. 2qr. 3na. to nails.

8. If 2yd. 1qr. of cloth are required for making one coat, how many yards will be used in making 8 coats?

9. What cost 25yd. 3qr. of cloth at $2 per quarter?

10. Reduce 7824 nails to yards.

---

97. For what is Cloth Measure used? Table? Scale? Note?

LONG MEASURE.

98. Long Measure is used in measuring distances, i. e. where length is required without regard to breadth or thickness.

TABLE.

3	Barleycorns (b. c.)	make	1 Inch,	in.
12	Inches	"	1 Foot,	ft.
3	Feet	"	1 Yard,	yd.
$5\frac{1}{2}$	Yards or $16\frac{1}{2}$ Feet	"	1 Rod,	rd.
40	Rods	"	1 Furlong,	fur.
8	Furlongs	"	1 Mile,	m.
3	Miles	"	1 League,	l.
$69\frac{1}{2}$	Statute miles, nearly,	"	1 Degree on Circ. of the Earth,	1°
360	Degrees	"	1 Circumference,	circ.

m.	fur.	rd.	yd.	ft.	in.	b. c.
					1	= 3
				1	= 12	= 36
			1	= 3	= 36	= 108
		1	= $5\frac{1}{2}$	= $16\frac{1}{2}$	= 198	= 594
	1	= 40	= 220	= 660	= 7920	= 23760
1	= 8	= 320	= 1760	= 5280	= 63360	= 190080

Note 1. The earth not being an exact sphere, the distance round it in different directions is not exactly the same. By the most exact measurements made, a degree is a little *less* than $69\frac{1}{2}$ miles.

Note 2. The barleycorn is but little used.

Note 3. The 3 before *miles* in the table is not a part of the scale.

Ex. 1. How many rods in 8m. 3fur. 30rd.?

OPERATION.

```
 8 m. 3fur. 30rd.
 8
 ----
 6 7 fur.
 4 0
 -------
 2 7 1 0 rd., Ans.
```

Ex. 2. Reduce 2710rd. to higher denominations.

OPERATION.

```
4 0 ) 2 7 1 0 rd.
      8 ) 6 7 fur. + 30rd.
          8 m. + 3fur.
```

Ans. 8m. 3fur. 30rd.

3. In 4yd. 2ft. 8in. how many barleycorns? Ans. 528.

---

98. For what is Long Measure used? Table? Scale? A degree upon the earth, how long?

4. Reduce 473b.c. to higher denominations.

5. The distance through the center of the earth is about 7912 miles; how many rods is it?

6. The distance round the earth is about 8000000 rods; how many miles is it?

CHAIN MEASURE.

99. Chain Measure is used by engineers and surveyors in measuring roads, canals, boundaries of fields, etc.

TABLE.

$7\frac{92}{100}$	Inches (in.)	make	1 Link,	li.
25	Links	"	1 Rod, Perch, or Pole,	rd.
4	Rods	"	1 Chain,	ch.
10	Chains	"	1 Furlong,	fur.
8	Furlongs	"	1 Mile,	m.

m.		fur.		ch.		rd.		li.		in.
								1	=	$7\frac{92}{100}$
						1	=	25	=	198
				1	=	4	=	100	=	792
		1	=	10	=	40	=	1000	=	7920
1	=	8	=	80	=	320	=	8000	=	63360

Note. To measure roads, etc., engineers often use a chain 100 feet long.

Ex. 1. Reduce 5m. 7fur. 8ch. 3rd. 15li. to links.

OPERATION.

5 m. 7fur. 8ch. 3rd. 15li.
8
47 fur.
10
478 ch.
4
1915 rd.
25
9590
3830
47890 li., Ans.

Ex. 2. Reduce 47890 links to higher denominations.

OPERATION.

25) 47890 li.
4) 1915 rd. + 15 li.
10) 478 ch. + 3 rd.
8) 47 fur. + 8 ch.
5 m. + 7 fur.

Ans. 5m. 7fur. 8ch. 3rd. 15li.

99. For what is Chain Measure used? Table? Scale? Note?

3. In 6fur. 2ch. 3rd. 18li. how many links? Ans. 6293.

4. Reduce 3879 links to higher denominations.
Ans. 3fur. 8ch. 3rd. 4li.

5. Reduce 17m. 3fur. 5ch. 2rd. 24li. to links.

6. Reduce 13475 links to higher denominations.

7. From Boston to Andover is 23 miles; how many links is it?

8. From Boston to Fitchburg is 400000 links; how many miles is it?

9. The distance round a field is 7fur. 6ch. 3rd.; what will it cost to fence the field at $2 per rod?

10. How many miles, etc., in 637482 links?

SQUARE MEASURE.

100. Square Measure is used for measuring surfaces.

TABLE.

144	Square Inches (sq. in.)	make	1	Square Foot,	sq. ft.
9	Square Feet	"	1	Square Yard,	sq. yd.
$30\frac{1}{4}$ / $272\frac{1}{4}$	Square Yards or / Square Feet	"	1	Square Rod,	sq. rd.
40	Square Rods	"	1	Rood,	r.
4	Roods	"	1	Acre,	a.
640	Acres	"	1	Square Mile,	sq. m.

(a) Also in Chain Measure,

10000 / 16	Square Links or / Square Rods	make	1	Square Chain,	sq. ch.
10	Square Chains	"	1	Acre,	a.

sq. m.	a.	r.	sq. rd.	sq. yd.	sq. ft.	sq. in.
					1=	144
				1=	9=	1296
			1=	$30\frac{1}{4}$=	$272\frac{1}{4}$=	39204
		1=	40=	1210=	10890=	1568160
	1=	4=	160=	4840=	43560=	6272640
1=	640=	2560=	102400=	3097600=	27878400=	4014489600

Note. In measuring land, surveyors use a 4-rod chain composed of 100 links. Sometimes the half-chain of 50 links is used.

---

100. For what is Square Measure used? Table? Scale? Table in Chain Measure? Note?

Ex. 1. In 2sq. m. 625a. 2r. 25sq. rd. how many sq. rods?

OPERATION.

2 sq. m. 625 a. 2r. 25sq. rd.
640
1905 a.
4
7622 r.
40
304905 sq. rd., Ans.

Ex. 2. Reduce 304905sq. rd. to higher denominations.

OPERATION.

40) 304905 sq. rd.
4) 7622 r. + 25sq. rd.
640) 1905 a. + 2r.
2 sq.m. + 625a.

Ans. 2sq. m. 625a. 2r. 25sq. rd.

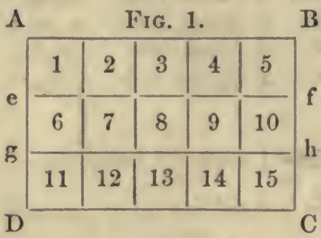
3. In 14sq. m. 25a. 3r. 30sq. rd. how many square rods?
Ans. 1437750.

4. Reduce 624873sq. rd. to higher denominations.

5. Bought a field containing 3a. 2r. 25sq. rd. at $2 per rod; what did it cost?

101. The manner of finding the area of a surface like Fig. 1, may be understood from the following explanation. Let A B represent (on a reduced scale) a line 5 inches long; then, evidently, if we pass from A to e, a distance of 1 inch, and draw the line e f, the figure A B f e will contain 5 square inches, i. e. 5 × 1 square inches. So, in like manner, A B h g will contain 10, or 5 × 2 square inches; and A B C D will contain 15, or 5 × 3 square inches, i. e. we *multiply together the numbers representing the length and breadth, and the product will be the number of square inches in the surface.*

Fig. 1.



Note. A surface like Fig. 1 is called a *rectangle.* If the length and breadth are equal, the rectangle is a *square.* The *angles* of a rectangle or square are all *equal to each other*, and each angle is called a *right angle.*

---

101. How is the area of a rectangle or square ascertained? What is said of the angles of a rectangle or square? What is each angle called?

102. The *area* of a rectangle *divided* by the *length* will give the *breadth*, and the *area divided* by the *breadth* will give the *length;* thus, in Fig. 1, $15 \div 5 = 3$ and $15 \div 3 = 5$.

Ex. 6. How many square rods in a field that is 7 rods wide and 9 rods long? Ans. 63.

7. How many square rods in a field that is 25 rods wide and 48 rods long? How many acres? 2d Ans. 7a. 2r.

8. A board containing 36 square feet, is 12 feet long; how wide is it?

9. A flower garden containing 300 square feet is 12 feet wide; how long is it?

10. How many acres in a field that is 20 rods wide and 56 rods long? Ans. 7.

SOLID OR CUBIC MEASURE.

103. SOLID OR CUBIC MEASURE is used in measuring things which have length, breadth, and thickness.

TABLE.

1728 Cubic Inches (c. in.)	make	1	Cubic Foot,	cu. ft.
27 Cubic Feet	"	1	Cubic Yard,	c. yd.
16 Cubic Feet	"	1	Cord Foot,	c. ft.
8 Cord Feet or 128 Cubic Feet	"	1	Cord,	c.

c. yd.		cu. ft.		c. in.
		1	=	1728
1	=	27	=	46656

NOTE 1. The *scale* in this table only includes 1728 and 27; the other numbers are irregular.

NOTE 2. Transportation companies often estimate freight, especially of light articles, by the space occupied, rather than by the actual weight. In this estimate, from 25 or 30 to 150 or 175 cubic feet are called a ton. This is called *arbitrary* weight, and it varies with different transportation companies, and somewhat according to the risks of carriage. The Boston and Maine Railroad Co., e. g., considers a thousand of bricks a ton, whereas the actual weight is more than *two* tons. Again, a horse is estimated at 3000lb.,

---

102. How is the breadth of a rectangle found when the area and length are known? How the length, when the area and breadth are known? **103.** For what is Solid Measure used? Table? Scale? Note 2?

though the average weight of horses is not far from 1000 lb. Masts, shi timber, hard-wood boards, etc., are estimated at the rate of 3000 lb. for 4 cubic feet, which gives 26⅔ feet per ton. The old distinction between squar and round timber is practically abolished. Furniture and other light an bulky articles are estimated at 150 feet to the ton, which gives about 8 ton to a full freight car-load.

Ex. 1. How many cubic feet in 34c. yd. 26cu. ft.?

OPERATION.

```
34 c. yd. 26 cu. ft.
27
---
264
68
---
944 cu. ft., Ans.
```

Ex. 2. Reduce 944cu. ft. t cubic yards and feet.

OPERATION.

```
27 ) 944 cu. ft.
     34 c. yd. + 26 cu. ft.
```

Ans. 34c. yd. 26cu. ft.

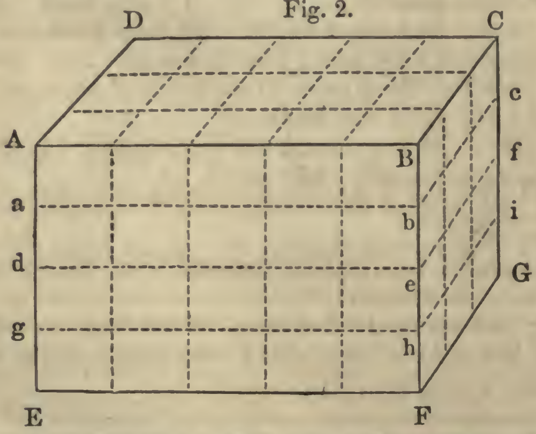
3. In 3c. 6c. ft. 15cu. ft. 156c. in. how many cubic inches?

Ans. 855516.

4. If 40cu. ft. make one ton, how many tons, cubic feet, etc. in 389664 cubic inches?

104. A body like Fig. 2 is called a *prism*. Each side, a

Fig. 2.



A B C D or A B F E, is called a *face* of the prism. If eac

---

104. What is a body like Fig. 2 called? What is one side of the prism called angle of the faces is a *right angle* the prism is *rectangular*, and if each face is a *square* the prism is a *cube*. To determine the contents of a rectangular prism, first find the area of the upper face, A B C D, as in Art. 101; then going from A, B, and C downward 1 inch to a, b, and c, and passing a plane through a, b, and c, we shall cut off 15 solid inches, i. e. $5 \times 3 \times 1$ solid inches. So if a plane be passed through d, e, and f it will cut off 30, or $5 \times 3 \times 2$ inches, etc.; i. e. the continued product of the numbers expressing the length, breadth, and depth, will give the solid contents of the prism.

105. So also, the solid contents divided by the area of the top face will give the depth; the contents divided by the area of one end will give the length; and the contents divided by the area of one side will give the breadth or width.

What are the solid contents of Fig. 2?

Ex. 5. How many cubic inches in a rectangular prism or block of wood which is 12 inches long, 8 inches wide, and 6 inches thick? Ans. $12 \times 8 \times 6 = 576$.

6. How many cubic feet in a room which is 18 feet long, 15 feet wide, and 9 feet high?

7. A rectangular block of marble which contains 96 cubic feet, is 8 feet long and 4 feet wide; how thick is it? Ans. 3 feet.

8. A grain-bin which holds 24 cubic feet of grain is 3 feet deep and 2 feet wide; how long is it?

9. A lady's work-box contains 480 cubic inches; it is 12 inches long and 5 inches deep; how wide is it?

10. In a pile of wood 16 feet long, 4 feet wide, and 6 feet high, how many cords? Ans. 3.

11. If a load of wood be 8 feet long and 4 feet wide, how high must it be to make a cord?

12. My bedroom is 15 feet long, 12 feet wide, and 9 feet high; in how many minutes shall I breathe the room full of air, if I breathe 1 cubic foot in 2 minutes?

---

103. When is a prism rectangular? When is it a cube? How are the contents of a rectangular prism found? 105. How the depth, length, or breadth, if we know the contents of the body and the area of one face?

LIQUID MEASURE.

106. Liquid Measure is used in measuring all liquids. The U. S. Standard Unit of Liquid Measure is the old English wine gallon, which contains 231 cubic inches.

TABLE.

4 Gills (gi.)	make	1 Pint,	pt.
2 Pints	"	1 Quart,	qt.
4 Quarts	"	1 Gallon,	gal.

gal.		qt.		pi.		gi.
				1	=	4
		1	=	2	=	8
1	=	4	=	8	=	32

Note 1. It has been customary to measure milk, and also beer, ale, and other malt liquors, by beer measure, the gallon containing 282 cubic inches, but this custom is fast going out of use.

Note 2. Casks of various capacities, from 50 to 150 or more gallons, are indiscriminately called hogsheads, pipes, butts, tuns, etc.

Ex. 1. In 6gal. 3qt. 1pt. 2gi. how many gills?

OPERATION.

```
 6 gal. 3qt. 1pt. 2gi.
 4
27 qt.
 2
55 pt.
 4
222 gi., Ans.
```

Ex. 2. Reduce 222 gills to gallons, quarts, etc.

OPERATION.

```
4 ) 222 gi.
  2 ) 55 pt. + 2 gi.
    4 ) 27 qt. + 1 pt.
         6 gal. + 3 qt.
```

Ans. 6gal. 3qt. 1pt. 2gi.

3. Reduce 8gal. 2qt. 1pt. 3gi. to gills. Ans. 279gi.

4. Reduce 7496 gills to higher denominations.

5. How many demijohns, each containing 2gal. 1qt. 1pt. 3gi. may be filled from a cask which contains 98 gallons and 3 quarts?

6. How many gallons of molasses in 24 jugs, each containing 2gal. 3qt. 1pt.?

---

106. For what is Liquid Measure used? Table? Scale? Note 1? Note 2?

DRY MEASURE.

107. DRY MEASURE is used in measuring grain, fruit, potatoes, salt, charcoal, etc.

TABLE.

2 Pints (pt.)	make	1 Quart,	qt.
8 Quarts	"	1 Peck,	pk.
4 Pecks	"	1 Bushel,	bush.

bush.		pk.		qt.		pt.
				1	=	2
		1	=	8	=	16
1	=	4	=	32	=	64

NOTE. The bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains a little less than $2150\frac{1}{2}$ solid inches, or nearly $9\frac{1}{4}$ wine gallons.

Ex. 1. In 3bush. 3pk. 7qt. 1pt. how many pints?

OPERATION.

```
  3 bush. 3pk. 7qt. 1pt.
  4
 ---
 1 5 pk.
   8
 ----
 1 2 7 qt.
     2
 ----
 2 5 5 pt., Ans.
```

Ex. 2. Reduce 255 pints to bushels, pecks, etc.

OPERATION.

```
2 ) 2 5 5 pt.
8 ) 1 2 7 qt. + 1pt.
  4 ) 1 5 pk. + 7qt.
        3 bush. + 3pk.
```

Ans. 3bush. 3pk. 7qt. 1pt.

3. Reduce 8bush. 2pk. 3qt. 1pt. to pints. Ans. 551pt.
4. Reduce 7893pt. to higher denominations.
5. Reduce 4698pt. to higher denominations.
6. How many pints in 15bush. 3pk. 6qt. 1pt.?
7. How many pints in 24bush. 1pk. 7qt. 1pt.?
8. What is the cost of 3bush. 2pk. of grass seed at \$1 a peck?
9. Reduce 34569 pints to higher denominations.
10. Reduce 63bush. 2pk. 7qt. 1pt. to pints.

---

107. For what is Dry Measure used? Table? Scale? What are the dimensions of the bushel measure? How many cubic inches does it contain? How many wine gallons?

TIME.

108. Time is used in measuring duration. The natural divisions of time are days, months (moons), seasons, and years. The artificial divisions are seconds, minutes, hours, weeks, etc.

TABLE.

60 Seconds (sec.)	make	1 Minute,	m.
60 Minutes	"	1 Hour,	h.
24 Hours	"	1 Day,	d.
7 Days	"	1 Week,	wk.
4 Weeks	"	1 Lunar Month,	l. m.
13 Months, 1 Day, and 6 Hours	"	1 Julian Year,	J. yr.
12 Calendar Months (=365 or 366 Days),		1 Civil Year,	c. yr.
100 Years	make	1 Century,	C.

J. yr.		l. m.		wk.		d.		h.		m.		sec.
										1	=	60
								1	=	60	=	3600
						1	=	24	=	1440	=	86400
				1	=	7	=	168	=	10080	=	604800
		1	=	4	=	28	=	672	=	40320	=	2419200
1	=	$13\frac{5}{112}$	=	$52\frac{5}{28}$	=	$365\frac{1}{4}$	=	8766	=	525960	=	31557600

Note 1. The twelve calendar months have the following number of days: January (Jan.) has 31 days; February (Feb.), 28 (in leap year, 29); March (Mar.), 31; April (Apr.), 30; May, 31; June, 30; July, 31; August (Aug.), 31; September (Sept.), 30; October (Oct.), 31; November (Nov.), 30; December (Dec.), 31.

Note 2. The number of days in each month may be easily remembered by committing the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save the second month alone,
Which has just eight and a score
Till leap year gives it one more."

Note 3. A solar year, i. e. a year by the sun, is very nearly 365 days, 5 hours, 48 minutes, and 50 seconds.

---

108. For what is time used? What are its natural divisions? Artificial divisions? Table? Scale? What are the names of the calendar months? How many days in each? Length of a solar year?

Ex. 1. Reduce 3wk. 6d. 23h. 59m. to minutes.

OPERATION.

```
  3wk. 6d. 23h. 59m.
    7
  ---
  27d.
  24
 ----
 131
 54
 ----
 671h.
    60
 ------
40319m., Ans.
```

Ex. 2. Reduce 40319m. to weeks, days, etc.

OPERATION.

```
60 ) 40319m.
     ------
  24 ) 671h. + 59m.
       ---
     7 ) 27d. + 23h.
         --
          3wk. + 6d.
```

Ans. 3wk. 6d. 23h. 59m.

3. Reduce 1wk. 4d. 16h. 8m. to minutes. Ans. 16808m.
4. Reduce 376487 seconds to higher denominations.
5. Reduce 365d. 5h. 48m. 50sec. to seconds.
6. In 342698 minutes how many days, hours, etc.?
7. In 5C. 56yr. 8m. how many calendar months?
8. Reduce 37846 calendar months to centuries, years, etc.
9. Reduce 2419199 seconds to weeks, days, etc.
10. Reduce 34d. 20h. 40m. 50sec. to seconds.

CIRCULAR MEASURE.

109. Circular Measure is used in surveying, navigation, geography, astronomy, etc., for measuring angles, determining latitude, longitude, etc.

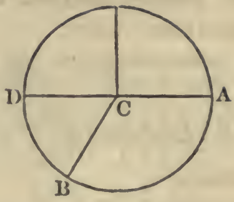
TABLE.

60 Seconds (60″)	make	1 Minute,	1′
60 Minutes	"	1 Degree,	1°
30 Degrees	"	1 Sign,	s
12 Signs, or 360°	"	1 Circumference,	circ.

circ.		s.		°		′		″
						1′	=	60″
				1°	=	60	=	3600
		1	=	30	=	1800	=	108000
1	=	12	=	360	=	21600	=	1296000

109. For what is Circular Measure used? Table? Scale?

7*



NOTE. A *Circle* is a figure bounded by a curved line, all parts of the curve being equally distant from the center of the circle.

The *Circumference* is the curve which bounds the circle. An *Arc* is any portion of the circumference, as A B or B D. An arc equal to a quarter of the circumference, or 90°, is called a *quadrant*. A *Radius* is a line drawn from the center to the circumference, as C A or C B. A *Diameter* is a line drawn *through* the center and limited by the curve, as A D.

Ex. 1. How many seconds in 5s. 25° 48′ 54″?

OPERATION.

5s. 25° 48′ 54″.
30
175°
60
10548′
60
632934″, Ans.

Ex. 2. Reduce 632934″ to higher denominations.

OPERATION.

60) 632934″
60) 10548′ + 54″
30) 175° + 48′
5 s. + 25°

Ans. 5s. 25° 48′ 54″.

3. Reduce 9s. 20° 55′ 47″ to seconds. Ans. 1047347″.
4. In 7484925″ how many circumferences, signs, etc.?
5. In 3 quadrants, 10° 8′ 5″ how many seconds?
6. Reduce 984627″ to quadrants, degrees, etc.

MISCELLANEOUS TABLE.

110. This table embraces a few terms in common use, and may be indefinitely extended.

12	Single things	make	1 Dozen.
12	Dozen	"	1 Gross.
12	Gross	"	1 Great Gross.
20	Single things	"	1 Score.
24	Sheets of paper	"	1 Quire.
20	Quires	"	1 Ream.
196	Pounds	"	1 Barrel of Flour.
200	Pounds	"	1 Barrel of Beef or Pork.

109. What is a Circle? Circumference? Arc? Quadrant? Radius? Diameter?

Ex. 1. How many dozen bottles, each bottle holding 1qt. 1pt. 3gi. will be sufficient to bottle 61gal. 3qt. 1pt. of wine?

2. How many sheets of paper in 3 reams, 18 quires, and 23 sheets?

Miscellaneous Examples in Reduction.

1. Reduce 27£. 14s. 6d. 3qr. to farthings.
2. Reduce 18bush. 3pk. 7qt. 1pt. to pints.
3. Reduce 7t. 14cwt. 2qr. 12lb. 8oz. 6dr. to drams.
4. How many tons, etc., in 574692 ounces?
5. Reduce 1577048 seconds to minutes, hours, etc.
6. Reduce 24838 grains to scruples, drams, etc.
7. Reduce 2circ. 4s. 20° 25′ 30″ to seconds.
8. Reduce 3m. 5fur. 7ch. 2rd. 20li. to links.
9. Reduce 14lb. 7oz. 15dwt. 23gr. to grains.
10. Reduce 6℔ 4℥ 3ʒ 1℈ 6gr. to grains.
11. Reduce 2548 square inches to higher denominations.
12. Reduce 411 nails to quarters and yards.
13. Reduce 7432 farthings to pence, etc.
14. Reduce 18469874 drams, Avoirdupois, to ounces, etc.
15. Reduce 54896 grains to pennyweights, etc.
16. Reduce 4sq. m. 25a. 3r. 34sq. rd. to square rods.
17. Reduce 8c. yd. 1727c. in. to cubic inches.
18. Reduce 4sq. yds. to square inches.
19. Reduce 4gal. 1pt. to gills.
20. Reduce 2wk. 6d. 8h. 16sec. to seconds.
21. Reduce 4m. 7fur. 39rd. to rods.
22. Reduce 3795 rods to furlongs, etc.
23. Reduce 17yd. 2qr. 3na. to nails.
24. Reduce 10881 links to miles, furlongs, etc.
25. Reduce 6598 pints to quarts, pecks, etc.
26. Reduce 4368294″ to higher denominations.
27. Reduce 4680 gills to higher denominations.
28. Reduce 195261 cubic inches to feet and yards.
29. Reduce 310556 square rods to roods, acres, and miles.

Note. This subject will receive further attention in the articles on Fractions.

DEFINITIONS AND GENERAL PRINCIPLES.

111. All numbers are *even* or *odd*.

An Even Number is a number that is divisible by 2 (Art. 74); as 2, 4, 8, 12.

An Odd Number is a number that is *not* divisible by 2; as 1, 3, 5, 11, 19.

112. All numbers are *prime* or *composite*.

A Prime Number is a number that is divisible by no whole number except *itself* and *one;* as 1, 2, 3, 5, 7, 11, 19.

Note 1. *Two* is the only *even* prime number, *for all even numbers are divisible by* 2.

Note 2. Two numbers are *mutually* prime (i. e. *prime to each other*) when no whole number but *one* will divide each of them; thus, 8 and 9 are *mutually* prime, although neither 8 nor 9 is *absolutely* prime.

A Composite Number is a number (Art. 61) that is divisible by other numbers besides itself and one; thus, 6 is composite, because it is divisible by 2 and by 3; 12 is composite, because it is divisible by 2, 3, 4, and 6; 25 is composite, because it is divisible by 5 and 5.

Note 3. A composite number that is composed of *any number of* EQUAL *factors* is called a *power*, and the *equal factors* are called the *roots* of the power; thus, 9, which equals 3×3 is the *second power* or *square* of 3, and 3 is the *second* or *square root* of 9; 64, which equals $4 \times 4 \times 4$, is the *third power* or *cube* of 4, and 4 is the *third* or *cube root* of 64.

Note 4. The *power* of a number is usually indicated by a figure, called an *index* or *exponent*, placed at the *right* and a little *above* the number; thus, the *second power* or *square* of 4 is written 4^2, which equals $4 \times 4 = 16$; the *third power* or *cube* of 4 is 4^3, which equals $4 \times 4 \times 4 = 64$.

Note 5. A *root* may be indicated by the *radical sign*, $\sqrt{\ }$; thus, $\sqrt{9}$ indicates the *second* or *square root* of 9, which is 3. So $\sqrt[3]{8}$ indicates the *third* or *cube root* of 8, which is 2. The *square* root of a number is one of its *two equal factors;* the *cube* root is one of the *three* equal factors of the number.

Note 6. *Every number is both the first power and the first root of itself.*

---

111. What is an Even Number? An Odd Number? **112.** A Prime Number? What is the only *even* prime number? When are numbers mutually prime? What is a Composite Number? A *power?* A *root?* How is a power indicated? A root? A number is what power of itself? What root?

FACTORING NUMBERS.

113. The FACTORS of a number are those numbers whose continued product is the number; thus, 3 and 7 are the factors of 21; 3 and 6, or 3, 3, and 2 are the factors of 18; etc.

NOTE 1. Every number is a factor of itself, the other factor being 1.

The *prime* factors of a number are those *prime* numbers whose continued product is the number; thus, the prime factors of 12 are 2, 2, and 3; the prime factors of 36 are 2, 2, 3, and 3; etc.

NOTE 2. Since 1, as a factor, is useless, it is not here enumerated.

114. To *factor* a number is to resolve or separate it into its factors. In resolving a number into its factors,

The following facts will be found convenient:

(a) Every number whose unit figure is 0, or an *even* number, is itself *even*, and ∴ divisible by 2.

(b) Any number is divisible by 3 when the *sum of its digits* (Art. 7) is divisible by 3; thus, 4257 is divisible by 3 because the sum of its digits, $4 + 2 + 5 + 7 = 18$, is divisible by 3.

(c) Any number is divisible by 4 when 4 will divide the number expressed by the *two right-hand figures;* thus, 4 will divide 32, ∴ it will divide 7532.

(d) Any number whose unit figure is 0 or 5 is divisible by 5; as 90, 1740, 35, 34975, etc.

(e) Any *even* number which is divisible by 3 is also divisible by 6; thus, 3528 is divisible by 3 and ∴ by 6.

NOTE 1. For 7 no *general* rule is known.

(f) Any number is divisible by 8 when 8 will divide the number expressed by the *three right-hand figures;* thus, 8 will divide 816, ∴ it will divide 175816.

---

113. What are the Factors of a number? Is a number a factor of *itself?* What are the *prime* factors of a number? **114.** What is it to factor a number? What number is divisible by 2? By 3? 4? 5? 6? What is said of 7? What number is divisible by 8?

(g) Any number is divisible by 9 when the *sum of its digits* is divisible by 9; thus, 7146 is divisible by 9 because the sum of its digits, $7 + 1 + 4 + 6 = 18$, is divisible by 9.

(h) Any number ending with 0 is divisible by 10.

(i) Any number is divisible by 11 when the *sum* of the digits in the *odd* places is equal to the *sum* of the digits in the *even* places; also when the *difference of these sums* is divisible by 11; thus, 8129, in which $9 + 1 = 2 + 8$, is divisible by 11; also 6280714, in which the sum of the digits in the *odd places*, $4 + 7 + 8 + 6$, differs from the sum of the digits in the *even places*, $1 + 0 + 2$, by 22, a number divisible by 11.

(j) Any number divisible by 3 and also by 4, is divisible by 12; and, *generally*, any number that is divisible by each of several numbers that are mutually prime, is divisible by the product of those numbers; thus, 84 is divisible by 2, 3, and 7, separately, and ∴ 84 is divisible by $2 \times 3 \times 7 = 42$; so also 108 is divisible by 4 and 9, and ∴ by $4 \times 9 = 36$.

NOTE 2. Every *prime* number, but 2 and 5, has 1, 3, 7, or 9 for its unit figure.

For further aid in determining the factors of numbers, we have the following

TABLE OF PRIME NUMBERS FROM 1 TO 997.

1	41	101	167	239	313	397	467	569	643	733	823	911
2	43	103	173	241	317	401	479	571	647	739	827	919
3	47	107	179	251	331	409	487	577	653	743	829	929
5	53	109	181	257	337	419	491	587	659	751	839	937
7	59	113	191	263	347	421	499	593	661	757	853	941
11	61	127	193	269	349	431	503	599	673	761	857	947
13	67	131	197	271	353	433	509	601	677	769	859	953
17	71	137	199	277	359	439	521	607	683	773	863	967
19	73	139	211	281	367	443	523	613	691	787	877	971
23	79	149	223	283	373	449	541	617	701	797	881	977
29	83	151	227	293	379	457	547	619	709	809	883	983
31	89	157	229	307	383	461	557	631	719	811	887	991
37	97	163	233	311	389	463	563	641	727	821	907	997

114. What number is divisible by 9? By 10? 11? 12? General principle?

115. A PROBLEM is something to be *done;* or, it is a question which requires a *solution.* The *solution* of a problem consists of the operations necessary for finding the *answer* to the question. To *solve* a problem is *to perform* the operations for finding the answer.

116. PROBLEM 1. To resolve or separate a number into its prime factors:

RULE. *Divide the given number by any prime number greater than one, that will divide it; divide the* QUOTIENT *by any prime number greater than one that will divide* IT, *and so on till the quotient is prime. The several divisors and last quotient will be the prime factors sought.*

Ex. 1. What are the prime factors of 30? Ans. 2, 3, and 5.

OPERATION.

$$\begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

It is immaterial in what order the prime factors are taken, though it will usually be most convenient to take the smaller factors first.

2. What are the prime factors of 24? Ans. 2, 2, 2, and 3.
3. Resolve 84 into its prime factors. Ans. 2, 2, 3, and 7.
4. Resolve 375 into its prime factors. Ans. 3, 5, 5, and 5.
5. What are the prime factors of 3465?
6. What are the prime factors of 19800?
7. What are the prime factors of 1440?
8. What are the prime factors of 3150?
9. What are the prime factors of 2310?
10. What are the prime factors of 1728?
11. What are the prime factors of 1800?
12. What are the prime factors of 2448?
13. What are the prime factors of 4824?
14. What are the prime factors of 3648?
15. What are the prime factors of 8696?
16. What are the prime factors of 7264?
17. What are the prime factors of 5075?

---

115. What is a Problem? The *solution* of a problem? What is it to *solve* a problem? 116. Rule for finding the prime factors of a number?

117. If a number has *composite factors*, they may be found by multiplying together two or more of its *prime factors;* thus, the prime factors of 12 are 2, 2, and 3, and the composite factors of 12 are 2×2, 2×3, and $2 \times 2 \times 3$, i. e. the composite factors of 12 are 4, 6, and 12.

GREATEST COMMON DIVISOR.

118. A COMMON DIVISOR of two or more numbers is *any number that will divide each of them without remainder;* thus 3 is a common divisor of 12, 18, and 30.

119. The GREATEST COMMON DIVISOR of two or more numbers is the *greatest* number that will divide each of them without remainder; thus, 6 is the greatest common divisor of 12, 18, and 30.

NOTE. A *divisor* of a number is often called *a measure* of the number, also an *aliquot part* of the number.

120. PROBLEM 2. To find the greatest common divisor of two or more numbers.

Ex. 1. What is the greatest common divisor of 18, 30, and 48?

Ans. $2 \times 3 = 6$.

OPERATION.

$18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$
$48 = 2 \times 3 \times 2 \times 2 \times 2$

We see that 2 and 3 are factors *common* to all the numbers, and, furthermore, they are the *only* common factors; hence their product, $2 \times 3 = 6$, is the greatest common divisor of the given numbers.

2. What is the greatest common divisor of 60, 72, 48, and 84?

Ans. $2 \times 2 \times 3 = 12$.

OPERATION.

$60 = 2 \times 2 \times 3 \times 5$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
$84 = 2 \times 2 \times 3 \times 7$

Although 2 is a factor *more than twice* in *some* of the given numbers, yet, as it is a factor *only* twice in *others*, we are *not* at liberty to take 2 *more than* twice

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117. Composite factors, how formed? **118.** What is a Common Divisor? **119.** Greatest Common Divisor? Other names for divisor?

in finding the greatest common divisor. The same remark applies to other factors. Hence,

RULE 1. *Resolve each number into its prime factors, and the continued product of all the prime factors that are common to all the given numbers will be the common divisor sought.*

3. What is the greatest common divisor of 24, 40, 64, 80, 96, 120, and 192? Ans. $2 \times 2 \times 2 = 8$.

4. Find the greatest common divisor of 15, 45, 75, 105, 135, 150, and 300. Ans. 15.

5. Find the greatest common divisor of 25, 45, and 70. Ans. 5.

6. Find the greatest common divisor of 24, 36, and 64. Ans. 4.

7. Find the greatest common divisor of 24, 48, 72, and 88.

8. Find the greatest common divisor of 45, 75, 90, 135, 150, and 180.

9. I have three rooms, the first 11ft. 3in. wide, the second 15ft. 9in. wide, and the third 18ft. wide; how wide is the widest carpeting which will exactly fit each room? How many breadths will be required to cover each room?

1st Ans. 27 inches.

121. When the given numbers are not readily resolved into their prime factors, their greatest common divisor may be more easily found by

RULE 2. *Divide the greater of two numbers by the less, and, if there be a remainder, divide the divisor by the remainder, and continue dividing the last divisor by the last remainder until nothing remains; the last divisor is the greatest common divisor of the two numbers.*

If more than two numbers are given, find the greatest divisor of two of them, then of this divisor and a third number, and so on until all the numbers have been taken; the last divisor will be the divisor sought.

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120. Rule for finding the greatest common divisor of two or more numbers?
121. Second rule for finding greatest common divisor?

10. What is the greatest common divisor of 14 and 20?

OPERATION.

```
        14)20(1
           14
           --
            6)14(2
              12
              --
Ans.           2)6(3
                 6
                 -
                 0
```

Before explaining this operation, *four* principles may be stated, viz.:

(a) Every number is a divisor of itself, the quotient being *one;* thus, 3 is contained in 3 *once;* 7 in 7 *once.*

(b) If one number divides another, the 1st will divide any number of times the 2d; thus, since 3 divides 12, it will divide 5 times 12, or *any number* of times 12.

(c) If a number divides each of two numbers, it will divide their *sum* and also their *difference;* thus, since 6 is contained *five* times in 30, and *twice* in 12, it is contained $5 + 2 = 7$ times in $30 + 12 = 42$; and $5 - 2 = 3$ times in $30 - 12 = 18$.

(d) Not only will the greatest common divisor of two numbers divide their difference, but unless one of the numbers is a divisor of the other, it will also divide what remains after one of the numbers has been taken from the other as many times as possible; thus, the greatest divisor of 6 and 22 will divide $22 - 3 \times 6 = 4$.

122. It may now be shown, 1st, that 2 is *a common divisor* of 14 and 20, and 2d, that it is their *greatest* common divisor.

First, 2 divides 6, ∴ (Art. 121, b) 2 divides $6 \times 2 = 12$, and (Art. 121, c) 2 divides $2 + 12 = 14$; again, since 2 divides 6 and 14 (Art. 121, c) it divides $6 + 14 = 20$; i. e. 2 divides both 14 and 20.

Second, The greatest divisor of 14 and 20 (Art. 121, c) must divide $20 - 14 = 6$, ∴ it *cannot be greater* than 6; again, the greatest divisor of 6 and 14 (Art. 121, d) must divide 14 —

---

121. First principle? Second? Third? Fourth? **122.** Explain why 2 is *a* common divisor of 14 and 20. Why it is their *greatest* common divisor.

$6 \times 2 = 2$, ∴ the greatest common divisor of 14 and 20 *cannot exceed* 2, and, as it has been previously shown that 2 *is* a divisor of 14 and 20, *it is their greatest common divisor.*

A similar explanation is applicable in all cases.

123. It will be seen that, in finding the common divisor of 14 and 20, we are led to find the divisor of 6 and 14, then of 2 and 6; i. e. in any example we seek to find the measure of the remainder and divisor, then of the *next* remainder and divisor, and so on, until the greatest measure of the last remainder and the divisor which gave that remainder is found, and this measure will be the greatest common divisor of the two given numbers. Thus the question becomes more and more simple as each successive step in the operation is taken.

11. What is the greatest common divisor of 3432 and 4760?

OPERATION.

	Quotients.	4760
3432	× 1 =	3432
2656	= 2 ×	1328
776	× 1 =	776
552	= 1 ×	552
224	× 2 =	448
208	= 2 ×	104
16	× 6 =	96
16	= 2 ×	8
0	.	

The *plan* of the operation in Ex. 10 requires more space and more time than this in Ex. 11, though the principle and the reasoning are precisely the same in both.

In Ex. 11 we first divide 4760 by 3432, and obtain 1 for quotient and 1328 for remainder; then divide 3432 by 1328, obtaining 2 for quotient, and 776 for remainder; and so proceed, dividing the last divisor by the last remainder, as directed in Rule 2, until the remainder is 0. The last divisor, 8, is the greatest common divisor of 3432 and 4760.

12. What is the greatest common divisor of 1430 and 3549?

Ans. 13.

13. What is the greatest common divisor of 3640 and 5733?

14. What is the greatest common divisor of 1440 and 3696?

15. What is the greatest common divisor of 2520 and 6237?

---

123. Explain the operation in Ex. 11.

16. What is the greatest common divisor of 16, 24, and 36?

FIRST OPERATION.

```
16)24(1
   16
   --
    8)16(2
      16
      --
       0

Again,  8)36(4
          32
          --
     Ans. 4)8(2
            8
            -
            0
```

SECOND OPERATION.

```
24)36(1
   24
   --
   12)24(2
      24
      --
       0

Again, 12)16(1
          12
          --
     Ans. 4)12(3
            12
            --
             0
```

In solving Ex. 16, we first find the divisor of 16 and 24, viz. 8, and then find the divisor of 8 and 36; or first find the divisor of 24 and 36, viz. 12, and then of 12 and 16; or we might first find the divisor of 16 and 36, and then of that divisor and 24.

17. What is the greatest common divisor of 84, 96, 144, and 174?

18. What is the greatest common divisor of 77, 105, and 140?

19. What is the greatest common divisor of 9 and 16?

Ans. 1.

20. What is the greatest common divisor of 9, 12, and 20?

LEAST COMMON MULTIPLE.

124. A MULTIPLE of a number is any number which is *divisible* by that number; thus, 15 is a multiple of 5 and also of 3; 21 is a multiple of 7 and of 3.

NOTE. Every number is both a divisor and a multiple of itself.

125. A COMMON MULTIPLE of two or more numbers, is any number which is divisible by each of the given numbers; thus, 48 is a common multiple of 4, 6, and 8.

---

123. How is Ex. 16 solved? **124.** What is a Multiple of a number? **125.** A Common Multiple of two or more numbers?

126. The LEAST COMMON MULTIPLE of two or more numbers, is the *least* number that is divisible by each of the given numbers; thus, 24 is the least common multiple of 4, 6, and 8.

NOTE. There is no such thing as a least common divisor, or greatest common multiple.

127. PROBLEM 3. To find the least common multiple of two or more numbers.

Ex. 1. What is the least common multiple of 20, 24, and 36?

Ans. $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

OPERATION.

$20 = 2 \times 2 \times 5$
$24 = 2 \times 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$

Since 360 contains all the factors of 20, 24, and 36, respectively, it, evidently, is divisible by each of those numbers. It is also evident that no number less than 360 will contain 20, 24, and 36, for if one of the 2's in the common multiple were omitted, it would not contain 24; if one of the 3's, it would not contain 36; and if the 5 were omitted, it would not contain 20.

Similar reasoning applies in all examples. Hence,

RULE 1. *Resolve each number into its prime factors, and the continued product of all the different prime factors, each taken the greatest number of times it occurs in either of the given numbers, will be the least common multiple.*

2. What is the least common multiple of 12, 16, 20, and 30?

Ans. $2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240$.

3. What is the least common multiple of 22, 33, and 55?
4. What is the least common multiple of 16, 36, 40, and 48?
5. What is the least common multiple of 20, 30, 50, and 80?
6. What is the least common multiple of 15, 25, 45, and 75?
7. What is the least common multiple of 35, 50, 75, and 90?
8. What is the least common multiple of 24, 36, 48, and 64?
9. What is the least common multiple of 72, 80, 84, and 96?
10. What is the least common multiple of 42, 49, 72, and 88?

---

126. The Least Common Multiple? May numbers have a *least common divisor? Greatest* common multiple? **127.** Rule for finding the least common multiple? Reason?

128. The same result is sometimes more easily attained by

RULE 2. *Having set the given numbers in a line, divide by any* PRIME *number that will divide two or more of them, and set the quotients and undivided numbers in a line beneath; proceed with this line as with the first, and so continue until no two of the numbers can be divided by any number greater than one; the continued product of the divisors and numbers in the last line will be the multiple sought.*

The second rule may be illustrated by the example already employed in explaining the first rule, viz.:

What is the least common multiple of 20, 24, and 36?

Ans. $2 \times 2 \times 3 \times 5 \times 2 \times 3 = 360$.

OPERATION.

$$\begin{array}{r|rrr} 2 & 20, & 24, & 36 \\ \hline 2 & 10, & 12, & 18 \\ \hline 3 & 5, & 6, & 9 \\ \hline & 5, & 2, & 3 \end{array}$$

If the process by the 1st rule be examined it will be seen that the factor 2 is found 7 times in the given numbers, and as 2 is taken but 3 times in finding the multiple, it is rejected 4 times. By the 2d rule, also, 2 is rejected 4 times, viz. twice in the 1st division by 2 and twice in the 2d division by 2. The learner may think 2 is rejected *three* times in each of the two first divisions, but he must remember that the *divisor*, 2, *is retained* as a factor in the common multiple in each instance.

Similar remarks are applicable to all rejected factors in like examples, ∴ the two rules give the same result.

11. What is the least common multiple of 5, 16, 24, 32, and 48?

Ans. $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 480$.

OPERATION.

By Rule 1.

$$\begin{aligned} 5 &= 5 \\ 16 &= 2 \times 2 \times 2 \times 2 \\ 24 &= 2 \times 2 \times 2 \times 3 \\ 32 &= 2 \times 2 \times 2 \times 2 \times 2 \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3 \end{aligned}$$

By Rule 2.

$$\begin{array}{r|rrrrr} 2 & 5, & 16, & 24, & 32, & 48 \\ \hline 2 & 5, & 8, & 12, & 16, & 24 \\ \hline 2 & 5, & 4, & 6, & 8, & 12 \\ \hline 2 & 5, & 2, & 3, & 4, & 6 \\ \hline 3 & 5, & 1, & 3, & 2, & 3 \\ \hline & 5, & 1, & 1, & 2, & 1 \end{array}$$

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128. Second Rule? Explanation?

NOTE 1. The principle, which is the same in the two rules, is most readily perceived by the first operation.

12. What is the least common multiple of 30, 40, 45, and 75?

13. What is the smallest sum of money with which I can buy horses at \$50 each, cows at \$30 each, or sheep at \$8 each, using the same sum in each case? Ans. \$600.

14. I have 4 wine measures; the first holds 4 quarts, the second 5 quarts, the third 6 quarts, and the fourth 8 quarts; what is the size of the smallest cask that can be exactly measured by means of each of these measures? Ans. 120 quarts.

15. What is the least common multiple of 10, 15, 45, 75, and 90?

In solving Ex. 15, it is evident that 10, 15, and 45 may at once be struck out, for each of these numbers is a measure of 90, and ∴ whatever multiple of 75, and 90 is found, *it*, certainly, must be a multiple of 10, 15, and 45; hence, the question is reduced to this: What is the least common multiple of 75 and 90?

NOTE 2. Many other abbreviations of this and other rules may be effected, but a delicate perception of the relations of numbers, and a skillful application of principles, will much more facilitate the progress of the learner than any set of formal rules.

(a) If the numbers are prime, or even mutually prime, their product is their least common multiple.

16. What is the least common multiple of 9 and 10?

Ans. $9 \times 10 = 90$.

17. What is the least common multiple of 8, 9, and 25?

(b) The least common multiple of *two* numbers is equal to their product divided by their greatest common divisor.

18. What is the least common multiple of 12 and 20?

The greatest common divisor of 12 and 20 is 4, and

The least common multiple is $12 \times 20 \div 4 = 60$, Ans.

19. What is the least common multiple of 63 and 72?

20. What is the least common multiple of 33 and 77?

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128. Ex. 15, how solved? What of other abbreviations? Least common multiple of *mutually prime* numbers? Of *two* numbers?

COMMON FRACTIONS.

129. A FRACTION is an expression representing one or more of the equal parts of a unit.

NOTE. A unit, or any other whole number, is often called an *Integer;* it is also called an *Integral* or *Entire Number.*

130. A COMMON or VULGAR FRACTION is expressed by two numbers, one above and the other below a line; thus $\frac{1}{2}$ (one half), $\frac{2}{5}$ (two fifths), etc.

(a) The number below the line shows *into how many parts the unit is divided,* and is called the DENOMINATOR, because it *denominates* or *gives name* to the parts; thus, if a unit is divided into 3 equal parts, each part is one *third;* if into 8, each part is one *eighth;* etc?

(b) The number above the line is called the NUMERATOR, because it *numerates* or *numbers* the parts *taken.*

(c) The numerator and the denominator are the TERMS of the fraction.

131. A fraction is nothing more nor less than *unexcuted division,* i. e. *division indicated but not performed,* the *numerator* being the *dividend,* and the *denominator* the *divisor.* Hence,

(a) The value of a fraction is the quotient of the numerator, divided by the denominator; thus, $\frac{12}{4} = 12 \div 4 = 3$; and, $\therefore$,

(b) *Any change in the* NUMERATOR *causes a* LIKE *change in the value of the fraction, and any change in the* DENOMINATOR *causes an* OPPOSITE *change in the value of the fraction* (Art. 84).

These principles are developed in the following *Problems.*

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129. What is a Fraction? Other names for a whole number? **130.** A Common Fraction, how expressed? Number below the line, what called? Why? Number above, what called? Why? Terms of a fraction, what? **131.** A fraction, what is it? Value of a fraction? What follows?

132. A PROPER FRACTION is one whose numerator is *less* than the denominator; as, $\frac{2}{3}$, $\frac{7}{11}$, $\frac{8}{21}$.

133. An IMPROPER FRACTION is one whose numerator *equals* or *exceeds* its denominator; as, $\frac{4}{4}$, $\frac{7}{7}$, $\frac{8}{5}$, $\frac{26}{13}$. An improper fraction *equals or exceeds a unit;* hence its *name,* IMPROPER *fraction.*

134. A SIMPLE FRACTION has but one numerator and one denominator, and is either *proper* or *improper;* as, $\frac{3}{5}$, $\frac{6}{6}$, $\frac{12}{7}$.

135. A COMPOUND FRACTION is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{7}{11}$, $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{6}{9}$.

136. A MIXED NUMBER is a whole number and a fraction united; as, $3\frac{4}{5}$, $20\frac{3}{8}$.

137. A COMPLEX FRACTION is one that has a fraction or a mixed number for one or for each of its terms; as, $\frac{3\frac{1}{4}}{7}$, $\frac{\frac{2}{5}}{6}$, $\frac{3}{2\frac{1}{2}}$, $\frac{7}{\frac{2}{4}}$, $\frac{8\frac{1}{3}}{7\frac{2}{6}}$, $\frac{\frac{3}{5}}{8\frac{3}{11}}$, $\frac{\frac{4}{7}}{\frac{2}{5}}$.

138. The RECIPROCAL of a number is a fraction whose numerator is 1, and whose denominator is the number itself; thus, the reciprocals of 4, 9, and $\frac{5}{7}$ are $\frac{1}{4}$, $\frac{1}{9}$, and $\frac{1}{\frac{5}{7}}$.

PROBLEM 1.

139. To reduce a mixed number to an improper fraction.

Ex. 1. In $3\frac{1}{4}$ how many fourths? Ans. $\frac{13}{4}$.

OPERATION.

$$\begin{array}{r} 3\frac{1}{4} \\ 4 \\ \hline \frac{13}{4}, \text{ Ans.} \end{array}$$

Since 4 fourths make a unit, there will be 4 times as many fourths as units, therefore, in three units there will be 4 times 3 fourths = 12 fourths, and the 1 fourth in the example added to the 12 fourths, gives 13 fourths, i. e. $\frac{13}{4}$. Hence,

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132. A Proper Fraction, what? 133. An Improper Fraction? 134. A Simple Fraction? 135. A Compound Fraction? 136. A Mixed Number? 137. A Complex Fraction? 138. The Reciprocal of a Number? 139. Explain the Operation in Ex. 1.

RULE. *Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.*

2. In $5\frac{4}{7}$ how many sevenths? Ans. $\frac{39}{7}$.
3. In $8\frac{3}{5}$ how many fifths? Ans. $\frac{43}{5}$.
4. In $\$7\frac{3}{4}$ how many fourths of a dollar? Ans. $\frac{31}{4}$.
5. Reduce $6\frac{5}{7}$ to an improper fraction. Ans. $\frac{47}{7}$.
6. Reduce $9\frac{5}{7}$ to an improper fraction. Ans. $\frac{68}{7}$.
7. Reduce $5\frac{7}{8}$.
8. Reduce $9\frac{3}{5}$.
9. Reduce $11\frac{5}{7}$.
10. Reduce $13\frac{5}{8}$.
11. Reduce $12\frac{7}{9}$.
12. Reduce $15\frac{5}{6}$.
13. Reduce $17\frac{3}{8}$.
14. Reduce $14\frac{7}{12}$.
15. Reduce $16\frac{4}{25}$.
16. Reduce $18\frac{5}{9}$.
17. Reduce $19\frac{3}{10}$.
18. Reduce $16\frac{8}{9}$.
19. Reduce $20\frac{6}{7}$.
20. Reduce $25\frac{13}{24}$.
21. Reduce $37\frac{3}{8}$.
22. Reduce $46\frac{7}{18}$.
23. Reduce $54\frac{3}{26}$.
24. Reduce $84\frac{5}{24}$.
25. Reduce $92\frac{5}{18}$.
26. Reduce $99\frac{27}{34}$.

(a) To reduce an integer to a fraction having any given denominator:

Multiply the integer by the proposed denominator, and under the product write the denominator (Art. 84, c).

27. Reduce 12 to a fraction whose denominator is 7. Ans. $\frac{84}{7}$.
28. Reduce 9 to a fraction whose denominator is 8.
29. Reduce 9 to a fraction whose denominator is 5.
30. Reduce 7 to a fraction whose denominator is 1. Ans. $\frac{7}{1}$.
31. Reduce 87 to a fraction whose denominator is 87.
32. Reduce 16 to a fraction whose denominator is 1.
33. Reduce 16 to a fraction whose denominator is 4.
34. Reduce 20 to a fraction whose denominator is 4.
35. Reduce 14 to five different fractional forms.

---

139. Rule for reducing a mixed number to an improper fraction? Reason? An integer, how reduced to a fractional form?

PROBLEM 2.

140. To reduce an improper fraction to a whole or mixed number.

Ex. 1. How many units in $\frac{13}{4}$? Ans. $3\frac{1}{4}$.

$\frac{13}{4} = 13 \div 4 = 3\frac{1}{4}$, Ans.

Since the numerator is a dividend and the denominator a divisor (Art. 131), the fraction is reduced to an equivalent whole or mixed number by the following

RULE. *Divide the numerator by the denominator; if there is any remainder, place it over the divisor, and annex the fraction so formed to the quotient.*

2. Reduce $\frac{37}{12}$ to a whole or mixed number. Ans. $3\frac{1}{12}$.
3. Reduce $\frac{66}{22}$ to a whole or mixed number. Ans. 3.
4. Reduce $\frac{84}{37}$ to a whole or mixed number. Ans. $2\frac{10}{37}$.
5. Reduce $\frac{776}{29}$ to a whole or mixed number. Ans. $26\frac{22}{29}$.
6. Reduce $\frac{837}{134}$.
7. Reduce $\frac{364}{354}$.
8. Reduce $\frac{783}{89}$.
9. Reduce $\frac{74}{1}$.
10. Reduce $\frac{86}{32}$.
11. Reduce $\frac{1728}{24}$.

PROBLEM 3.

141. To reduce a fraction to its lowest terms.

Ex. 1. Reduce $\frac{36}{48}$ to its lowest terms. Ans. $\frac{3}{4}$.

FIRST OPERATION.

$\frac{36}{48} = \frac{12}{16} = \frac{3}{4}$, Ans.

Dividing both terms of a fraction by any number does not alter the value of the fraction (Art. 84, b, and 131); ∴ dividing each term of $\frac{36}{48}$ by 3 gives the equal fraction $\frac{12}{16}$; then dividing each term of this result by 4 gives $\frac{3}{4}$, and as 3 and 4 are mutually prime (Art. 112), $\frac{36}{48}$, in its lowest terms, equals $\frac{3}{4}$.

SECOND OPERATION.

$12\,)\,\frac{36}{48} = \frac{3}{4}$, Ans.

In this operation both terms of the fraction $\frac{36}{48}$ are divided by their greatest common divisor, 12 (Art. 119), and thus the fraction is reduced at once to its lowest terms. Hence,

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140. Rule for reducing an improper fraction to a whole or mixed number? Reason?

RULE 1. *Divide each term by any factor common to them; then divide these quotients by any factor common to* THEM, *and so proceed till the quotients are mutually prime.* Or,

RULE 2. *Divide each term by their greatest common divisor.*

2. Reduce $\frac{66}{44}$ to its lowest terms. Ans. $\frac{3}{2}$.
3. Reduce $\frac{49}{56}$ to its lowest terms. Ans. $\frac{7}{8}$.
4. Reduce $\frac{8}{32}$ to its lowest terms. Ans. $\frac{1}{4}$.
5. Reduce $\frac{144}{1728}$ to its lowest terms. Ans. $\frac{1}{12}$.
6. Reduce $\frac{96}{144}$.
7. Reduce $\frac{72}{81}$.
8. Reduce $\frac{844}{972}$.
9. Reduce $\frac{565}{875}$.
10. Reduce $\frac{333}{698}$.
11. Reduce $\frac{81}{243}$.
12. Reduce $\frac{12}{288}$.
13. Reduce $\frac{864}{984}$.
14. Reduce $\frac{625}{3125}$.
15. Reduce $\frac{896}{484}$.

PROBLEM 4.

142. To multiply a fraction by a whole number.

Ex. 1. Multiply $\frac{2}{15}$ by 3. Ans. $\frac{6}{15}$ or $\frac{2}{5}$.

FIRST OPERATION.

$\frac{2}{15} \times 3 = \frac{6}{15}$, Ans.

It is just as evident that 3 times $\frac{2}{15}$ are $\frac{6}{15}$ as that 3 times 2 cents are 6 cents, or that 3 times 2 are 6; i. e. when the numerator is multiplied by 3 the fraction represents 3 times as many parts as before, and each part continues of the same size; ∴ the fraction is multiplied by 3.

SECOND OPERATION.

$\frac{2}{15} \times 3 = \frac{2}{5}$, Ans.

If the denominator is divided by 3, the fraction represents just as many parts as before, *but each part is three times as great*, and ∴ the whole fraction is three times as great. Hence,

RULE 1. *Multiply the numerator by the whole number.* Or,

RULE 2. *Divide the denominator by the whole number.*

NOTE 1. The correctness of Rule 1 is also evident from Art. 83 (a), and Art. 131. Rule 2 also depends on Art. 83 (d).

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141. First rule for reducing a fraction to its lowest terms? Second rule? Reason? **142.** First rule for multiplying a fraction by a whole number? Why? Second rule? Why? Another reason?

2. Multiply $\frac{3}{12}$ by 3. Ans. $\frac{9}{12}$ or $\frac{3}{4}$.

NOTE 2. The second rule is preferable in this and all similar examples, because it gives the fraction in *smaller terms*.

3. Multiply $\frac{7}{15}$ by 5. Ans. $\frac{7}{3}$.
4. Multiply $\frac{9}{77}$ by 11. Ans. $\frac{9}{7}$ or $1\frac{2}{7}$.
5. Multiply $\frac{3}{17}$ by 4.

$$\tfrac{3}{17} \times 4 = \tfrac{12}{17}, \text{ by Rule 1; or,}$$

$$\tfrac{3}{17} \times 4 = \frac{3}{4\frac{1}{4}}, \text{ by Rule 2.}$$

NOTE 3. The first rule is preferable for this and all similar examples, because the second gives *a complex fraction*.

6. Multiply $\frac{3}{19}$ by 4. Ans. $\frac{12}{19}$ or $\dfrac{3}{4\frac{3}{4}}$.
7. Multiply $\frac{5}{37}$ by 6. Ans. $\frac{30}{37}$.
8. Multiply $\frac{9}{37}$ by 4. Ans. $\frac{36}{37}$.
9. Multiply $\frac{13}{85}$ by 3. Ans. $\frac{39}{85}$.
10. Multiply $\frac{15}{79}$ by 5. Ans. $\frac{75}{79}$.
11. Multiply $\frac{7}{29}$ by 4.
12. Multiply $\frac{9}{53}$ by 5.
13. Multiply $\frac{19}{1667}$ by 15. Ans. $\frac{285}{1667}$.
14. Multiply $\frac{8}{25}$ by 15. $15 = 5 \times 3$.

$$\tfrac{8}{25} \times 5 = \tfrac{8}{5}\text{; and } \tfrac{8}{5} \times 3 = \tfrac{24}{5}, \text{ Ans.}$$

NOTE 4. We may here, as in whole numbers (Art. 61), use the factors of the multiplier, and in using these factors we may apply the 1st or the 2d rule, or both.

15. Multiply $\frac{12}{78}$ by 66. $66 = 6 \times 11$.

$$\tfrac{12}{78} \times 6 = \tfrac{12}{13}\text{; and } \tfrac{12}{13} \times 11 = \tfrac{132}{13}, \text{ Ans.}$$

16. Multiply $\frac{23}{36}$ by 42. Ans. $\frac{161}{6}$.
17. Multiply $\frac{73}{108}$ by 84.
18. Multiply $\frac{156}{33}$ by 44.

(a) If we multiply a fraction by its denominator, the product will be the numerator.

19. Multiply $\frac{7}{8}$ by 8. Ans. $\frac{7}{8} \times 8 = \frac{7}{1} = 7$, by Rule 2.
20. Multiply $\frac{63}{44}$ by 44.

---

142. May the factors of the multiplier be used? What is the product if a fraction is multiplied by its denominator?

(b) To multiply a mixed number by an integer:

Multiply the fractional part and the entire part separately, and add the products together; or, *reduce the mixed number to an improper fraction* (Art. 139), *and then multiply.*

21. Multiply $3\frac{4}{5}$ by 5. Ans. 19.

First multiply $\frac{4}{5}$ by 5 and the product is 4; then multiply 3 by 5 and the product is 15. These partial products added give $15 + 4 = 19$ for the true product. Or, first reduce $3\frac{4}{5}$ to $\frac{19}{5}$ and then multiply by 5 and the product is 19, as before.

22. Multiply $8\frac{3}{7}$ by 9.

$\frac{3}{7} \times 9 = 3\frac{6}{7}$; $8 \times 9 = 72$; and $72 + 3\frac{6}{7} = 75\frac{6}{7}$., Ans.

23. Multiply $9\frac{5}{11}$ by 12. Ans. $113\frac{5}{11}$.

24. Multiply $18\frac{4}{5}$ by 20.

25. Multiply $23\frac{4}{7}$ by 7.

PROBLEM 5.

143. To divide a fraction by a whole number.

Ex. 1. Divide $\frac{8}{9}$ by 4. Ans. $\frac{2}{9}$ or $\frac{8}{36}$.

FIRST OPERATION.

$\frac{8}{9} \div 4 = \frac{2}{9}$, Ans.

It is just as evident that one fourth of $\frac{8}{9}$ is $\frac{2}{9}$ as that one fourth of 8 cents is 2 cents, or that one fourth of 8 is 2; i. e. when the numerator is divided by 4 the fraction represents only one fourth as many parts as before, and each part continues of the same size; ∴ the fraction is divided by 4.

SECOND OPERATION.

$\frac{8}{9} \div 4 = \frac{8}{36}$, Ans.

If the denominator is multipled by 4, the fraction represents just as many parts as before, *but each part is only one fourth as great,* and ∴ the whole fraction is only one fourth as great. Hence,

RULE 1. *Divide the numerator by the whole number.* Or,

RULE 2. *Multiply the denominator by the whole number.*

NOTE 1. These rules may also be explained by Art. 83 (b) and (c).

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142. How is a mixed number multiplied by an integer? Another way? **143.** First rule for dividing a fraction by a whole number? Why? Second rule? Why? Another explanation?

2. Divide $\frac{12}{17}$ by 2. Ans. $\frac{6}{17}$ by Rule 1 ; $\frac{12}{34}$ by Rule 2.

NOTE 2. The 1st rule is preferable in this example. Why?

3. Divide $\frac{24}{29}$ by 6. Ans. $\frac{4}{29}$.
4. Divide $\frac{33}{95}$ by 11.
5. Divide $\frac{75}{83}$ by 25.
6. Divide $\frac{144}{253}$ by 12.
7. Divide $\frac{23}{15}$ by 4.

$$\tfrac{23}{15} \div 4 = \frac{5\frac{3}{4}}{15}, \text{ by Rule 1 ; or,}$$

$$\tfrac{23}{15} \div 4 = \tfrac{23}{60}, \text{ by Rule 2.}$$

NOTE 3. The 2d rule is preferable in this example. Why?

8. Divide $\frac{17}{25}$ by 5. Ans. $\frac{17}{125}$.
9. Divide $\frac{63}{25}$ by 11. Ans. $\frac{63}{275}$.
10. Divide $\frac{12}{44}$ by 6.
11. Divide $\frac{23}{24}$ by 4.
12. Divide $\frac{8}{25}$ by 20. $20 = 4 \times 5$.

$\frac{8}{25} \div 4 = \frac{2}{25}$, and $\frac{2}{25} \div 5 = \frac{2}{125}$, Ans.

NOTE 4. See Art. 142, Note 4.

13. Divide $\frac{15}{73}$ by 35. $35 = 5 \times 7$.

$\frac{15}{73} \div 5 = \frac{3}{73}$, and $\frac{3}{73} \div 7 = \frac{3}{511}$, Ans.

14. Divide $\frac{27}{35}$ by 18. Ans. $\frac{3}{70}$.
15. Divide $\frac{35}{46}$ by 14. Ans. $\frac{5}{92}$.
16. Divide $\frac{624}{7}$ by 44.

(a) To divide a mixed number by a whole number.

17. Divide $23\frac{1}{5}$ by 4. Ans. $5\frac{4}{5}$.

OPERATION.

$$4\,)\,\underline{23\tfrac{1}{5}}$$

Quo., $5 \ldots 3\frac{1}{5}$ Rem.

$3\frac{1}{5} = \frac{16}{5}$, and $\frac{16}{5} \div 4 = \frac{4}{5}$,

$\therefore 5 + \frac{4}{5} = 5\frac{4}{5}$, Ans.

First divide as in Art. 74, Ex. 35, and obtain the quotient, 5, and the remainder, $3\frac{1}{5}$. Then reduce $3\frac{1}{5}$ to the improper fraction, $\frac{16}{5}$, divide it by 4, and add or annex the result, $\frac{4}{5}$, to the partial quotient, 5, and we have $5\frac{4}{5}$ for the true quotient.

---

143. May the factors of the divisor be used separately? A mixed number, how divided by an integer?

18. Divide $27\frac{3}{7}$ by 6. Ans. $4\frac{4}{7}$.
19. Divide $17\frac{3}{5}$ by 9. Ans. $1\frac{43}{45}$.
20. Divide $65\frac{4}{11}$ by 8.
21. Divide $5\frac{3}{8}$ by 7.

$$5\frac{3}{8} = \frac{43}{8};\ \frac{43}{8} \div 7 = \frac{43}{56}, \text{ Ans.}$$

NOTE 5. In Ex. 21, the dividend is less than the divisor; hence the quotient is a proper fraction.

22. Divide $7\frac{3}{11}$ by 9. Ans. $\frac{80}{99}$.
23. Divide $5\frac{3}{4}$ by 11.
24. Divide $\$6\frac{3}{4}$ equally between 9 boys.

PROBLEM 6.

144. To multiply a fraction by a fraction.

Ex. 1. Multiply $\frac{2}{7}$ by $\frac{3}{5}$. Ans. $\frac{6}{35}$.

To multiply $\frac{2}{7}$ by $\frac{3}{5}$, 1st, $\frac{2}{7} \times 3 = \frac{6}{7}$ (Art. 142, Rule 1); but the multiplier, 3, is 5 times $\frac{3}{5}$, $\therefore$ the product, $\frac{6}{7}$, is 5 times the product sought; hence, 2d, $\frac{6}{7} \div 5 = \frac{6}{35}$ (Art. 143, Rule 2) is the product sought; i. e.

$$\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}. \quad \text{Hence,}$$

RULE. *Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

2. Multiply $\frac{3}{11}$ by $\frac{2}{7}$. Ans. $\frac{6}{77}$.
3. Multiply $\frac{5}{13}$ by $\frac{7}{8}$. Ans. $\frac{35}{104}$.
4. Multiply $\frac{4}{9}$ by $\frac{7}{15}$. Ans. $\frac{28}{135}$.
5. Multiply $\frac{13}{15}$ by $\frac{7}{11}$.
6. Multiply $\frac{23}{17}$ by $\frac{18}{25}$.

(a) To multiply by a fraction is only to multiply by the numerator, and then divide the product by the denominator.

In Ex. 7 we multiply $\frac{12}{35}$ by 5, and obtain $\frac{12}{7}$ (Art. 142, Rule 2), and then $\frac{12}{7}$ divided by 6 gives $\frac{2}{7}$ (Art. 143, Rule 1), the result sought.

---

144. Rule for multiplying one fraction by another? Reason? To multiply by a fraction, what is it? What principles in the operation in Ex. 7?

7. Multiply $\frac{12}{35}$ by $\frac{5}{6}$.

$$\frac{\overset{2}{\cancel{12}}}{\underset{7}{\cancel{35}}} \times \frac{\cancel{5}}{\cancel{6}} = \frac{2}{7},\ \text{Ans.}$$

In this simple operation is involved THE WHOLE PRINCIPLE OF CANCELING. To *cancel* (i. e. *strike out*, or *reject*) any factor of a number, is to *divide* the number by the rejected factor; thus, 35 is the same as 5×7, and if the 5 is *canceled*, there will remain only 7, which is the *quotient* of 35 *divided* by 5.

8. Multiply $\frac{12}{35}$ by $\frac{14}{27}$.

$$\frac{\overset{4}{\cancel{12}}}{\underset{5}{\cancel{35}}} \times \frac{\overset{2}{\cancel{14}}}{\underset{9}{\cancel{27}}} = \frac{8}{45},\ \text{Ans.}$$

The 8th example is solved on the same principle as the 7th. It may be written thus, $\frac{12 \times 14}{35 \times 27}$, which is the same as $\frac{4 \times 3 \times 2 \times 7}{5 \times 7 \times 9 \times 3}$, and then *canceling* 3 and 7, i. e. *dividing* both numerator and denominator by 3 and 7 (Art. 84, b, and 131) we have $\frac{4 \times 2}{5 \times 9} = \frac{8}{45}$.

NOTE. There can be no difficulty in canceling so long as we remember the simple *principle*, that it rests upon rejecting equal factors from dividend and divisor (Art. 84, b). The *process* is only to *strike out* or *cancel* the same factors from numerator and denominator, and it often saves much labor. It can be profitably applied whenever the product of two or more numbers is to constitute a dividend, and the product of other numbers is to constitute a divisor, provided that there are equal factors in the dividend and divisor.

9. Multiply $\frac{46}{85}$ by $\frac{25}{23}$.

$$\frac{\overset{2}{\cancel{46}}}{\underset{17}{\cancel{85}}} \times \frac{\overset{5}{\cancel{25}}}{\cancel{23}} = \frac{10}{17},\ \text{Ans.}$$

In this example, cancel 23 with 46, giving 2 in the numerator; and then cancel 5 in 25 and 85, giving 5 in the numerator and 17 in the denominator.

---

144. Explain Ex. 7. On what principles does canceling rest? When should it be applied?

10. Multiply $\frac{25}{44}$ by $\frac{8}{75}$. Ans. $\frac{2}{33}$.
11. Multiply $\frac{24}{35}$ by $\frac{28}{84}$.
12. Multiply $\frac{33}{64}$ by $\frac{12}{44}$.
13. Multiply $\frac{36}{125}$ by $\frac{75}{84}$.

(b) In canceling 3 and 5 in Example 14, we obtain the quotients 1 and 1 in the numerators, and whenever an entire term cancels we obtain 1 to place instead of the term canceled; but since 1, as a multiplier or divisor, is valueless, there is no need of retaining it under any circumstances, except where all the numerators are canceled; in such a case, 1 *is the true numerator, and must be retained.*

14. Multiply $\frac{3}{25}$ by $\frac{5}{12}$.

$$\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{12}}} = \frac{1}{20}, \text{ Ans.}$$

15. Multiply $\frac{144}{365}$ by $\frac{73}{288}$.

$$\frac{\cancel{144}}{\underset{5}{\cancel{365}}} \times \frac{\overset{1}{\cancel{73}}}{\underset{2}{\cancel{288}}} = \frac{1}{10}, \text{ Ans.}$$

16. Multiply $\frac{25}{3}$ by $\frac{12}{5}$.

$$\frac{\overset{5}{\cancel{25}} \times \overset{4}{\cancel{12}}}{\cancel{3} \times \cancel{5}} = 20, \text{ Ans.}$$

17. Multiply $\frac{66}{77}$ by $\frac{231}{43}$.
18. Multiply $\frac{64}{85}$ by $\frac{17}{192}$.
19. Multiply $\frac{81}{25}$ by $\frac{75}{27}$. Ans. 9.
20. Multiply $\frac{25}{81}$ by $\frac{27}{75}$. Ans. $\frac{1}{9}$.
21. Multiply $\frac{32}{125}$ by $\frac{25}{96}$.

(c) To reduce a compound fraction to a simple one.

22. What part of an apple is $\frac{5}{7}$ of $\frac{3}{4}$ of it? Ans. $\frac{15}{28}$.

If $\frac{1}{4}$ of an apple be divided into 7 equal parts, one of those parts will be $\frac{1}{28}$ of the whole apple; and if $\frac{1}{7}$ of $\frac{1}{4}$ is $\frac{1}{28}$, then $\frac{1}{7}$

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144. In canceling when should the quotient 1 be retained?

of $\frac{3}{4}$ will be $\frac{3}{28}$, and $\frac{5}{7}$ of $\frac{3}{4}$ will be $\frac{15}{28}$; i. e. *a compound fraction may be reduced to a simple one by the rule for multiplying a fraction by a fraction.*

23. Multiply $\frac{3}{8}$ by $\frac{7}{11}$, i. e. reduce $\frac{3}{8}$ of $\frac{7}{11}$ to a simple fraction. Ans. $\frac{21}{88}$.

24. Reduce $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{11}{13}$ to a simple fraction. Ans. $\frac{165}{728}$.

25. Reduce $\frac{8}{9}$ of $\frac{5}{7}$ of $\frac{11}{17}$ to a simple fraction.

26. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$?

$$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{8}} \times \frac{\cancel{8}}{9} = \frac{1}{9}, \text{ Ans.}$$

27. Reduce $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{5}{11}$ of $\frac{7}{6}$ to a simple fraction.

28. Reduce $\frac{5}{7}$ of $\frac{13}{17}$ of $\frac{17}{16}$ of $\frac{8}{13}$ to a simple fraction.

29. What cost $\frac{3}{4}$ of a yard of cloth at $\frac{4}{6}$ of a dollar per yard? Ans. $\frac{1}{2}$ of a dollar.

30. If a man builds $\frac{8}{9}$ of a rod of wall in a day, how much will he build in $\frac{3}{8}$ of a day?

31. A man owning $\frac{5}{8}$ of a farm sold $\frac{3}{5}$ of his share; what part of the farm did he sell?

(d) To multiply a whole number by a fraction.

32. At \$8 a barrel what will $\frac{3}{4}$ of a barrel of flour cost? Ans. \$6.

FIRST OPERATION.

```
4 ) $ 8, Price of 1 bbl.
    $ 2, Cost of 1/4 bbl.
      3
    $ 6, Cost of 3/4 bbl.
```

If a barrel costs \$8, then 1 fourth of a barrel will cost $\frac{1}{4}$ of \$8, viz. \$2, and 3 fourths will cost 3 times \$2 = \$6, Ans.

SECOND OPERATION.

```
    $ 8, Price of 1bbl.
      3
4 ) $ 2 4, Cost of 3 bbl.
    $ 6, Cost of 3/4 bbl.
```

If 1 bbl. costs \$8, then 3 bbl. will cost 3 times \$8 = \$24, and since $\frac{1}{4}$ of 3 bbl. is the same as $\frac{3}{4}$ of 1 bbl. we divide the cost of 3 bbl. by 4, and so find the cost of $\frac{3}{4}$ of a barrel, viz. \$6, which is the same result as by the first operation.

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144. How is a compound fraction reduced to a simple one? How many ways to multiply an integer by a fraction? First method? Second?

33. Multiply 24 by $\frac{5}{8}$; i. e. find $\frac{5}{8}$ of 24. Ans. 15.

34. If an acre of land costs \$45, what will $\frac{3}{5}$ of an acre cost?

35. What is the value of $\frac{3}{4}$ of a bushel of clover seed, at \$7 per bushel? Ans. \$$5\frac{1}{4}$.

(e) To multiply a mixed number by a fraction or mixed number:

Reduce each factor to the form of a fraction and then multiply the fractions together.

36. Multiply $2\frac{3}{4}$ by $1\frac{4}{5}$.

$$2\frac{3}{4} \times 1\frac{4}{5} = \frac{11}{4} \times \frac{9}{5} = \frac{99}{20} = 4\frac{19}{20}, \text{ Ans.}$$

37. What cost $2\frac{3}{8}$ yards of cloth, at \$$1\frac{3}{5}$ per yard? Ans. \$$3\frac{4}{5}$.

38. What cost $1\frac{3}{4}$ cords of wood, at \$$6\frac{1}{2}$ per cord?

39. How many square rods of land in a garden that is $6\frac{3}{8}$ rods long and $5\frac{2}{3}$ rods wide?

Problem 7.

145. To divide a fraction by a fraction.

Ex. 1. Divide $\frac{2}{3}$ by $\frac{5}{7}$. Ans. $\frac{14}{15}$.

To divide $\frac{2}{3}$ by $\frac{5}{7}$, 1st, $\frac{2}{3} \div 5 = \frac{2}{15}$ (Art. 143, Rule 2); but the divisor, 5, is 7 times $\frac{5}{7}$, $\therefore$ (Art. 83, f) the quotient $\frac{2}{15}$ is only $\frac{1}{7}$ of the quotient sought; hence, 2d, $\frac{2}{15} \times 7 = \frac{14}{15}$ (Art. 142, Rule 1) is the quotient sought; i. e.

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}. \quad \text{Hence,}$$

Rule. *Invert the divisor, and then proceed as in multiplication* (Art. 144).

The rule may be otherwise explained as follows:

First, *To divide by any number is the same as to multiply by its reciprocal* (Art. 138).

Thus, $12 \div 4 = 3$, and also $12 \times \frac{1}{4} = 3$.

Again, $\frac{5}{7} \div 4 = \frac{5}{28}$, and also $\frac{5}{7} \times \frac{1}{4} = \frac{5}{28}$; i e. dividing by 4

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144. Rule for multiplying a mixed number by a mixed number? **145.** Rule for dividing a fraction by a fraction? Reason? Second explanation?

and multiplying by the reciprocal of 4, viz. $\frac{1}{4}$, we have the *quotient equal to the product.*

Second, *The reciprocal of a fraction is the fraction inverted;* thus, the reciprocal of $\frac{5}{7}$ is $\frac{1}{\frac{5}{7}}$ (Art. 138), and, multiplying both numerator and denominator of this complex fraction, $\frac{1}{\frac{5}{7}}$, by 7, we obtain $\frac{7}{5}$; but multiplying both terms of a fraction by the same number does not change its value (Art. 84, a), $\therefore \frac{1}{\frac{5}{7}} = \frac{7}{5}$; i. e. the reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$; and, *generally, the reciprocal of any fraction is that fraction inverted.* Hence, to divide by a fraction, *invert the divisor and multiply.*

Ex. 2. Divide $\frac{3}{8}$ by $\frac{2}{11}$. Ans. $\frac{33}{16} = 2\frac{1}{16}$.

3. Divide $\frac{7}{9}$ by $\frac{5}{8}$.

4. Divide $\frac{8}{11}$ by $\frac{9}{13}$.

5. Divide $\frac{8}{21}$ by $\frac{13}{26}$. Ans. $\frac{208}{273}$.

6. Divide $\frac{7}{15}$ by $\frac{11}{13}$.

7. Divide $\frac{3}{7}$ of $\frac{7}{4}$ by $\frac{11}{5}$ of $\frac{5}{9}$.

$$\frac{3}{7} \times \frac{7}{4} \div \frac{11}{5} \times \frac{5}{9} = \frac{3}{\cancel{7}} \times \frac{\cancel{7}}{4} \times \frac{\cancel{5}}{11} \times \frac{9}{\cancel{5}} = \frac{27}{44}, \text{ Ans.}$$

8. Divide $\frac{3}{5}$ of $\frac{7}{9}$ of $\frac{5}{6}$ by $\frac{3}{8}$ of $\frac{4}{7}$. Ans. $\frac{49}{27} = 1\frac{22}{27}$.

9. Divide $\frac{5}{8}$ of $\frac{4}{9}$ by $\frac{4}{3}$ of $\frac{8}{9}$ of $\frac{5}{6}$.

10. Divide $\frac{4}{5}$ of $\frac{3}{9}$ of $\frac{7}{6}$ by $\frac{7}{8}$ of $\frac{3}{4}$.

(a) If the denominator of the divisor is like that of the dividend, as in Ex. 11, they may both be disregarded; for, evidently, $\frac{6}{27}$ are contained in $\frac{24}{27}$ just as many times as 6 apples are contained in 24 apples, or 6 in 24; i. e. $\frac{24}{27} \div \frac{6}{27} = 24 \div 6 =$ numerator of dividend $\div$ numerator of divisor; and this is equally true when the numerator of the dividend is not a multiple of the numerator of the divisor; thus, $\frac{5}{7} \div \frac{3}{7} = 5 \div 3 = \frac{5}{3}$.

11. Divide $\frac{24}{27}$ by $\frac{6}{27}$. Ans. 4.

12. Divide $\frac{36}{19}$ by $\frac{3}{19}$. Ans. 12.

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145. How is the division performed when the denominators are alike?

13. Divide $\frac{3}{11}$ by $\frac{7}{11}$. Ans. $\frac{3}{7}$.
14. Divide $\frac{18}{23}$ by $\frac{9}{23}$.
15. Divide $\frac{84}{97}$ by $\frac{21}{97}$.
16. Divide $\frac{848}{329}$ by $\frac{424}{329}$.
17. Divide $\frac{423}{967}$ by $\frac{846}{967}$. Ans. $\frac{423}{846} = \frac{1}{2}$.
18. Divide $\frac{144}{896}$ by $\frac{1728}{896}$.

(b) When the numerator and denominator of the divisor are respectively factors of the corresponding terms of the dividend, as in Ex. 19, it is best to divide numerator by numerator, and denominator by denominator. This mode is *true* in *all* examples but *not always convenient.* Why true? Why not convenient?

19. Divide $\frac{36}{77}$ by $\frac{4}{7}$ Ans. $\frac{9}{11}$.
20. Divide $\frac{96}{1728}$ by $\frac{24}{144}$.
21. If $\frac{7}{8}$ of a yard of cloth cost $\frac{21}{32}$ of a dollar, what costs 1 yard?
22. If I earn $\frac{6}{15}$ of a dollar in $\frac{3}{5}$ of a day, what shall I earn in 1 day?
23. If I pay $\frac{9}{8}$ of a dollar for $\frac{3}{4}$ of a bushel of corn, what shall I pay for 1 bushel? Ans. \$$1\frac{1}{2}$.

(c) To divide a whole or mixed number by a fraction or mixed number:

Reduce divisor and dividend each to the form of a simple fraction, and then divide by the rule already given.

24. Divide $8\frac{3}{4}$ by $3\frac{1}{2}$.

$$8\tfrac{3}{4} \div 3\tfrac{1}{2} = \tfrac{35}{4} \div \tfrac{7}{2} = \tfrac{5}{2} = 2\tfrac{1}{2}, \text{ Ans.}$$

25. Divide 8 by $3\frac{5}{8}$.

$$8 \div 3\tfrac{5}{8} = \tfrac{8}{1} \div \tfrac{29}{8} = \tfrac{8}{1} \times \tfrac{8}{29} = \tfrac{64}{29} = 2\tfrac{6}{29}, \text{ Ans.}$$

26. When $3\frac{1}{2}$ lb. of beef cost $43\frac{3}{4}$ cents, what is the price per pound? Ans. $12\frac{1}{2}$ cents.
27. B traveled $19\frac{13}{14}$ miles in $5\frac{1}{7}$ hours; how far did he travel per hour?
28. B traveled $19\frac{13}{14}$ miles, going at the rate of $3\frac{7}{8}$ miles per hour; how many hours did he travel?

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145. Mode of dividing when the terms of the divisor are factors of the terms of the dividend? To divide a mixed number by a mixed number?

PROBLEM 8.

146. To reduce a complex fraction to a simple one.

Ex. 1. The complex fraction $\frac{\frac{3}{4}}{\frac{5}{7}}$ equals what simple fraction?

The operation required is only to divide a fraction by a fraction; thus, $\frac{\frac{3}{4}}{\frac{5}{7}} = \frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$. Hence,

RULE. *First, if necessary, reduce the numerator and denominator of the complex fraction each to a simple fraction; then divide the fractional numerator by the fractional denominator* (Art. 145).

NOTE. A complex fraction may also be made simple by multiplying each term of the complex fraction by the least common multiple of their denominators; thus, in Ex. 1, the least common multiple of the two denominators, 4 and 7, is 28, whose factors are 4 and 7. Multiplying the numerator, $\frac{3}{4}$, by 4, gives 3 (Art. 142, a), and multiplying 3 by 7, the other factor of the multiple, gives 21 for the numerator of the reduced fraction. In like manner, multiplying the denominator, $\frac{5}{7}$, by 7, and that product by 4, gives 20 for the denominator of the reduced fraction.

Ex. 2. Reduce $\frac{1\frac{4}{7}}{2\frac{2}{21}}$ to a simple fraction.

$$\frac{1\frac{4}{7}}{2\frac{2}{21}} = \frac{\frac{11}{7}}{\frac{44}{21}} = \frac{11}{7} \div \frac{44}{21} = \frac{11}{7} \times \frac{21}{44} = \frac{3}{4}, \text{ Ans.}$$

3. Reduce $\frac{4\frac{3}{5}}{6\frac{9}{10}}$ to a simple fraction. Ans. $\frac{2}{3}$.

4. Reduce $\frac{8\frac{4}{9}}{12\frac{2}{3}}$.

5. Reduce $\frac{4\frac{3}{8}}{1\frac{3}{8}}$.

6. Reduce $\frac{3\frac{5}{8}}{6\frac{3}{4}}$.

7. Reduce $\frac{5\frac{3}{8}}{7\frac{5}{6}}$.

8. Reduce $\frac{5\frac{3}{5}}{\frac{3}{5}}$.

9. Reduce $\frac{\frac{2}{5} \text{ of } \frac{3}{7} \text{ of } 2\frac{1}{3}}{3\frac{4}{5}}$.

10. Reduce $\frac{7\frac{2}{3}}{4\frac{4}{9}}$.

11. Reduce $\frac{18}{2\frac{1}{3}}$.

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146. Rule for reducing a complex fraction to a simple one? Reason? Another mode?

12. Reduce $\frac{\frac{5}{7}}{6}$ to a simple fraction.

$$\frac{\frac{5}{7}}{6} = \tfrac{5}{7} \div 6 = \tfrac{5}{42}, \text{ Ans., by Art. 143, Rule 2; or,}$$

$$\frac{\frac{5}{7}}{6} = \frac{\frac{5}{7}}{6} \times \frac{7}{7} = \frac{5}{42}, \text{ Ans., by Art. 84 (a) and Art. 142 (a).}$$

13. Reduce $\frac{\frac{3}{8}}{7}$ to a simple fraction.

14. Reduce $\frac{8}{\frac{5}{9}}$ to a simple fraction.

$$\frac{8}{\frac{5}{9}} = \frac{8}{\frac{5}{9}} \times \frac{9}{9} = \frac{72}{5}, \text{ Ans., by Art. 145 (c).}$$

15. Reduce $\dfrac{\frac{3}{5} \text{ of } \frac{2}{6} \text{ of } \frac{7}{8} \text{ of } \frac{5}{4}}{\frac{14}{6} \text{ of } \frac{3}{16} \text{ of } \frac{1}{4} \text{ of } 2}$ to its simplest form. Ans. 1.

Problem 9.

147. To reduce fractions that have not a common denominator to equivalent fractions that have a common denominator.

Ex. 1. Reduce $\frac{2}{3}$ and $\frac{5}{7}$ to equivalent fractions having a common denominator. Ans. $\frac{14}{21}$ and $\frac{15}{21}$.

OPERATION.

$$\frac{2}{3} \times \frac{7}{7} = \frac{14}{21}$$

$$\frac{5}{7} \times \frac{3}{3} = \frac{15}{21}$$

Multiplying both terms of each fraction by the denominator of the other fraction will not alter the value of either fraction (Art. 84, a), but it will necessarily make the denominators alike, for each new denominator is the product of the two given denominators.

Similar reasoning applies, however many fractions are to be reduced. Hence,

Rule 1. *Multiply all the denominators together for a common denominator, and multiply each numerator into the continued product of all the denominators, except its own, for new numerators.*

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147. Common denominator, how found by Rule 1? How the numerators? Explanation?

2. Reduce $\frac{3}{4}$, $\frac{5}{7}$, and $\frac{1}{9}$ to equivalent fractions having a common denominator.

OPERATION.

$$4 \times 7 \times 9 = 252, \text{ common denominator,}$$
$$3 \times 7 \times 9 = 189, \text{ 1st numerator,}$$
$$5 \times 4 \times 9 = 180, \text{ 2d numerator,}$$
$$1 \times 4 \times 7 = 28, \text{ 3d numerator;}$$
$$\therefore \tfrac{3}{4}, \tfrac{5}{7}, \text{ and } \tfrac{1}{9} = \tfrac{189}{252}, \tfrac{180}{252}, \text{ and } \tfrac{28}{252}, \text{ Ans.}$$

3. Reduce $\frac{3}{5}$, $\frac{3}{4}$, and $\frac{3}{7}$. Ans. $\frac{84}{140}$, $\frac{105}{140}$, and $\frac{60}{140}$.
4. Reduce $\frac{7}{8}$, $\frac{3}{9}$, and $\frac{5}{7}$. Ans. $\frac{441}{504}$, $\frac{168}{504}$, and $\frac{360}{504}$.
5. Reduce $\frac{2}{5}$, $\frac{5}{6}$, $\frac{3}{7}$, and $\frac{1}{2}$. Ans. $\frac{168}{420}$, $\frac{350}{420}$, $\frac{180}{420}$, and $\frac{210}{420}$.
6. Reduce $\frac{5}{12}$, $\frac{2}{7}$, and $\frac{4}{5}$.
7. Reduce $\frac{5}{11}$, $\frac{3}{7}$, and $\frac{5}{6}$.
8. Reduce $\frac{7}{15}$, $\frac{5}{18}$, and $\frac{4}{13}$.
9. Reduce $\frac{2}{9}$, $\frac{3}{5}$, and $\frac{4}{15}$.
10. Reduce $\frac{7}{11}$, $\frac{5}{16}$, and $\frac{8}{23}$.
11. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{9}$, and $\frac{11}{13}$.
12. Reduce $\frac{2}{8}$, $\frac{5}{7}$, $\frac{1}{3}$, and $\frac{7}{11}$.
13. Reduce $\frac{4}{5}$, $\frac{8}{11}$, $\frac{1}{9}$, and $\frac{16}{17}$.
14. Reduce $\frac{3}{25}$, $\frac{4}{27}$, $\frac{5}{34}$, and $\frac{9}{23}$.
15. Reduce $\frac{8}{13}$, $\frac{4}{17}$, $\frac{2}{25}$, and $\frac{5}{18}$.

(a) The foregoing rule will always give *a* common denominator, but not always *the least integral* common denominator; this, however, may always be effected by

RULE 2. *Reduce each fraction, if necessary, to its lowest terms* (Art. 141). *Find the least common multiple of the denominators* (Art. 127) *for a common denominator. Divide this multiple by each given denominator, and multiply the several quotients by the respective numerators for new numerators.*

NOTE 1. Each of these rules is founded on the principle that multiplying both terms of a fraction by the same number does not alter its value.

16. Reduce $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$.

OPERATION BY THE SECOND RULE.

$$\begin{array}{r|lll} & \frac{3}{8}, & \frac{5}{6}, & \frac{7}{12} \\ 2 & 8, & 6, & 12 \\ 2 & 4, & 3, & 6 \\ 3 & 2, & 3, & 3 \\ & 2, & 1, & 1, \end{array}$$

$$2 \times 2 \times 3 \times 2 = 24, \text{ least common multiple of denominators,}$$
$$\tfrac{24}{8} \times 3 = 9, \text{ 1st numerator,}$$
$$\tfrac{24}{6} \times 5 = 20, \text{ 2d numerator,}$$
$$\tfrac{24}{12} \times 7 = 14, \text{ 3d numerator;}$$
$$\therefore \tfrac{3}{8}, \tfrac{5}{6}, \text{ and } \tfrac{7}{12} = \tfrac{9}{24}, \tfrac{20}{24}, \text{ and } \tfrac{14}{24}, \text{ Ans.}$$

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147. Rule for finding the *least* common denominator? Rule for finding the numerators? Principle?

17. Reduce $\frac{7}{10}$, $\frac{5}{8}$, $\frac{4}{5}$, and $\frac{3}{4}$. Ans. $\frac{28}{40}$, $\frac{25}{40}$, $\frac{32}{40}$, $\frac{30}{40}$.
18. Reduce $\frac{3}{20}$, $\frac{5}{16}$, $\frac{7}{40}$, and $\frac{9}{80}$.
19. Reduce $\frac{4}{15}$, $\frac{3}{16}$, $\frac{17}{80}$, and $\frac{13}{40}$.

NOTE 2. The first clause of Rule 2 is omitted by many authors, but its necessity is apparent from the following example:

20. Reduce $\frac{6}{8}$, $\frac{4}{12}$, and $\frac{3}{18}$ to equivalent fractions having the least common denominator.

Disregarding the first clause of the rule, we find 72 to be the least common multiple of the denominators, and the fractions $\frac{6}{8}$, $\frac{4}{12}$, and $\frac{3}{18}$, reduce to $\frac{54}{72}$, $\frac{24}{72}$, and $\frac{12}{72}$; but, regarding the first clause, we have $\frac{6}{8}$, $\frac{4}{12}$, and $\frac{3}{18} = \frac{3}{4}$, $\frac{1}{3}$, and $\frac{1}{6} = \frac{9}{12}$, $\frac{4}{12}$, and $\frac{2}{12}$, which have a common denominator *less than* 72.

21. Reduce $\frac{6}{10}$, $\frac{4}{8}$, $\frac{9}{12}$, and $\frac{4}{5}$. Ans. $\frac{12}{20}$, $\frac{10}{20}$, $\frac{15}{20}$, and $\frac{16}{20}$.
22. Reduce $\frac{6}{9}$, $\frac{12}{15}$, $\frac{4}{12}$, and $\frac{9}{18}$.
23. Reduce $\frac{6}{24}$, $\frac{15}{25}$, $\frac{8}{12}$, and $\frac{10}{30}$.
24. Reduce $\frac{18}{30}$, $\frac{15}{45}$, $\frac{9}{18}$, and $\frac{18}{72}$.

NOTE 3. In this and the following problems, each fraction should be in its simplest form before applying the rule.

25. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ and $\dfrac{3\frac{1}{5}}{1\frac{23}{25}}$.

$$\tfrac{2}{3} \text{ of } \tfrac{3}{5} = \tfrac{2}{5}\,;\quad \frac{3\frac{1}{5}}{1\frac{23}{25}} = \tfrac{16}{5} \div \tfrac{48}{25} = \tfrac{5}{3}\,; \text{ but}$$

$$\tfrac{2}{5} \text{ and } \tfrac{5}{3} = \tfrac{6}{15} \text{ and } \tfrac{25}{15}, \text{ Ans.}$$

26. Reduce $\frac{1}{3}$ of $\frac{8}{7}$, $2\frac{3}{7}$, $\dfrac{3\frac{1}{4}}{5\frac{5}{12}}$, and $\frac{3}{10}$.

27. Reduce $\dfrac{\frac{7}{3}}{\frac{3}{4}}$, $\frac{1}{3}$ of $\frac{5}{6}$, and $\frac{7}{24}$.

REMARK. The *numerators*, as well as the denominators, of fractions, *may be made alike* by reduction; thus, $\frac{2}{3}$ and $\frac{5}{7}$ are equal in value to $\frac{10}{15}$ and $\frac{10}{14}$; also $\frac{4}{7}$ and $\frac{6}{11} = \frac{12}{21}$ and $\frac{12}{22}$; also $\frac{4}{7}$, $\frac{6}{11}$, and $\frac{8}{5} = \frac{24}{42}$, $\frac{24}{44}$, and $\frac{24}{15}$; etc. The process is simple, but of little practical importance, and therefore seldom presented in Arithmetic.

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147. May the *numerators* of fractions be made alike? How?

Problem 10.

148. To reduce a fraction of a higher denomination to a fraction of a lower denomination.

Ex. 1. Reduce $\frac{1}{5}$ of a penny, to the fraction of a farthing.

As 1 penny is equal to 4 farthings, so *any fraction* of a penny will be 4 times *as great* a fraction of a farthing; $\therefore \frac{1}{5}$d. $=$ 4 times $\frac{1}{5}$qr. $= \frac{4}{5}$qr., Ans.

2. Reduce $\frac{1}{84}$ of a shilling to the fraction of a farthing.

As 1s. is equal to 12d., so $\frac{1}{84}$s. $=$ 12 times $\frac{1}{84}$d. $= \frac{1}{7}$d., and $\frac{1}{7}$d. $=$ 4 times $\frac{1}{7}$qr. $= \frac{4}{7}$qr., Ans. Hence,

Rule. *Multiply the fraction by such numbers as are necessary to reduce the given to the required denomination.*

3. Reduce $\frac{7}{36}$s. to the fraction of a farthing.

$\frac{7}{36}$s. $(= \frac{7}{36}$d. $\times 12) = \frac{7}{3}$d. $(= \frac{7}{3}$qr. $\times 4) = \frac{28}{3}$qr., Ans.; or,

$$\frac{7 \times 12 \times 4}{36} = \frac{7 \times \cancel{12} \times 4}{\cancel{36} \quad 3} = \frac{28}{3}\text{qr., Ans., as before.}$$

Note 1. The sign of multiplication, in these examples, is written only between the numbers which are given before the canceling is begun; thus, in Ex. 3, no sign is written between 36 and 3, *for they are not to be multiplied together*, but the 3 is obtained by canceling 12 in 36. So in Ex. 4, the 12 comes from canceling 20 in 240, and the 3 from canceling 4 in 12.

4. Reduce $\frac{7}{240}$ of a ton to the fraction of a dram.

$$\frac{7 \times 2\cancel{0} \times \cancel{4} \times 25 \times 16 \times 16}{24\cancel{0} \quad \cancel{12} \quad 3} = \frac{44800}{3}\text{dr., Ans.}$$

5. Reduce $\frac{10}{21}$ of a rod to the fraction of a barleycorn.

$$\frac{10 \times 16\frac{1}{2} \times 12 \times 3}{21} = \frac{10 \times \overset{11}{\cancel{33}} \times \overset{6}{\cancel{12}} \times 3}{\underset{7}{\cancel{21}} \times \cancel{2}} = \frac{1980}{7}\text{b. c., Ans.}$$

Note 2. In the first statement of Ex. 5, the $16\frac{1}{2}$, in the numerator, is equal to $\frac{33}{2}$, and, in the second statement, the 33 is retained in the numerator as a factor in the dividend, and the 2 is put in the denominator as a factor in the divisor.

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148. Rule for reducing a fraction from a higher to a lower denomination? Explanation? How is Ex. 5 solved?

6. Reduce $\frac{3}{800}$ of a pound, Troy Weight, to the fraction of a grain. Ans. $\frac{108}{5}$.

7. Reduce $\frac{3}{800}$ of a pound, Apothecaries' Weight, to the fraction of a grain. Ans. $\frac{108}{5}$.

8. Reduce $\frac{1}{4200}$ of a day to the fraction of a second. Ans. $\frac{144}{7}$.

9. Reduce $\frac{2}{15}$ of a bushel to the fraction of a pint. Ans. $\frac{128}{15}$.

10. Reduce $\frac{5}{12}$ of a gallon to the fraction of a gill.

11. Reduce $\frac{7}{8640}$c. yd. to the fraction of a cubic inch.

12. Reduce $\frac{1}{84}$ of a sign to the fraction of a second.

13. Reduce $\frac{3}{42400}$sq. m. to the fraction of a rod. Ans. $\frac{384}{53}$.

14. Reduce $\frac{3}{2800}$fur. to the fraction of a link. Ans. $\frac{15}{14}$.

15. Reduce $\frac{4}{363}$ of an acre to the fraction of a square yard.

16. Reduce $\frac{3}{28}$yd. of cloth to the fraction of an inch.

17. Reduce $\frac{1}{21}$circ. to the fraction of a second.

18. Reduce $\frac{5}{48}$ of a ton to the fraction of an ounce.

19. Reduce $\frac{5}{3024}$ of a day to the fraction of a second.

20. Reduce $\frac{1}{6720}$£ to the fraction of a farthing.

21. Reduce $\frac{5}{72}$ of a bushel to the fraction of a pint.

Problem 11.

149. To reduce a fraction of a lower denomination to a fraction of a higher denomination.

Ex. 1. Reduce $\frac{2}{5}$ of a barleycorn to the fraction of an inch.

In 15 barleycorns there is only $\frac{1}{3}$ of 15 inches, so in $\frac{2}{5}$ of a barleycorn there is only $\frac{1}{3}$ of $\frac{2}{5}$ of an inch $= \frac{2}{15}$ of an inch, Ans.

2. Reduce $\frac{20}{21}$ of a gill to the fraction of a quart.

As 1 gill is $\frac{1}{4}$ of a pint, so $\frac{20}{21}$gi. is $\frac{1}{4}$ of $\frac{20}{21}$pt. $= \frac{5}{21}$pt. and, for a like reason, $\frac{5}{21}$pt. is $\frac{1}{2}$ of $\frac{5}{21}$qt. $= \frac{5}{42}$qt., Ans. Hence,

Rule. *Divide the given fraction by such numbers as are required to reduce the given to the required denomination.*

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149. Rule for reducing a fraction from a lower to a higher denomination? Explanation?

3. Reduce $\frac{28}{3}$qr. to the fraction of a shilling.

$\frac{28}{3}$qr. $(= \frac{28}{3}\text{d.} \div 4) = \frac{7}{3}\text{d.}\ (= \frac{7}{3}\text{s.} \div 12) = \frac{7}{36}$s., Ans.; or,

$$\frac{\cancel{28}\ \ 7}{3\times\cancel{4}\times12}=\frac{7}{36}\text{s. Ans., as before.}$$

4. Reduce $\frac{44800}{3}$dr. to the fraction of a ton.

$$\frac{\cancel{44800}\ \ \cancel{2800}\ \ \cancel{175}\ \ 7}{3\times\cancel{16}\times\cancel{16}\times\cancel{25}\times4\times20}=\frac{7}{240}\text{ tons, Ans.}$$

5. Reduce $\frac{1980}{7}$b.c. to the fraction of a rod.

$$\frac{1980}{7\times3\times12\times3\times5\frac{1}{2}}=\frac{\cancel{1980}\ \ \cancel{180}\ \ \overset{10}{\cancel{30}}\ \ \cancel{2}}{7\times3\times\underset{6}{\cancel{12}}\times\cancel{3}\times\cancel{11}}=\frac{10}{21}\text{rd. Ans.}$$

6. Reduce $\frac{168}{5}$gr. to the fraction of a pound, Apothecaries' Weight. Ans. $\frac{7}{1200}$.

7. Reduce $\frac{168}{5}$gr. to the fraction of a pound, Troy Weight.

8. Reduce $\frac{25200}{11}$sec. to the fraction of a day.

9. Reduce $\frac{4}{25}$in. to the fraction of a yard, Cloth Measure.

10. Reduce $\frac{168}{5}$sec. to the fraction of a week.

11. Reduce $\frac{432}{7}$sq. in. to the fraction of a yard.

12. Reduce $\frac{450}{21}$ links to the fraction of a furlong.

13. Reduce $\frac{363}{13}$yd. to the fraction of an acre. Ans. $\frac{3}{520}$.

14. Reduce $\frac{720}{14}$ seconds to the fraction of a sign.

15. Reduce $\frac{16}{3}$ gills to the fraction of a gallon.

Problem 12.

150. To reduce a fraction of a higher denomination to whole numbers of lower denominations.

Ex. 1. Reduce $\frac{1}{6}$£ to shillings and pence. Ans. 3s. 4d.

$\frac{1}{6}$£ $(= \frac{1}{6}\text{s.} \times 20) = \frac{10}{3}\text{s.} = 3\frac{1}{3}$s.; again $\frac{1}{3}$s. $(= \frac{1}{3}\text{d.} \times 12) =$ 4d.; $\therefore\ \frac{1}{6}$£ $=$ 3s. 4d., Ans. Hence,

Rule. *Reduce the given fraction to a fraction of the next lower denomination* (Art. 148); *then, if the fraction is improper, reduce it to a whole or mixed number* (Art. 140). *If the result is*

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150. Rule for reducing a fraction of a higher denomination to integers of lower denominations? Explanation?

a mixed number, reduce the fractional part of it to the next lower denomination, as before, and so proceed as far as desirable.

NOTE. If, at any time, the reduced fraction is *proper*, there will be no whole number of that denomination.

2. Reduce $\frac{13}{64}$£ to whole numbers of lower denominations.

$\frac{13}{64}$£ $(= \frac{13}{64}\text{s.} \times 20) = \frac{65}{16}\text{s.} = 4\frac{1}{16}\text{s.}$; $\frac{1}{16}\text{s.} (= \frac{1}{16}\text{d.} \times 12) = \frac{3}{4}\text{d.}$, a proper fraction; $\frac{3}{4}\text{d.} (= \frac{3}{4}\text{qr.} \times 4) = 3\text{qr.}$; $\therefore$ $\frac{13}{64}$£ $=$ 4s. 0d. 3qr., Ans.

3. Reduce $\frac{9}{25}$ of an acre to lower denominations.

Ans. 1r. 17rd. 18yd. 1ft. $50\frac{2}{5}$in.

4. Reduce $\frac{7}{15}$ of a furlong to rods, yards, etc.

Ans. 18rd. 3yd. 2ft.

5. Reduce $\frac{3}{8}$ of a week to days, etc.

6. Reduce $\frac{325}{297}$ of a rod, Long Measure, to yards, etc.

7. Reduce $\frac{11687}{21600}$ of a circumference to signs, etc.

8. Reduce $\frac{7}{24}$ of a ton to hundred weights, etc.

9. Reduce $\frac{173}{576}$lb to ounces, drams, scruples, etc.

10. Reduce $\frac{2309}{10368}$circ. to signs, degrees, etc.

11. Reduce $\frac{14}{27}$ of a civil year (365 days) to days, etc.

12. What is the value of $\frac{769}{1440}$ of a pound Troy?

13. What is the value of $\frac{19}{24}$ of a bushel?

14. What is the value of $\frac{17}{48}$ of a gallon?

15. What is the value of $\frac{5}{24}$ of a pound, Apothecaries' Weight?

16. Reduce $\frac{5}{18}$ of a mile to furlongs, chains, etc.

17. Reduce $\frac{8}{15}$ of a cord to cord feet, cubic feet, etc.

18. Reduce $\frac{7}{18}$ of a yard to quarters, nails, etc.

PROBLEM 13.

151. To reduce whole numbers of lower denominations to the fraction of a higher denomination.

Ex. 1. One farthing is what part of a penny? Ans. $\frac{1}{4}$.

Since 4 farthings make a penny, 1 farthing is $\frac{1}{4}$ of a penny.

2. Six pence and 1 farthing are what part of a shilling?

6d. + 1qr. = 25qr; and 1s. = 48qr.; $\therefore$ 6d. and 1qr. $= \frac{25}{48}$s., Ans.

To determine what part one thing is of another, considered as a *unit* or *whole thing, the part is always made the numerator of a fraction, and the unit or whole thing is put for the denominator;* thus, the fraction $\frac{3}{5}$ expresses the part that 3 miles is of 5 miles. Before the comparison can be made, the part and the whole must be of the same kind or denomination; thus, 3 pecks is not $\frac{3}{5}$ of 5 bushels, but, reducing the 5 bushels to 20 pecks, we have 3 pecks equal to $\frac{3}{20}$ of 20 pecks, i. e. $\frac{3}{20}$ of 5 bushels. Hence,

RULE 1. *Reduce the given quantity to the lowest denomination it contains, for a numerator; and reduce a unit of the higher denomination to the same denomination as the numerator, for a denominator.*

3. Reduce 6rd. 5ft. 9in. to the fraction of a furlong.

$$6\text{rd. } 5\text{ft. } 9\text{in.} = 1257\text{in. and } 1\text{fur.} = 7920\text{in.}$$

$$\therefore 6\text{rd. } 5\text{ft. } 9\text{in.} = \tfrac{1257}{7920}\text{fur.} = \tfrac{419}{2640}\text{fur., Ans.}$$

4. Reduce 7oz. 4dwt. to the fraction of a pound. Ans. $\frac{3}{5}$.

5. Reduce 9 rods, 1 foot, and 6 inches to the fraction of a furlong.

$$9\text{rd. } 1\text{ft. } 6\text{in.} = 1800\text{in. and } 1\text{fur.} = 7920\text{in.};$$

$$\therefore 9\text{rd. } 1\text{ft. } 6\text{in.} = \tfrac{1800}{7920}\text{fur.} = \tfrac{5}{22}\text{fur., Ans.}$$

(a) In Ex. 5, 6in. $= \frac{1}{2}$ft.; $1\frac{1}{2}$ft. $= \frac{1}{2}$yd. $= \frac{1}{11}$rd. and $9\frac{1}{11}$rd. $= \frac{100}{11}$rd. $= \frac{5}{22}$fur., Ans., as by Rule 1. Hence,

RULE 2. *Divide the number of the lowest denomination given by the number required to reduce it to the next higher denomination, and annex the fractional quotient so obtained to the given number of that higher denomination; divide the mixed number so formed by the number required to reduce it to the* NEXT *higher denomination, annex the quotient to the given number of that denomination, and so proceed as far as necessary.*

NOTE 1. This rule is frequently preferable to the 1st, because it enables us to use smaller numbers and gives the result in lower terms.

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131. Rule for reducing the lower denominations of a compound number to a fraction of a higher denomination? Explanation? Principle? Second rule for reducing integers of lower denominations to the fraction of a higher denomination? Explanation? Why preferable to Rule 1?

6. Reduce 1r. 2sq. rd. 20sq. yd. 1sq. ft. 72sq. in. to the fraction of an acre. Ans. $\frac{4}{15}$.

7. Reduce 4oz. 6dwt. $9\frac{3}{5}$gr. to the fraction of a pound. Ans. $\frac{9}{25}$.

Note 2. In Example 7, by Rule 1, reduce 4oz. 6dwt. $9\frac{3}{5}$gr. to *fifths* of a grain for a numerator, and 1lb. to fifths of a grain for a denominator. How shall it be done by Rule 2? Which mode is preferable? Why?

8. Reduce 1pk. 3qt. 1pt. to the fraction of a bushel.

9. Reduce 6s. 20° 20′ 30″ to the fraction of a circumference.

10. Reduce 1m. 2fur. 11rd. 2yd. 1ft. $2\frac{4}{7}$b. c. to the fraction of a league.

11. Reduce 1qr. 2na. $\frac{8}{10}$in. to the fraction of a yard.

12. Reduce 3wk. 6d. 9h. 27m. to the fraction of a Julian year.

13. Reduce 1qt. 1pt. $1\frac{4}{7}$gi. to the fraction of a gallon.

14. Reduce 4 cord feet, 12 cubic feet, and $1382\frac{2}{5}$ cubic inches to the fraction of a cord. Ans. $\frac{3}{8}$.

15 Reduce 3oz. 4dr. 1sc. 10gr. to the fraction of a pound.

16. Reduce 4fur. 5ch. 2rd. 20li. to the fraction of a mile.

17. Reduce 11cwt. 11lb. 1oz. $12\frac{4}{5}$dr. to the fraction of a ton.

18. Reduce 3 bushels, 1 peck, 4 quarts, and 1 pint to the fraction of a bushel. Ans. $\frac{217}{64}$.

Note 3. Sometimes, as in Ex. 18, the number called the *part* is *greater* than the unit with which it is compared; sometimes it is *equal* to the unit.

Problem 14.

152. If numbers of the same kind are added together, their sum will be of the same kind as the numbers added; thus, 3 books + 4 books = 7 books; 3 hats + 4 hats = 7 hats; and for a like reason, $\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$; $\frac{5}{13}+\frac{4}{13}=\frac{9}{13}$, etc., etc.

(a) Numbers of different kinds cannot be united by addition; thus, 3 hats + 4 books are neither 7 hats nor 7 books; so $\frac{3}{5}+\frac{4}{9}$ are neither $\frac{7}{5}$ nor $\frac{7}{9}$; but numbers that are unlike may sometimes be made alike by reduction, and then added; thus,

$$\frac{3}{5}+\frac{4}{9}=\frac{27}{45}+\frac{20}{45}\text{ (Art. 147)}=\frac{47}{45}.$$

(b) Again, 2bush. + 3pk. are neither 5bush. nor 5pk.; but 2bush. = 8pk., and then 8pk. + 3pk. = 11pk.; so $\frac{2}{7}$bush. +

$\frac{3}{7}$pk. are neither $\frac{5}{7}$bush. nor $\frac{5}{7}$pk.; but $\frac{2}{7}$bush. $=\frac{8}{7}$pk. (Art. 148), and then $\frac{8}{7}$pk. $+\frac{3}{7}$pk. $=\frac{11}{7}$pk. Hence,

To add fractions:

RULE. *Reduce the fractions, if necessary, first to the same denomination, then to a common denominator; after which write the sum of the new numerators over the common denominator.*

Ex. 1. Add $\frac{3}{15}$ and $\frac{4}{15}$ together. Ans. $\frac{7}{15}$.

2. Add $\frac{4}{19}$, $\frac{3}{19}$, and $\frac{7}{19}$ together. Ans. $\frac{14}{19}$.

3. Add $\frac{8}{17}$, $\frac{4}{17}$, $\frac{9}{17}$, and $\frac{15}{17}$ together. Ans. $\frac{36}{17}=2\frac{2}{17}$.

4. Add $\frac{5}{8}$ and $\frac{7}{8}$ together. Ans. $\frac{12}{8}=1\frac{4}{8}=1\frac{1}{2}$.

5. Add $\frac{4}{15}$, $\frac{7}{15}$, $\frac{8}{15}$, and $\frac{2}{15}$ together. Ans. $1\frac{2}{5}$.

6. Add $\frac{5}{24}$, $\frac{3}{24}$, $\frac{7}{24}$, $\frac{9}{24}$, and $\frac{5}{24}$ together.

7. Add $\frac{23}{32}$, $\frac{9}{32}$, $\frac{6}{32}$, $\frac{10}{32}$, and $\frac{8}{32}$ together.

8. Add together $\frac{5}{18}$, $\frac{7}{18}$, $\frac{12}{18}$, $\frac{14}{18}$, and $\frac{4}{18}$. Ans. $2\frac{1}{3}$.

9. Add together $\frac{4}{36}$, $\frac{10}{36}$, $\frac{12}{36}$, $\frac{10}{36}$, and $\frac{9}{36}$.

10. Add together $\frac{63}{96}$, $\frac{84}{96}$, $\frac{48}{96}$, and $\frac{72}{96}$.

11. Add together $\frac{4}{125}$, $\frac{87}{125}$, $\frac{67}{125}$, $\frac{7}{125}$, and $\frac{10}{125}$.

12. Add together $\frac{7}{6}$, $\frac{9}{6}$, $\frac{6}{6}$, and $\frac{4}{6}$. Ans. $4\frac{1}{3}$.

13. Add together $\frac{24}{20}$, $\frac{18}{20}$, $\frac{16}{20}$, and $\frac{44}{20}$.

14. Add together $\frac{3}{8}$, $\frac{7}{12}$, and $\frac{5}{16}$.

$\frac{3}{8}+\frac{7}{12}+\frac{5}{16}=\frac{18}{48}+\frac{28}{48}+\frac{15}{48}$ (Art. 147, Rule 2) $=$
$\frac{61}{48}=1\frac{13}{48}$, Ans.

15. Add together $\frac{3}{5}$ and $\frac{5}{6}$.

$\frac{3}{5}+\frac{5}{6}=\frac{18}{30}+\frac{25}{30}$ (Art. 147, Rule 1) $=\frac{43}{30}=1\frac{13}{30}$, Ans.

16. Add together $\frac{5}{16}$, $\frac{3}{9}$, and $\frac{4}{7}$.

17. Add together $\frac{3}{12}$, $\frac{4}{16}$, and $\frac{6}{8}$.

$\frac{3}{12}+\frac{4}{16}+\frac{6}{8}=\frac{1}{4}+\frac{1}{4}+\frac{3}{4}=\frac{5}{4}=1\frac{1}{4}$, Ans.

18. Add $\frac{3}{18}$, $\frac{5}{40}$, $\frac{3}{12}$, and $\frac{10}{25}$.

19. Add $\frac{3}{5}$ of $\frac{5}{8}$ to $\frac{4}{7}$ of $\frac{14}{16}$. $\frac{3}{8}+\frac{1}{2}=\frac{7}{8}$, Ans.

20. Add $\frac{3}{5}$ of $\frac{25}{18}$ to $\frac{4}{9}$ of $\frac{27}{36}$.

21. Add $\dfrac{8\frac{4}{9}}{16\frac{8}{9}}$ to $\frac{1}{2}$ of $\frac{4}{5}$. Ans. $\frac{9}{10}$.

22. Add $5\frac{2}{11}$ to $8\times\frac{3}{48}$.

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152. Rule for adding fractions? Can unlike numbers be added? Of what kind is the sum of two or more numbers?

23. Add $\frac{2}{5}$s. to $\frac{2}{3}$d.

$\frac{2}{5}$s. $+ \frac{2}{3}$d. $= \frac{24}{5}$d. $+ \frac{2}{3}$d. $= \frac{72}{15}$d. $+ \frac{10}{15}$d. $= \frac{82}{15}$d. $= 5\frac{7}{15}$d., Ans.;
or, $\frac{2}{5}$s. $+ \frac{2}{3}$d. $= \frac{2}{5}$s. $+ \frac{1}{18}$s. $= \frac{36}{90}$s. $+ \frac{5}{90}$s. $= \frac{41}{90}$s., 2d Ans. $=$ 1st Ans.

24. Add $\frac{3}{5}$gal. to $\frac{1}{3}$qt. Ans. $\frac{41}{15}$qt. or $\frac{41}{60}$gal.
25. Add together $\frac{1}{3}$bush. $\frac{2}{5}$pk. and $\frac{5}{6}$qt.
26. Add together $\frac{2}{5}$ton $\frac{3}{5}$cwt. and $\frac{1}{2}$qr.

(c) To add two fractions that have a common numerator:

Multiply the sum of the denominators by either numerator, and place the product over the product of the denominators.

27. What is the sum of $\frac{3}{7}$ and $\frac{3}{8}$?

$$\frac{1}{7} + \frac{1}{8} = \frac{8+7}{7 \times 8} = \frac{15}{56}; \therefore \frac{3}{7} + \frac{3}{8} = \frac{3 \times 15}{56} = \frac{45}{56}, \text{ Ans.}$$

28. What is the sum of $\frac{5}{6}$ and $\frac{5}{9}$? $\frac{75}{54} = 1\frac{7}{18}$, Ans.
29. What is the sum of $\frac{4}{5}$ and $\frac{4}{21}$?

(d) To add mixed numbers:

Add the sum of the fractions to the sum of the integers.

30. What is the sum of $3\frac{4}{5}$ and $4\frac{3}{8}$?

$\frac{4}{5} + \frac{3}{8} = \frac{32}{40} + \frac{15}{40} = \frac{47}{40} = 1\frac{7}{40}$; $3 + 4 = 7$;
$\therefore 3\frac{4}{5} + 4\frac{3}{8} = 7 + 1\frac{7}{40} = 8\frac{7}{40}$, Ans.

31. What is the sum of $5\frac{4}{9}$, $3\frac{5}{6}$, and $12\frac{2}{3}$? Ans. $21\frac{17}{18}$.
32. What is the sum of $18\frac{8}{15}$, $5\frac{4}{9}$, and $24\frac{5}{8}$?
33. What is the sum of $15\frac{2}{3}$, 24, $7\frac{4}{9}$, and $\frac{5}{12}$?
34. What is the sum of $3\frac{4}{15}$, $6\frac{9}{16}$, $4\frac{5}{12}$, and $24\frac{3}{8}$?
35. What is the sum of $\frac{1}{2}$ of $\frac{2}{3}$ of $6\frac{3}{8}$, $\dfrac{4\frac{1}{2}}{9}$, and $4\frac{3}{8}$?
36. What is the sum of $\dfrac{8\frac{4}{5}}{3\frac{1}{7}}$, $3\frac{8}{9}$, $6\frac{2}{3}$, and $\frac{1}{2}$ of $\frac{3}{5}$?
37. What is the sum of $3\frac{8}{11}$, $4\frac{5}{22}$, $8\frac{4}{33}$, and 25?
38. How many are $8\frac{2}{3} + 3\frac{5}{6} + 8\frac{7}{9} + 14$?

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152. Mode of adding two fractions that have like numerators? Mode of adding mixed numbers?

PROBLEM 15.

153. To subtract a less fraction from a greater:

RULE. *Prepare the fractions as in addition, and then write the difference of the numerators over the common denominator.*

Ex. 1. From $\frac{8}{15}$ take $\frac{3}{15}$. $\frac{8}{15}-\frac{3}{15}=\frac{5}{15}=\frac{1}{3}$, Ans.
2. From $\frac{12}{37}$ take $\frac{7}{37}$. Ans. $\frac{5}{37}$.
3. From $\frac{16}{83}$ take $\frac{4}{83}$.
4. From $\frac{63}{97}$ take $\frac{14}{97}$.
5. From $\frac{46}{54}$ take $\frac{18}{54}$.
6. From $\frac{46}{27}$ take $\frac{37}{27}$.
7. Take $\frac{3}{23}$ from $\frac{10}{23}$. Ans. $\frac{7}{23}$.
8. Take $\frac{63}{85}$ from $\frac{84}{85}$.
9. Take $\frac{47}{98}$ from $\frac{91}{98}$.
10. Take $\frac{42}{125}$ from $\frac{87}{125}$.
11. From $\frac{4}{5}$ take $\frac{2}{3}$.
(a) $\frac{4}{5}-\frac{2}{3}=\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$, Ans. (See Art. 152, a).
12. From $\frac{8}{9}$ take $\frac{3}{5}$. Ans. $\frac{13}{45}$.
13. From $\frac{23}{47}$ take $\frac{3}{11}$.
14. From $\frac{11}{12}$ take $\frac{3}{8}$. $\frac{11}{12}-\frac{3}{8}=\frac{22}{24}-\frac{9}{24}=\frac{13}{24}$, Ans.
15. From $\frac{8}{15}$ take $\frac{4}{9}$. Ans. $\frac{4}{45}$.
16. From $\frac{11}{16}$ take $\frac{5}{12}$.
17. From $\frac{17}{20}$ take $\frac{13}{30}$.
18. From $\frac{27}{84}$ take $\frac{6}{40}$.
$\frac{27}{84}-\frac{6}{40}=\frac{9}{28}-\frac{3}{20}=\frac{45}{140}-\frac{21}{140}=\frac{24}{140}=\frac{6}{35}$, Ans.
19. From $\frac{77}{336}$ take $\frac{5}{300}$. Ans. $\frac{17}{80}$.
20. From $\frac{87}{252}$ take $\frac{25}{330}$.
21. From $\frac{3}{4}$ of $\frac{4}{5}$ take $\frac{1}{2}$ of $\frac{4}{7}$.
$\frac{3}{4}\times\frac{4}{5}-\frac{1}{2}\times\frac{4}{7}=\frac{3}{5}-\frac{2}{7}=\frac{21}{35}-\frac{10}{35}=\frac{11}{35}$, Ans.
22. From $\frac{3}{4}$ of $\frac{5}{6}$ take $\frac{1}{3}$ of $\frac{6}{11}$. Ans. $\frac{39}{88}$.
23. From $\frac{5}{7}$ of $\frac{14}{15}$ take $\frac{3}{5}$ of $\frac{10}{18}$.
24. From $\frac{8}{9}$ of $\frac{10}{24}$ take $\frac{9}{11}$ of $\frac{22}{45}$.
25. From $\frac{9}{14}$ of $\frac{7}{6}$ take $\frac{5}{8}$ of $\frac{12}{25}$.
26. From $\frac{3}{5}$ of $\frac{10}{21}$ of $\frac{7}{8}$ take $\frac{4}{9}$ of $\frac{3}{16}$ of $\frac{4}{3}$.

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153. Rule for subtracting one fraction from another? How are the fractions prepared in addition?

27. From $\dfrac{9\frac{3}{4}}{4\frac{1}{3}}$ take $\dfrac{7\frac{5}{9}}{5\frac{2}{3}}$. Ans. $\frac{11}{12}$.

$\dfrac{9\frac{3}{4}}{4\frac{1}{3}} = \dfrac{\frac{39}{4}}{\frac{13}{3}} = \frac{39}{4} \div \frac{13}{3} = \frac{39}{4} \times \frac{3}{13} = \frac{9}{4}$;

$\dfrac{7\frac{5}{9}}{5\frac{2}{3}} = \dfrac{\frac{68}{9}}{\frac{17}{3}} = \frac{68}{9} \div \frac{17}{3} = \frac{4}{3}$ (Art. 145, b);

Complex fractions reduced to simple ones.

$\frac{9}{4} - \frac{4}{3} = \frac{27}{12} - \frac{16}{12} = \frac{11}{12}$, Ans.

28. From $\dfrac{6\frac{3}{5}}{2\frac{16}{25}}$ take $\dfrac{4\frac{2}{7}}{7\frac{1}{2}}$. Ans. $\frac{27}{14} = 1\frac{13}{14}$.

29. From $\dfrac{9\frac{3}{8}}{6\frac{1}{4}}$ take $\dfrac{\frac{2}{3}}{\frac{5}{3}}$.

30. From $\frac{1}{7}$s. take $\frac{1}{5}$d. (See Art. 152, b).

(b) $\frac{1}{7}$s. — $\frac{1}{5}$d. = $\frac{12}{7}$d. — $\frac{1}{5}$d. = $\frac{60}{35}$d. — $\frac{7}{35}$d. = $\frac{53}{35}$d. = $1\frac{18}{35}$d., Ans.
or, $\frac{1}{7}$s. — $\frac{1}{5}$d. = $\frac{1}{7}$s. — $\frac{1}{60}$s. = $\frac{60}{420}$s. — $\frac{7}{420}$s. = $\frac{53}{420}$s., 2d Ans.

31. From $\frac{2}{3}$qt. take $\frac{1}{6}$pt. Ans. $\frac{7}{12}$qt. or $1\frac{1}{6}$pt.

32. From $\frac{2}{3}$ ton take $\frac{3}{7}$cwt.

33. From $\frac{3}{7}$ acre take $\frac{3}{4}$ rod. Ans. $\frac{1897}{4480}$a. or $67\frac{23}{28}$rd.

Note. The answer to these examples may be in any denomination of the table.

34. From $\frac{1}{5}$ of a week take $\frac{3}{7}$ of an hour.

(c) To subtract when the fractions have a common numerator:

Multiply the difference of the denominators by either numerator, and write the product over the product of the denominators.

35. From $\frac{4}{5}$ take $\frac{4}{8}$.

$$\frac{1}{5} - \frac{1}{8} = \frac{8-5}{5 \times 8} = \frac{3}{40};\ \therefore\ \frac{4}{5} - \frac{4}{8} = \frac{4 \times 3}{40} = \frac{12}{40} = \frac{3}{10}, \text{ Ans.}$$

36. From $\frac{5}{7}$ take $\frac{5}{11}$. Ans. $\frac{20}{77}$.

37. From $\frac{7}{6}$ take $\frac{7}{9}$.

(d) To take a mixed number from a whole or mixed number.

38. From $6\frac{4}{5}$ take $2\frac{3}{5}$.

$6\frac{4}{5} - 2\frac{3}{5} = 4\frac{1}{5}$, Ans. (See Art. 152, d).

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153. Mode of subtracting when the fractions have a common numerator?

39. From $8\frac{4}{11}$ take $2\frac{7}{11}$.

$$8\tfrac{4}{11} - 2\tfrac{7}{11} = 7\tfrac{15}{11} - 2\tfrac{7}{11} = 5\tfrac{8}{11}, \text{ Ans.}$$

In Ex. 38, take $\frac{3}{8}$ from $\frac{5}{8}$, and 2 units from 6 units; but in Ex. 39 we cannot take $\frac{7}{11}$ from $\frac{4}{11}$, $\therefore$ reduce *one* of the 8 units to $\frac{11}{11}$, and add it to the $\frac{4}{11}$, making $\frac{15}{11}$, and then take the $\frac{7}{11}$ from $\frac{15}{11}$, and the 2 units from the remaining 7 units.

40. From $9\frac{2}{5}$ take $3\frac{5}{6}$.

$$9\tfrac{2}{5} - 3\tfrac{5}{6} = 8\tfrac{7}{5} - 3\tfrac{5}{6} = 8\tfrac{42}{30} - 3\tfrac{25}{30} = 5\tfrac{17}{30}, \text{ Ans.}$$

41. From $12\frac{3}{8}$ take $4\frac{3}{5}$.
42. From 9 take $5\frac{2}{7}$. Ans. $3\frac{5}{7}$.
43. From 8 take $2\frac{5}{9}$.

Miscellaneous Examples in Fractions.

1. Multiply $\frac{3}{17}$ by 5. Ans. $\frac{15}{17}$.
2. Multiply $\frac{5}{24}$ by 6.
3. Reduce $\frac{32}{48}$ to its lowest terms.
4. Add $8\frac{3}{17}$ to $6\frac{11}{12}$.
5. Subtract $18\frac{13}{32}$ from $25\frac{17}{18}$.
6. Reduce $23\frac{5}{9}$ to an improper fraction.
7. Reduce 8 to a fraction whose denominator is 27.
8. Reduce 9 to 6 fractional forms.
9. Divide $\frac{18}{26}$ by $\frac{2}{13}$.
10. Divide $\frac{16}{8}$ by $\frac{23}{8}$.
11. Divide $\frac{6}{11}$ by $\frac{5}{8}$.
12. Reduce $\frac{3}{7}$ of a day to hours, minutes, and seconds.
13. Reduce 3pk. 5qt. 1pt. to the fraction of a bushel.
14. Multiply $8\frac{3}{5}$ by 10.
15. Divide $9\frac{3}{5}$ by 4.
16. Divide $\frac{27}{64}$ by 9.
17. Divide 18 by $\frac{3}{5}$.
18. Reduce $\frac{374}{25}$ to a mixed number.
19. Reduce $\frac{1728}{48}$ to a whole number.
20. Multiply $\frac{32}{27}$ by $\frac{81}{48}$.
21. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{16}{21}$ to a simple fraction.
22. Subtract $\frac{3}{25}$ from $\frac{3}{5}$.

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153. Mode of taking a mixed number from a whole or mixed number?

23. Reduce $\frac{3}{11}$, $\frac{4}{5}$, and $\frac{5}{7}$ to equivalent fractions that have a common denominator.

24. Reduce $\frac{12}{15}$, $\frac{18}{24}$, and $\frac{27}{30}$ to equivalent fractions having the least common denominator.

25. Reduce $\frac{5}{7}$ and $\frac{3}{8}$ to equivalent fractions having a common numerator.

26. Reduce $\frac{6}{8}$, $\frac{8}{16}$, and $\frac{16}{30}$ to equivalent fractions having the least common numerator.

27. Reduce $\dfrac{4\frac{3}{8}}{6\frac{4}{5}}$ to a simple fraction.

28. Add $\frac{5}{24}$ to $\frac{5}{32}$.

29. Divide $\frac{3}{16}$ by 4.

30. Reduce $\frac{3}{8}$ of a gallon to the fraction of a quart.

31. Reduce $\frac{3}{8}$ of an hour to the fraction of a week.

32. Reduce $\dfrac{\frac{4}{5} \text{ of } \frac{2}{3} \text{ of } \frac{7}{8} \text{ of } 1\frac{4}{11}}{\frac{8}{9} \text{ of } \frac{3}{4} \text{ of } \frac{5}{11} \text{ of } \frac{7}{12}}$ to its simplest form.

33. Multiply $\frac{25}{33}$ by 33.

34. Multiply 25 by $\frac{3}{5}$.

35. Multiply 25 by $\frac{3}{8}$.

36. Divide $\frac{6}{24}$ by $\frac{6}{8}$.

37. Add $\frac{3}{8}$£, $\frac{1}{5}$s., and $\frac{4}{9}$d. together.

38. Subtract $\frac{3}{5}$ of a gill from $\frac{4}{9}$ of a gallon.

39. Add $\frac{3}{19}$, $\frac{5}{19}$, $\frac{4}{19}$, $\frac{2}{19}$, and $\frac{7}{19}$ together.

40. From $\frac{63}{63}$ take $\frac{33}{63}$.

41. Five gallons, 3 quarts, 1 pint, and 3 gills, are what part of 1 gallon? (See Art. 151, Note 3).

42. Three pecks are what part of 3 pecks?

Examples in Analysis.

154. We analyze an example when we proceed with it, step by step, according to its own conditions, without being guided by any particular rule.

Ex. 1. If 4 tons of hay cost \$48, what will 7 tons cost?

Solution. If 4 tons cost \$48, then 1 ton will cost $\frac{1}{4}$ of \$48, which is \$12; and if 1 ton cost \$12, then 7 tons will cost 7 times \$12, which is \$84, Ans.

2. What is the value of 12 acres of land, if 3 acres cost $81?
Ans. $324.

3. What is the cost of 16 barrels of flour, if 3 barrels cost $24?

4. If a man can cut 8 cords of wood in 4 days, how much will he cut in 7 days?

5. If 1 ton of hay costs $15, what will $\frac{4}{5}$ of a ton cost?

SOLUTION. One ton costs $15; $\therefore$ $\frac{1}{5}$ of a ton costs $\frac{1}{5}$ of $15 = $3, and $\frac{4}{5}$ cost 4 times $3 = $12, Ans.

6. What is the value of $\frac{7}{8}$ of an acre of land, at $40 per acre?
Ans. $35.

7. If 6 men mow 12 acres of grass in a day, how many acres will they mow in $\frac{5}{6}$ of a day?

8. If a man cradle 18 acres of wheat in 9 days, how many acres will he cradle in 5 days?

9. Paid $6 for $\frac{3}{4}$ of a yard of velvet; what was the price per yard?

SOLUTION. Since $6 were paid for $\frac{3}{4}$ of a yard, $\frac{1}{4}$ cost $\frac{1}{3}$ of $6 = $2, and $\therefore$ $\frac{4}{4}$, or a whole yard, cost 4 times $2 = $8, Ans.

10. If $\frac{7}{8}$ of a yard of ribbon cost 63 cents, what will a yard cost?

11. If $\frac{5}{8}$ of an acre of land cost $75, what is the price per acre?

12. If 234 bushels of potatoes grow on $\frac{3}{8}$ of an acre, how many bushels will grow on an acre?

13. If $\frac{3}{4}$ of a farm cost $4200, what cost $\frac{5}{7}$ of it?

SOLUTION. If $\frac{3}{4}$ cost $4200, then $\frac{1}{4}$ costs $\frac{1}{3}$ of $4200 = $1400, and $\frac{4}{4}$ cost 4 times $1400 = $5600. Now the whole farm costs $5600, $\therefore$ $\frac{1}{7}$ of it costs $\frac{1}{7}$ of $5600 = $800, and $\frac{5}{7}$ cost 5 times $800 = $4000, Ans.

14. If $\frac{5}{8}$ of a cord of wood are bought for 3\frac{3}{4}$, what will $\frac{3}{4}$ of a cord cost?

15. If $\frac{3}{16}$ of a ship are worth $8769, what is the value of $\frac{5}{8}$ of her?

16. If $\frac{4}{9}$ of the distance from A to B is 32 miles, what is $\frac{5}{12}$ of the distance from A to B?

17. If 3 men build $\frac{5}{8}$ of a rod of wall in an hour, how many rods will 4 men build in 6 hours?

18. If 6 men can do a piece of work in $3\frac{1}{2}$ days, how long will it take 4 men to do the same work?

19. What cost 6lb. of sugar, at $8\frac{1}{3}$c. per lb.?

20. What shall I pay for $16\frac{1}{2}$lb. of rice, at 4c. per lb.?

21. Bought 4lb. of raisins, at $12\frac{1}{2}$c. per lb., and paid for them in eggs, at $16\frac{2}{3}$c. per dozen; how many dozen did it take?

22. What cost $12\frac{1}{2}$lb. of pork, at 6c. per pound?

23. If $\frac{5}{8}$ of a bushel of wheat cost $\$1\frac{1}{4}$, what is the cost of $12\frac{1}{2}$ bushels?

24. If 7bbl. of flour cost \$56, what will $3\frac{1}{2}$bbl. cost?

25. If $2\frac{1}{4}$ cords of wood will pay for 27 gallons of molasses, how many cords will pay for 4 times 27 gallons?

Ans. 4 times $2\frac{1}{4}$ cords, viz. 9 cords.

26. What cost $12\frac{1}{2}$ yards of silk at $\$1\frac{1}{4}$ per yard?

27. How many times will a wheel that is 9 feet in circumference turn round in running $20\frac{1}{2}$ miles?

28. How many cubic feet in a box that is $6\frac{1}{2}$ft. long, $5\frac{1}{3}$ft. wide, and $3\frac{3}{8}$ft. deep? Ans. 117. (See Art. 104).

29. How many bottles containing $1\frac{3}{4}$ pints each are required to bottle 21 gallons of wine?

30. What costs a farm of $75\frac{1}{2}$ acres at $\$96\frac{1}{2}$ per acre?

31. If it costs $\$8\frac{1}{4}$ to carry 13cwt. 3qr. $5\frac{3}{8}$ lb. $8\frac{1}{2}$ miles, how far can the same be carried for $\$16\frac{1}{2}$?

32. Bought $\frac{3}{4}$ of a 20-acre lot, and sold $\frac{1}{3}$ of the part purchased; how much had I remaining?

33. If $3\frac{3}{4}$ bushels of oats will sow an acre, how many bushels will sow $7\frac{1}{5}$ acres?

34. A staff 3ft. long cast a shadow $\frac{3}{4}$ of a foot at 12 o'clock; what is the length of a shadow cast by a steeple $125\frac{1}{2}$ft. high, at the same time?

35. If a staff 3ft. long casts a shadow of $\frac{3}{4}$ of a foot at 12 o'clock, what is the hight of a steeple that casts a shadow $31\frac{3}{8}$ft., at the same time?

36. Sold a watch for $\$43\frac{3}{4}$, which was $\frac{7}{8}$ of its cost; what was its cost?

37. How many pounds of butter in 24 firkins containing $33\frac{1}{2}$ lb. each, and what is it worth at $\frac{1}{4}$ of a dollar per pound?

38. If 6 is $\frac{3}{4}$ of some number, what is $5\frac{1}{2}$ times that number? Ans. 44.

Analysis. If 6 is $\frac{3}{4}$, then $\frac{1}{4}$ is $\frac{1}{3}$ of 6, which is 2, and $\frac{4}{4}$ are 4 times $2 = 8$. Since 8 is the number, $5\frac{1}{2}$ times the number will be $5\frac{1}{2}$ times $8 = 44$, Ans.

39. If 12 is $\frac{3}{5}$ of some number, what is $7\frac{2}{5}$ times that number?

40. Fifteen is $\frac{3}{8}$ of how many times 10? Ans. 4.

Analysis. If 15 is $\frac{3}{8}$, then $\frac{1}{8}$ is $\frac{1}{3}$ of $15 = 5$, and $\frac{8}{8}$ are 8 times $5 = 40$. Now 40 is 4 times 10; $\therefore$ 15 is $\frac{3}{8}$ of *four* times 10, Ans.

41. Twenty-four is $\frac{6}{11}$ of how many times 2? Ans. 22.

42. Thirty-five is $\frac{7}{4}$ of how many times 5?

43. Seven ninths of 72 are $\frac{4}{5}$ of how many times 7? Ans. 10.

Analysis. One ninth of 72 is 8, and $\frac{7}{9}$ are 7 times $8 = 56$; if 56 is $\frac{4}{5}$, then $\frac{1}{5}$ is $\frac{1}{4}$ of 56, which is 14, and $\frac{5}{5}$ are 5 times $14 = 70$. Now 70 is 10 times 7; $\therefore$ $\frac{7}{9}$ of 72 are $\frac{4}{5}$ of *ten* times 7, Ans.

44. Three eighths of 40 are $\frac{3}{7}$ of how many times 5? Ans. 7.

45. Seven eighths of 48 are $\frac{21}{16}$ of how many times 8?

46. Six fifths of 30 are $\frac{9}{8}$ of how many sixths of 24? Ans. 8.

Analysis. One fifth of 30 is 6, and $\frac{6}{5}$ are 6 times $6 = 36$; if 36 is $\frac{9}{8}$, then $\frac{1}{8}$ is $\frac{1}{9}$ of $36 = 4$, and $\frac{8}{8}$ are 8 times $4 = 32$. Now $\frac{1}{6}$ of 24 is 4, and 4 is contained 8 times in 32; $\therefore$ $\frac{6}{5}$ of 30 are $\frac{9}{8}$ of *eight* sixths of 24, Ans.

47. Five eighths of 64 are $\frac{4}{5}$ of how many thirds of 75?

48. Four sevenths of 35 are $\frac{2}{10}$ of how many eighths of 40?

49. Of the inhabitants of a certain town, $\frac{3}{8}$ are farmers, $\frac{1}{4}$ mechanics, $\frac{1}{10}$ manufacturers, $\frac{1}{5}$ students and professional men, and the remainder, numbering 246, are engaged in various occupations. What is the population of the town? Ans. 3280.

50. What would be the population of the town mentioned in Ex. 49, all the conditions remaining the same except that 246 shall be changed to 123? Ans. 1640.

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51. A certain room is $16\frac{1}{2}$ft. long, 15ft. wide, and 9ft. high; how many square feet in the walls?

Ans. 567. (See Art. 101).

52. What would be the cost of carpeting the room mentioned in Ex. 51, the carpet being 1yd. wide, and costing $\$1\frac{1}{2}$ per yd.?

53. A merchant bought $48\frac{3}{4}$lb. of butter of one customer, $28\frac{3}{4}$ of another, $25\frac{3}{16}$ of another, and $56\frac{5}{16}$ of another; how many pounds did he buy, and what was the cost of the whole at 25c. per pound?

54. In a certain school $\frac{1}{2}$ the scholars study arithmetic, $\frac{1}{4}$ algebra, $\frac{1}{12}$ geometry, and the remainder of the school, viz. 14 scholars, study surveying; how many scholars are there in the school? Ans. 84.

55. How many scholars would there be in the school mentioned in Ex. 54, if only seven scholars studied surveying?

56. A fox has 16 rods the start of a hound, but the hound runs 22 rods while the fox runs 20; how many rods will the fox run before the hound overtakes him?

57. A fox has 18 rods the start of a hound, but the hound runs 25 rods while the fox runs 22; how far must the hound run to overtake the fox?

58. A boy being asked how many doves he had, replied that if he had as many more, $\frac{1}{2}$ as many more and 6 doves, he should have 56; how many doves had he?

59. A boy being asked how many lambs he had, replied that if he had twice as many more, $\frac{1}{2}$ as many more and $5\frac{1}{2}$ lambs, he should have 30; how many lambs had he?

60. If 2 be added to each term of the fraction $\frac{3}{4}$, will the value of the fraction be increased or diminished?

Ans. Increased by $\frac{1}{12}$.

61. If 2 be added to each term of the fraction $\frac{4}{3}$, will its value be increased or diminished? Ans. Diminished by $\frac{2}{15}$.

62. If 2 be added to each term of the fraction $\frac{3}{3}$, will its value be increased or diminished? Ans. Neither.

What principle is involved in the last three examples? How would the values of the several fractions in the last three examples be affected if 2 were *subtracted* from each term?

63. A merchant owning $\frac{3}{8}$ of a ship, sold $\frac{2}{3}$ of his share for \$3000; what was the value of the ship? Ans. \$12000.

64. A can do a piece of work in 6 days, and B in 12 days; in what time can A and B together do the work?

65. A, B, and C can do a piece of work in 4 days; A and B can do it in 5 days; in what time can C do it?

66. Bought a pair of oxen and a horse for \$180. The oxen cost $\frac{4}{5}$ of the price of the horse; what was the price of the horse?

67. Bought a pair of oxen and a horse for \$175, and a wagon for $\frac{3}{5}$ of the price of the horse. The horse cost $\frac{3}{4}$ as much as the oxen; what was the price of the wagon? Ans. \$45.

68. Six men are to be clothed with cloth that is $1\frac{1}{2}$yd. wide. Now if it takes $2\frac{2}{3}$yd. of this cloth for each man, how many yards of cloth $\frac{3}{4}$yd. wide will be sufficient to line all the garments?

69. A gentleman gave $\frac{1}{2}$ of his estate to his wife, $\frac{2}{3}$ of the remainder to his son, and $\frac{1}{2}$ of what then remained to his daughter, who received \$$376\frac{1}{8}$; what was the value of the estate?

70. Sold a watch for \$$37\frac{1}{2}$, which was $\frac{3}{4}$ of its cost; what was lost by the transactions?

71. If a man earn \$$1\frac{2}{3}$ per day, in how many days will he earn \$100?

72. How many miles of furrow will be turned in plowing an acre, if the furrows are $\frac{3}{4}$ of a foot wide?

73. If a man can do a piece of work in 9 days by working $14\frac{2}{3}$ hours per day, in how many days, of $8\frac{1}{4}$ hours each, can he do the same work?

74. How many pounds of butter, at $\frac{1}{4}$ of a dollar per pound, will pay for 9 pounds of coffee at $\frac{3}{8}$ of a dollar per pound?

75. If $1\frac{7}{8}$ yards of cloth are required for 1 coat, how many coats may be made from $16\frac{7}{8}$ yards?

76. If $15\frac{3}{4}$ yards of silk make a dress, and 3 dresses be made from a piece containing 50 yards, what remnant will be left?

77. How many square feet of boards will be required to make 3 dozen boxes whose inner dimensions shall be $2\frac{1}{2}$ feet in length and breadth, and $1\frac{3}{4}$ feet in depth, the boards being 1 inch in thickness? Ans. 1163.

78. How many feet will be required to make 36 boxes whose *outer* dimensions are the same as the *inner* dimensions given in Ex. 77, the boards being of the same thickness; and what is the difference in the capacity of the two sets of boxes in cubic inches. Ans. 1001ft.; 144144c. in.

DECIMAL FRACTIONS.

155. A DECIMAL FRACTION is a fraction whose denominator is 10, 100, 1000, or 1 with one or more ciphers annexed.

NOTE 1. The word *decimal* is derived from the Latin *decem*, which signifies *ten*.

NOTE 2. By the word *decimal* we usually mean a *decimal fraction*.

156. The denominator of a *Common Fraction* may be *any number whatever*. Every principle and every operation in Common Fractions is equally applicable to Decimals.

157. The denominator of a decimal fraction is not usually expressed, since it can be easily determined, it being 1 *with as many ciphers annexed as there are figures in the given decimal.*

158. A decimal fraction is distinguished from a whole number by a *period*, called the *decimal point* or *separatrix*, placed before the decimal; the first figure at the right of the point is *tenths;* the second, *hundredths;* the third, *thousandths;* etc.; thus, $.6 = \frac{6}{10}$, $.06 = \frac{6}{100}$, $.006 = \frac{6}{1000}$, etc., the figures in the decimal decreasing in value from left to right, as in whole numbers (Art. 15).

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155. What is a Decimal Fraction? Decimal, from what derived? What is usually meant by the word decimal? **156.** A Common Fraction, what is its denominator? Are the principles of common fractions applicable to decimals? **157.** Is the denominator of a decimal usually expressed? **158.** How is a decimal fraction distinguished from a whole number? What is the first figure at the right of the point? Second? Third?

159. Since whole numbers and decimal fractions both decrease by the same law from left to right, they may be expressed together in the same example, and numerated as in the following

NUMERATION TABLE.

Thousands,	Hundreds,	Tens,	Units,	.	Tenths,	Hundredths,	Thousandths,	Ten-Thousandths,	Hundred-Thousandths,	Millionths,	Ten-Millionths,	Hundred-Millionths,	Billionths,	Ten-Billionths,	Hundred-Billionths,	Trillionths,	Ten-Trillionths,	Etc., etc.
3	4	7	1	.	6	5	9	8	7	2	8	4	3	2	1	6	5	

160. A whole number and decimal fraction written together, as in the above table, form a *mixed number.* The integral part is numerated from the decimal point toward the *left*, and the fraction from the same point toward the *right*, each figure, both in the whole number and decimal, *taking its name and value by its distance from the decimal point.* Hence,

161. Moving the decimal point one place toward the *right*, *multiplies* the number by 10; moving the point two places multiplies the number by 100, etc. Also moving the point one place to the *left*, *divides* the number by 10; moving the point two places divides by 100, etc.

162. In reading a decimal. we may give the name to each figure separately, or we may read it as we read a whole number, and *give the name of the right-hand figure only;* thus, the expression .23 may be read $\frac{2}{10}$ and $\frac{3}{100}$, or it may be read $\frac{23}{100}$, for $\frac{2}{10}$ and $\frac{3}{100} = \frac{20}{100}$ and $\frac{3}{100} = \frac{23}{100}$.

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159. Read the Numeration Table. **160.** What is a mixed number? Which way is the integral part numerated? Which way the decimal? What determines the name and value of a figure? **161.** How does moving the decimal point to the *right* affect the value of a number? How moving it to the *left?* **162.** In what two ways may a decimal be read? Illustrate.

163. To read a decimal fraction as we read a whole number, requires two numerations; first, *from* the decimal point, to determine the *denominator*, and second, *towards* the point, to determine the *numerator;* thus, to read the following: .3578692, first, to determine *the denominator or name of the right-hand figure*, beginning at the 3, say, tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, ten-millionths; and then, to determine *the value of the numerator, or name of the left-hand figure considered as an integer*, beginning at the 2, say, units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, and then read, three million, five hundred and seventy-eight thousand, six hundred and ninety-two *ten-millionths.*

164. Since multiplying both terms of a fraction by the same number does not alter its value (Art. 147, a, Note 1), *annexing* one or more ciphers to a decimal does not affect its value; thus, $\frac{2}{10} = \frac{20}{100} = \frac{200}{1000}$, etc.; i. e. $.2 = .20 = .200$, etc.

165. *Prefixing* a cipher to a decimal, i. e. inserting a cipher between the separatrix and a decimal figure, *diminishes* the value of that figure to $\frac{1}{10}$ its previous value; for it removes the figure one place further from the decimal point (Art. 161); thus, $.3 = \frac{3}{10}$, but $.03 =$ only $\frac{3}{100}$, which is but $\frac{1}{10}$ of $\frac{3}{10}$.

What is the effect of prefixing two, three, or more ciphers to a decimal?

166. A common fraction is sometimes annexed to a decimal; thus, $.2\frac{1}{4}$. This is equivalent to the complex fraction $\frac{2\frac{1}{4}}{10}$. The common fraction is never to be counted as a decimal place, but it is always a fraction of a unit of that order represented by the preceding decimal figure; thus, in $.234\frac{1}{2}$, the $\frac{1}{2}$ is half of *a thousandth.*

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163. To read a decimal requires how many numerations? First, which way? For what purpose? Second, which way? For what? Illustrate. **164.** How is the value of a decimal affected by *annexing* a cipher? Why? **165.** How by *prefixing* a cipher? Why? **166.** A common fraction annexed to a decimal, what is it? Illustrate.

Notation and Numeration of Decimal Fractions.

167. Let the pupil express in figures the following numbers:

1. Fifty-two hundredths. Ans. .52.
2. Four hundred and sixteen thousandths. Ans. .416.
3. Three hundred and forty-two ten-thousandths.

Note 1. An ambiguity often arises in enunciating a whole number and a decimal in the same example; thus, .203 is two hundred and three thousandths, and 200.003 is two hundred, and three thousandths. This ambiguity may, however, be avoided by placing the word *decimal* before the fraction; thus, 200.003 may be read two hundred and *decimal* three thousandths.

Note 2. In decimals, as in whole numbers (Art. 16), ciphers are used to fill places that would otherwise be vacant.

4. Write the decimal six hundred and forty-one thousandths.
5. Decimal five hundred and eighteen ten-thousandths.
6. Eight hundred and decimal eight thousandths.
7. Six thousand and decimal six millionths.
8. Nine hundred and thirty and eight tenths.
9. Decimal two hundred and forty-six ten-millionths.
10. One thousand and decimal two hundred-thousandths.
11. Eleven and eleven ten-billionths.
12. Six hundred and sixteen and sixteen trillionths.
13. Ten thousand and decimal four ten-thousandths.
14. Decimal three hundred twenty-five thousand, four hundred and eighty-seven hundred-millionths.

168. Write the following numbers in words, or read them orally:

1.	42.56	7.	3694.876942
2.	3.789	8.	760.4070823
3.	892.6758	9.	4004.40040004
4.	987.23876	10.	3333.33333333
5.	29.00045	11.	46.00046482
6.	1.800647	12.	8769.27642935

167. What uncertainty often exists in reading mixed numbers? How can this ambiguity be avoided? For what are ciphers used in the notation of decimals?

NOTE 1. Addition, subtraction, multiplication, and division of decimal fractions are performed precisely as the same operations in whole numbers, no further explanation being necessary, except to determine the place of the decimal point in the several results.

NOTE 2. The *proofs* are the same as in whole numbers.

PROBLEM 1.

169. To add decimal fractions:

RULE. *Place tenths under tenths, hundredths under hundredths, etc.; then add as in whole numbers, and place the point in the sum directly under the points in the numbers added.*

	Ex. 1.	2.	3.
	36.472	356.842	564.987426
	84.926	387.646	42.86539
	28.047	984.285	874.827641
Sum,	149.445	1728.773	1482.680457
Proof,	149.445	1728.773	1482.680457

4.	5.
872143.872954	348209.15342687415
2410.402683	270.423
791842.2163	3429.87251342179
841.360498	865123.71942
724310.006843	319817.05841628347

6. Add 42.76, 934.247, 27.862. Ans. 1004.869.

7. Add 3.546, 44.8693, 2.8769, and 734.68723.

8. 872.34, 6789.3274, 22.987, and 346.42.

9. Add 3582.47, 62.84693, .47249, and 7.458.

10. Add five hundred and decimal six thousandths; forty-five millionths; eighty-four million and decimal twelve millionths; seventy thousandths; and decimal three hundred and fifty-four hundred-thousandths. Ans. 84000500.079597.

11. What is the sum of one thousand two hundred twenty-six

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168. How are addition, subtraction, multiplication, and division of decimals performed? Proofs? **169.** Rule for addition? The point, where placed?

and decimal one hundred and forty-four thousandths; twenty-five and sixty-two hundredths; and eight hundred forty-nine and sixty-three hundredths?

12. What is the sum of fifty hundred-thousandths; eighteen hundred and decimal sixty-three ten-thousandths; seventy-four and seventeen hundred-thousandths?

PROBLEM 2.

170. To subtract a less decimal from a greater:

RULE. *Place the less number under the greater, tenths under tenths, etc.; then subtract as in whole numbers, and place the point in the remainder, directly under the points in the minuend and subtrahend.*

	Ex. 1.	2.	3.
From	6.4279	47.42964	49.0684
Take	2.8946	18.16293	21.874693
Rem.	3.5333	29.26671	27.193707
Proof,	6.4279	47.42964	

NOTE. If, as in Ex. 3, there are more figures in the subtrahend than in the minuend, the deficiency may be supplied by annexing ciphers, or *supposing* them annexed, to the minuend (Art. 164).

4. From 65.8487 take 24.3869. Ans. 41.4618.
5. From 1684.469 take 368.8743352.
6. From 9846.2764 take 5427.9824.
7. From 2140.6872 take 1724.1943.
8. From one thousand eight hundred seventy-six and decimal three hundred sixty-four thousandths, take eight hundred sixteen and decimal three hundred and three thousandths. Ans. 1060.061.
9. From ten take six millionths.
10. A man owned eighty-seven hundredths of a railroad and sold forty-eight hundredths of it; what part of the road did he still own?

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170. Rule for subtraction of decimals? When the number of decimal places in the subtrahend exceeds the number of decimal places in the minuend?

PROBLEM 3.

171. To multiply one decimal by another:

RULE. *Multiply as in whole numbers, and point off as many figures for decimals in the product as there are decimal places in both factors, counted together.*

Ex. 1. Multiply .48 by .26.

	OPERATION.		PROOF.
Multiplicand,	.4 8		.2 6
Multiplier,	.2 6		.4 8
	2 8 8		2 0 8
	9 6		1 0 4
Product,	.1 2 4 8	=	.1 2 4 8

(a) If the number of figures in the product is less than the number of decimal places in the two factors, the deficiency must be supplied by *prefixing ciphers to the product*, as in Ex. 3.

	2.	3.
Multiplicand,	2 6.2 9 8 3	.3 2
Multiplier,	8.4	.2 3
	1 0 5 1 9 3 2	9 6
	2 1 0 3 8 6 4	6 4
Product,	2 2 0.9 0 5 7 2	.0 7 3 6

NOTE 1. The reason of the rule for pointing the product will be obvious if we change the decimals to the form of common fractions and then perform the multiplication;

Thus, $.48 \times .26 = \frac{48}{100} \times \frac{26}{100} = \frac{1248}{10000} = .1248$, as in Ex. 1.
Again, $.32 \times .23 = \frac{32}{100} \times \frac{23}{100} = \frac{736}{10000} = .0736$, as in Ex. 3.

NOTE 2. The reason of the rule for pointing the product may be explained in another manner, as follows:

The *smaller the factors are, the smaller is the product.* Now, by trial, we know that

$32 \times 23 = 736$; ∴, dividing one factor by 10 (Art. 161), we have
$32 \times 2.3 = 73.6 = \frac{1}{10}$ of the previous product; dividing again by 10,
$32 \times .23 = 7.36 = \frac{1}{10}$ of the 2d product; dividing the other factor by 10,
$3.2 \times .23 = .736 = \frac{1}{10}$ of the 3d product; dividing again by 10,
$.32 \times .23 = .0736 = \frac{1}{10}$ of the 4th product; dividing again by 10,
$.032 \times .23 = .00736 = \frac{1}{10}$ of the 5th product; and so on to any extent.

	4.	5.
Multiplicand,	4 2 3.6	.3 2 5 9
Multiplier,	.5 4	.0 0 0 0 2 5
	1 6 9 4 4	1 6 2 9 5
	2 1 1 8 0	6 5 1 8
Product,	2 2 8.7 4 4	.0 0 0 0 0 8 1 4 7 5

6. Multiply .5642 by .37. Ans. .208754.
7. Multiply 34.87 by 4.5. Ans. 156.915.
8. Multiply 2769 by .84. Ans. 2325.96.
9. Multiply .2436 by .034. Ans. .0082824.
10. Multiply .0068 by .003. Ans. .0000204.
11. Multiply 36.874 by .5421.
12. Multiply .14687 by .00054.
13. Multiply .17288 by .14403.
14. Multiply .00369 by .24683.
15. Multiply 8.756 by 10. Ans. 87.56 (See Art. 161).
16. Multiply 356.4 by 100. Ans. 35640.
17. Multiply 9.8765 by 1000.
18. Multiply 348.69 by 100000.
19. Multiply 236.487 by 100000.
20. Multiply 374.28 by 100000.
21. Multiply 4.68 by 20. Ans. 93.6.

In Ex. 21 multiply by the factors of 20, viz. 10 and 2; i. e. move the point one place to the right, and then multiply by 2.

22. Multiply 36.42 by 60. Ans. 2185.2.
23. Multiply 472.8 by 800. Ans. 378240.
24. Multiply 36.74 by 300.
25. Multiply 54.26 by 406000.
26. Multiply three hundred and fifty-six thousandths by one hundred and forty-five ten-thousandths. Ans. .005162.
27. Multiply thirty-four millionths by twenty-six ten-millionths.
28. Multiply eight hundred and forty-two thousandths by five hundred thousand.

---

171. Rule for multiplication of decimals? Suppose there are not figures enough in the product? Reason of the rule for pointing the product? Second explanation?

PROBLEM 4.

172. To divide one decimal fraction by another:

RULE. *Divide as in whole numbers, and point off as many figures for decimals in the quotient as the number of decimal places in the dividend exceeds those in the divisor.*

Ex. 1. Divide .625 by .25.

OPERATION.	PROOF.
.2 5) .6 2 5 (2.5	.2 5 Divisor.
5 0	2.5 Quotient.
1 2 5	1 2 5
1 2 5	5 0
0	.6 2 5 Dividend.

2. Divide 1.2575125 by 2.5. Ans. .503005.
3. Divide 8.43648108 by .06. Ans. 140.608018.

(a) If the number of figures in the quotient is less than the excess of decimal places in the dividend over those of the divisor, supply the deficiency by *prefixing ciphers to the quotient.*

4. Divide .000744 by .62. Ans. .0012.

NOTE 1. The dividend is a product, the divisor and quotient being the factors (Art. 77); hence the rule for pointing the quotient.

NOTE 2. The rule for determining the place of the point in the quotient may also be explained by changing the decimals to the form of common fractions and performing the division; thus,

$$.625 \div .25 = \tfrac{625}{1000} \div \tfrac{25}{100} = \tfrac{25}{10} = 2.5.$$

NOTE 3. By attending to the relative size of divisor and dividend (Art. 83), we have another mode of fixing the place of the decimal point in the quotient; thus,

$625 \div 25 = 25$; ∴, by dividing the *dividend* by 10 (Art. 161), we have
$62.5 \div 25 = 2.5 = \frac{1}{10}$ of the preceding quotient; dividing again by 10,
$6.25 \div 25 = .25 = \frac{1}{10}$ of the 2d quotient; dividing again by 10,
$.625 \div 25 = .025 = \frac{1}{10}$ of the 3d quotient. Now dividing the *divisor* by 10,
$.625 \div 2.5 = .25 = 10$ times the 4th quotient; dividing again by 10,
$.625 \div .25 = 2.5 = 10$ times the 5th quotient; and so on to any extent.

---

172. Rule for dividing decimals? What is said of ciphers in the quotient? Reason of the rule for pointing the quotient? Second explanation? Third?

5. Divide 38.7425 by .25. Ans. 154.97.
6. Divide .09936 by .276. Ans. .36.
7. Divide .000975 by .15.
8. Divide 17.472 by .48.
9. Divide 234.7744 by 62.44.
10. Divide 58.794 by 12.3.

(b) If there are more decimal places in the divisor than in the dividend, the number may be made equal by annexing one or more ciphers to the dividend. The quotient will then be a whole number; thus, $4.5 \div .18 = 4.50 \div .18 = 25$.

11. Divide 3647 by .125. Ans. 29176.
12. Divide 90321.6 by 3.642. Ans. 24800.
13. Divide 72 by .064.

(c) If there is a remainder after all the figures of the dividend have been used, the division may be continued by annexing ciphers to the dividend. Each cipher annexed becomes a decimal place in the dividend.

In some examples this operation may be continued until there is no remainder, but in others there will *necessarily* be a remainder, however far the operation may be continued. This latter class of examples gives rise to *circulating decimals*; thus, $.7 \div .9 = .7777$, etc. Again, $.8 \div .11 = .727272$, etc. In the first of these examples, the figure 7 will be repeated perpetually, and in the second example, the figures 7 and 2 will be repeated in like manner. Whenever the *remainder* consists of the same figure or figures as any preceding *dividend*, the quotient figures will begin to repeat.

It may be remarked, however, that, if the divisor contains no prime factors but 2's and 5's, the divison can *always* be continued until there shall be no remainder; but if there is any *other* prime factor in the divisor, the division *can never be completed* unless the *same* other factor is in the original dividend; for a

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172. What shall be done when there are more decimal places in the divisor than in the dividend? What is done when there is a remainder? The cipher annexed is what? When can the division be completed? When can it *not* be completed? Why?

dividend is not divisible by a divisor unless it contains *all* the factors of the divisor; whereas annexing ciphers to the dividend introduces no prime factor into it except 2's and 5's.

14. Divide .13 by 8.
15. Divide 7.2 by .16.
16. Divide 8.7 by .25.
17. Divide 3.6 by 7.5.

NOTE 4. When a decimal is not complete, we sometimes place the sign + after it, signifying that there is a remainder.

18. Divide .34 by .24. Ans. 1.4166+.
19. Divide .73 by 1.5.
20. Divide 4.63 by 2.9.
21. Divide 36.5 by 10. Ans. 3.65 (See Art. 161).
22. Divide 4.69 by 100. Ans. .0469.
23. Divide 846.9 by 100.
24. Divide 5.647 by 1000.
25. Divide 843.57 by 300. Ans. 2.8119.

In Ex. 25, divide by the factors of 300, viz. 100 and 3; i. e. move the point two places to the left and then divide by 3.

26. Divide 3.6412 by 400. Ans. .009103.
27. Divide 56.427 by 8000.
28. Divide 36.49 by 600.
29. Divide three thousand eight hundred and fifty-three hundred-thousandths by thirty-two millionths. Ans. 1204.0625.
30. Divide eighty-four and eighty-four hundredths by forty-eight thousandths.

PROBLEM 5.

173. To reduce a common fraction to a decimal.

Ex. 1. Reduce $\frac{3}{4}$ to a decimal fraction.

$$\tfrac{3}{4} \times 100 = \tfrac{300}{4} = 75\text{; and } 75 \div 100 = .75, \text{ Ans.}$$

If a number be multiplied by any number, and the product be divided by the multiplier, the quotient will be the multiplicand

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172. For what is the sign + sometimes used?

(Art. 84, c). Now, in the above example, $\frac{3}{4}$ is multiplied by 100 by annexing two ciphers to the numerator; the fraction $\frac{300}{4}$ is then reduced to the whole number 75, and, finally, 75 is divided by 100 by placing the decimal point before the 75; $\therefore \frac{3}{4} = .75$. Hence,

RULE. *Annex one or more ciphers to the numerator and divide the result by the denominator, continuing the operation until there is no remainder, or as far as is desirable. Point off as many decimal places in the quotient as there are ciphers annexed to the numerator.*

2. Reduce $\frac{3}{8}$ to a decimal fraction.

$\frac{3}{8} \times 1000 = \frac{3000}{8} = 375$; and $375 \div 1000 = .375$, Ans.

3. Reduce $\frac{7}{16}$ to a decimal. Ans. .4375.
4. Reduce $\frac{73}{64}$ to a decimal. Ans. 1.140625.
5. Reduce $\frac{77}{32}$ to a decimal.
6. Reduce $\frac{7}{12}$ to a decimal. Ans. .5833+.
7. Reduce $\frac{1}{3}$ to a decimal. Ans. .3333+.
8. Reduce $\frac{3}{7}$ to a decimal. Ans. .428571+.
9. Reduce $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{13}{16}$, $\frac{7}{64}$, $\frac{7}{25}$, and $\frac{17}{50}$ to decimals.

174. *Every decimal fraction is a common fraction*, and, if its denominator be written, it will *appear* as such. It may then be reduced to lower terms, or modified like any other common fraction. This proves the rule in Art. 173.

10. Reduce .48 to the form of a common fraction and then to its lowest terms. $.48 = \frac{48}{100} = \frac{12}{25}$, Ans.

11. Reduce .125 to its lowest terms.

$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$, Ans.

12. Reduce .17 to the form of a common fraction.

Ans. $\frac{17}{100}$.

13. Reduce .275, .325, .00025, and .00625.
14. Reduce 2.8. $2.8 = \frac{28}{10} = \frac{14}{5}$, Ans.
15. Reduce 1.5, 3.75, 8.25, 9.125, and 2.0125.

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173. Rule for reducing a common fraction to a decimal? Explanation?
174. Is a decimal also a common fraction? How is this made evident? How may the rule in Art. 173 be proved correct?

Problem 6.

175. To reduce whole numbers of lower denominations to the decimal of a higher denomination.

Ex. 1. Reduce 2pk. 3qt. to the decimal of a bushel.

1st. 3qt. $=\frac{3}{8}$pk. $=$ .375pk.; $\therefore$ 2pk. and 3qt. $=$ 2.375pk.

2d. 2.375pk. $=\frac{2.375}{4}$bush. $=$ .59375bush., Ans.

The principle is the same as in Art. 173. Hence,

Rule. *Having annexed one or more ciphers to the lowest denomination, divide by the number it takes of that denomination to make one of the next higher, and annex the quotient as a decimal to that next higher; then divide the result by the number it takes of* THIS *denomination to make one of the* NEXT *higher, and so continue till it is brought to the denomination required.*

2. Reduce 9s. 6d. 3qr. to the decimal of a pound.

OPERATION.

4	3.0 0 qr.
1 2	6.7 5 0 0 d.
2 0	9.5 6 2 5 0 0 s.
	.4 7 8 1 2 5 £, Ans.

3qr. $=$ .75d.; 6.75d. $=$ .5625s.; 9.5625s. $=$ .478125£, Ans.

Note. In dividing by 20 to reduce the decimal of a pound, and in all similar examples, we may point off the 0 in the divisor, and then divide by 2, but in such a case *the point in the dividend must be moved one place toward the left,* for by so doing both divisor and dividend are divided by 10, and $\therefore$ the quotient is unchanged (Art. 84, b).

3. Reduce 2ft. 9in. 1 b. c. to the decimal of a yard.

OPERATION.

3	1.0 0 0 0 0 0 b. c.
12	9.3 3 3 3 3 3 + in.
3	2.7 7 7 7 7 7 + ft.
	.9 2 5 9 2 5 + yd., Ans.

In this example there will be a remainder, however far the operation is carried.

4. Reduce 3cwt. 2qr. 20lb. 8oz. to the decimal of a ton.

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175. Rule for reducing the lower denominations of a compound number to the decimal of a higher denomination? Principle? Mode of dividing when the divisor is 20, 40, etc ? When the divisor is a mixed number?

5. Reduce 3oz. 12dwt. 18gr. to the decimal of a pound, Troy Weight. Ans. .303125.

6. Reduce 6℥ 4ʒ 1℈ 10gr. to the decimal of a pound.

7. Reduce 5yd. 2ft. 6in. to the decimal of a rod, Long Measure.

OPERATION.

12	6.0 in.
3	2.5 0 0 0 ft.
5½	5.8 3 3 3 + yd.
2	2
1 1	1 1.6 6 6 6 + half yd.
	1.0 6 0 6 + rods, Ans.

Since one of the divisors, in this example, is 5½, both divisor and dividend are reduced to halves. The feet and inches are more than a half yard; ∴ the sum of the given numbers is more than a rod.

8. Reduce 3s. 15° 30″ to the decimal of a circumference. Ans. .291689+.

9. Reduce 2d. 6h. 18m. 24sec. to the decimal of a week.

10. Reduce 2qt. 1pt. 1gi. to the decimal of a gallon.

11. Reduce 3fur. 8ch. 2rd. 10li. to the decimal of a mile.

12. Reduce 8cu. ft. 144c. in. to the decimal of a cubic yard.

13. Reduce 3r. 2rd. 20yd. to the decimal of an acre.

14. Reduce 5fur. 30rd. 5yd. 1ft. 9in. 2b. c. to the decimal of a mile.

Problem 7.

176. To reduce a decimal of a higher denomination to whole numbers of lower denominations.

Ex. 1. Reduce .428125£ to shillings, pence, and farthings.

OPERATION.

```
£.4 2 8 1 2 5
          2 0
8.5 6 2 5 0 0 s.
      1 2
6.7 5 0 0 d.
    4
3.0 0 qr.
```

Ans. 8s. 6d. 3qr.

This article is the reverse of Art. 175; ∴ first multiply by 20, because there will be 20 times as many shillings as pounds. For a like reason, multiply the *fractional part* of a shilling by 12, to reduce it to pence, etc. After having fixed the decimal point in the several products, *the ciphers at the* RIGHT *of the significant figures are disregarded.*

RULE. *Multiply the given decimal by the number it takes of the next lower denomination to make one of this higher, and place the decimal point as in multiplication of decimals; multiply the* DECIMAL PART *of this product by the number it takes of the* NEXT *lower denomination to make one of* THIS, *and so proceed as far as necessary. The several numbers at the left of the points will be the answer.*

2. Reduce .984375 of a bushel to pecks, quarts, and pints.
Ans. 3pk. 7qt. 1pt.

3. Reduce .40625 of a gallon to quarts, pints, and gills.

4. Reduce .902288 of a lunar month to weeks, days, hours, minutes, and seconds. Ans. 3w. 4d. 6h. 20m. 15.1296sec.

5. Reduce .90625 of a yard to quarters, nails, etc.

6. What is the value of .375°? Ans. 22′ 30″.

7. What is the value of .375 of a ton?

8. What is the value of .4658 of a pound, Troy Weight?

9. Reduce .3587 of a mile to furlongs, rods, yards, etc.

10. Reduce .562℔ to ℥, ʒ, etc.

MISCELLANEOUS EXAMPLES IN DECIMAL FRACTIONS.

1. What is the cost of 6.25 lb. of beef, at 12 cents per pound?
Ans. 75c.

2. Bought 4.5 tons of hay, at $12.50 per ton; what was the cost of the whole? Ans. $56.25.

3. What is the value of 8 acres of land, at $62.50 per acre?

4. Paid $500 for 8 acres of land; what was the price per acre?

5. Paid $500 for a piece of land at $62.50 per acre; how many acres were bought?

6. Bought land at $62.50 per acre, and sold it again at $75 per acre, thereby making $100; how many acres were bought?

7. Bought 8 acres of land at $62.50 per acre, and sold the lot for $600; was there a gain or a loss? How much total? How much per acre?

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176. Rule for reducing a decimal of a higher denomination to whole numbers of lower denominations? Explanation?

8. What cost 43a. 3r. 20rd. of land, at $40 per acre?

9. What cost 3t. 15cwt. 2qr. 12½lb. of coal, at $6 per ton?

10. What cost 12.25 cords of wood, at $6 per cord?

11. What cost 7¾ cords of wood, at $6.25 per cord?

12. What will it cost to build 24m. 3fur. 20rd. of railroad, at $5775 per mile?

13. A rectangular field is 40.5 rods long, and 30.5 rods wide; what will it cost to build a wall around it, at $1 per rod?

14. What cost 3yd. 3qr. 2na. of cloth, at 16c. per yard?

15. How much land in a rectangular field that is 40.5 rods long and 25.75 rods wide?

16. What would 16 bales of cotton cost, each bale weighing 4.5cwt., at $10.50 per cwt.?

17. What cost .825 of a ton of coal, at $7 per ton?

18. What cost .825 cwt. of coal, at $7 per ton?

19. What is the value of .25 of a ton of hay, at 2£ 5s. 6d. 1qr. per ton?

20. What is the value of .75 cwt. of hay, at 2£ 5s. 6d. 1qr. per ton?

21. Paid 3£ 9s. 6d. 1qr. per acre, for 5a. 2r. 15rd. of land; what was the entire cost?

22. If 365¼ days make a year, how many days, hours, etc., are there in .785 of a year?

23. What is the cost of 3 pieces of cloth, the first containing 15 yards, at $2.25 a yard; the second, 12.5 yards, at $3.50 a yard; and the third, 8.8 yards, at $3.25 a yard?

24. A three-sided plat of ground is inclosed by a railroad on one side, and highways on the other two sides; the side next the railroad is 4.1 rods long, and the other two sides are respectively 4 rods and .9 of a rod in length; what is the cost of fencing this plat, the fence costing $3.75 a rod?

25. If a boat sails 8.75 miles an hour, how far will it sail in 8.4 hours?

26. How many bins, each holding 37.5 bushels, will be filled with 1687.5 bushels of grain?

27. How many coats, each requiring 2.75 yards of cloth, may be made from 35.75 yards?

28. In how many days will a man earn \$20.125, if he earn \$1.75 a day?

29. How many square feet in a board which is 18.25 feet long and 2.8 feet wide?

30. Bought a load of straw that weighed 1t. 2cwt. 3qr. 12½lb., at \$8 a ton; what shall I pay for the load?

31. Paid \$7.175 for 35 gall. 3qrt. 1pt. of vinegar; what was the price per gallon?

32. If a pole 12.5 feet long casts a shadow 3.125 feet at 12 o'clock, what is the hight of a steeple that casts a shadow 33.28125 feet at the same time?

33. What is the cost of carpeting a room that is 16.5 feet long, and 15 feet wide, the carpet costing, \$1.25 per square yard?

UNITED STATES MONEY.

177. UNITED STATES MONEY, sometimes called *Federal Money*, is the currency of the United States.

TABLE.

10 Mills (m.)	make	1 Cent,	marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	\$
10 Dollars	"	1 Eagle,	"	e.

						Cents.		Mills.
				Dimes.		1	=	10
		Dollars.		1	=	10	=	100
Eagle.		1	=	10	=	100	=	1000
1	=	10	=	100	=	1000	=	10000

NOTE. The terms eagle and dime are seldom used in computation; eagles and dollars being read collectively and called dollars, and dimes and cents being called cents; thus, 3 eagles and 5 dollars are called \$35, and 4 dimes and 3 cents are called 43 cents.

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177. What is United States Money? Repeat the Table. Are the terms eagle and dime much used?

178. The currency of the United States being based upon the *Decimal Notation*, most of the necessary rules for operations in this currency, and also many examples, have already been given; but the importance of the subject justifies a separate consideration of it.

179. A *coin* is a piece of gold, silver, or other metal, stamped by authority of the General Government, to be used as money.

180. The coins authorized by our Government, and stamped at the U. S. Mint, are the following:

Gold.		Silver.	
Double Eagle,	\$20.00	Dollar,	\$1.00
Eagle,	10.00	Half Dollar,	.50
Half Eagle,	5.00	Quarter Dollar,	.25
Quarter Eagle,	2.50	Dime,	.10
Three-Dollar Piece,	3.00	Half Dime,	.05
One Dollar,	1.00	Three-Cent Piece,	.03

Also of Copper and Nickel, the Cent, .01

181. Gold and silver, for coinage, are *hardened* by being mixed with harder and cheaper metals. These cheaper metals, when combined with the gold and silver, are called *alloys.*

182. *Carat* is a term used in indicating the *purity* or *fineness* of gold. If a piece of metal is *pure* gold it is said to be 24 carats fine; if $\frac{23}{24}$ of it are gold, and the remaining $\frac{1}{24}$ is alloy, it is 23 carats fine; etc., etc.

183. The *standard* purity of gold and silver coin at the U. S. Mint, is $\frac{9}{10}$ of pure metal and $\frac{1}{10}$ alloy. The alloy in silver coin is pure copper. The alloy in gold coin is copper and silver, the silver not to exceed the copper.

(a) The new cent is composed of 88 parts of copper for 12 parts of nickel.

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178. On what is the currency of the U. S. based? **179.** What is a coin? **180.** What gold coins are authorized by our Government? What silver coins? Of what is the cent made? **181.** What is alloy? For what used? **182.** For what is the term carat used? Pure gold is how many carats fine? **183.** What is the standard purity of gold and silver coin? What is the alloy for silver? What for gold? What part of the new cent is nickel?

NOTE 1. The *copper cent* is still in use, but is no longer coined at the U. S. Mint.

NOTE 2. The *mill* is not coined.

NOTE 3. Other pieces of money, as the 50-dollar gold piece, the half and quarter dollar gold pieces, are in use to some extent, but are not *legal coin.*

NOTE 4. The greater part of the money in general use, consists of *bank bills*, which are much more convenient for most purposes than gold and silver.

184. The *weight* of the eagle is 258 grains, Troy. The silver dollar weighs 412½ grains, but the smaller coins are not so heavy in proportion to their value; thus, the half dollar weighs only 192 grains; the quarter, only 96 grains, etc. The new cent weighs 72 grains.

NOTE. These standards of weight and purity are regulated by Congress, and may be changed at any time.

185. In this currency, the *dollar* is the *unit*, cents and mills being *decimals* of a dollar; thus, $3.62 represents three dollars and sixty-two cents; $4.085 represents four dollars, eight cents, and five mills, etc.

NOTE. Figures at the right of the third decimal place, represent *parts of mills;* thus, $5.3627 = 5 dollars, 36 cents, 2 mills, and $\frac{7}{10}$ of a mill.

REDUCTION.

186. The *reduction* of U. S. Currency is very simple. *Dollars are reduced to cents by annexing two ciphers* (Art. 62), *and to mills by annexing three ciphers;* thus $4 = 400 cents = 4000 mills.

Dollars and cents are reduced to cents by removing the decimal point; thus, $3.56 = 356 cents. Dollars, cents, and mills

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183. Is the mill coined? What of other pieces of money? What of paper money? **184.** What is the weight of the eagle? Of the silver dollar? Half dollar? By whom is the standard of weight and purity fixed? **185.** What is the *unit* in this currency? What are cents and mills? What are figures at the right of the third decimal place? **186.** How are dollars reduced to cents? How to mills? How are dollars and cents reduced to cents? How dollars, cents, and mills to mills?

are reduced to mills in the same way; thus, $5.468 = 5468 mills.

Ex. 1. Reduce $47 to cents. Ans. 4700 cents.
2. Reduce $34.56 to cents. Ans. 3456 cents.
3. Reduce $3.456 to mills. Ans. 3456 mills.
4. Reduce $483 to cents. To mills.
5. Reduce $6.84 to cents. To mills.
6. Reduce $1.876 to mills.

187. *Cents are reduced to dollars by pointing off two decimal places* (Art. 81). *Mills are reduced to dollars by pointing off three decimal places;* thus, 3768 cents = $37.68; 3768 mills = $3.768.

7. Reduce 564 cents to dollars. Ans. $5.64.
8. Reduce 3692 mills to dollars. Ans. $3.692.
9. Reduce 87694 cents to dollars.
10. Reduce 76843 mills to dollars.

188. *Addition, Subtraction, Multiplication, and Division of U. S. currency, are performed precisely as the corresponding operations in Decimal Fractions.*

ADDITION.

	Ex. 1.	2.	3.
	$75.564	$876.542	$56487.33
	24.876	397.428	4296.87
	96.445	679.324	44.98
Sum,	$196.885		

4. Paid $87.50 for a horse, $145.25 for a pair of oxen, $14.25 for a wagon, and $45.75 for a cart; what did I pay for all? Ans. $292.75.

5. Bought a hat for $4.50, a coat for $18.75, a vest for $5.25, and a pair of boots for $5; what did I pay for all?

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187. How are cents reduced to dollars? How mills to dollars? **188.** How are Addition, Subtraction, Multiplication, and Division of U. S. Money performed?

SUBTRACTION.

	Ex. 1.	2.	3.
From	\$487.964	\$63.87	\$86.485
Take	\$268.788	\$47.43	\$44.368
Ans.	\$219.176		

4. A man who owed \$699.60, paid \$164.60; how much did he still owe? Ans. \$535.

5. Bought a farm for \$3684.75, and stock and tools for the farm for \$1476.25; how much more did I pay for the farm than for the stock and tools?

MULTIPLICATION.

	Ex. 1.	Ex. 2.
Multiply	\$348.765	\$3684.375
By	254	2437
	1395060	
	1743825	3.
	697530	\$4386.942
		369
Ans.	\$88586.310	

4. If 12 gentlemen have \$7497.84 apiece, what sum have they all? Ans. \$89974.08.

5. If 45 persons deposit \$346.25 each in a savings bank, how many dollars are deposited?

DIVISION.

Ex. 1. If \$225 are divided equally between 27 men, what sum will each receive?

OPERATION.

```
27)$225($8.33 1/3, Ans.
    216
    ---
      90
      81
      --
       90
       81
       --
        9
```

Dividing 225 by 27, gives 8 for quotient and 9 for remainder. Annexing ciphers and continuing the division, as in Decimal Fractions (Art. 172, c), we obtain \$8.33⅓ for the share of each man.

2. Divide $69345.36 equally between 18 men.

Ans. $3852.52.

3. Divide $4832.40 into 24 equal parts.

PRACTICAL EXAMPLES.

189. To find the cost of a number of things when the price of one thing is given.

1. If apples are worth $2.50 per barrel, what are 3 barrels worth?

Three barrels are worth 3 times as much as one barrel, ∴ 3 barrels are worth $2.50 × 3 = $7.50, Ans. Hence,

RULE. *Multiply the price of one by the number.*

2. What is the cost of 9 barrels of flour, at $7.75 per barrel?

Ans. $69.75.

3. Bought 25 sheep, at $6.25 each; what was the cost of the flock?

4. Bought 18 yards of broadcloth, at $3.875 per yard; what was the cost of the piece?

5. What is the value of 75 acres of land, at $37.50 per acre?

190. To find the price of an article when the cost of a given number of articles is known.

6. When eight cords of wood are worth $44, what is the value of 1 cord?

If 8 cords are worth $44, one cord is worth $\frac{1}{8}$ of $44; and $44 ÷ 8 = $5.50, Ans. Hence,

RULE. *Divide the cost by the number.*

7. If 24 yards of broadcloth cost $93, what is the price per yard? Ans. $3.87½.

8. Bought 37 pounds of butter for $8.51, what was the price?

Ans. 23c.

---

189. How is the cost of a number of things found when the price of one is known? **190.** How the price of one when the cost of a number is known?

NOTE. *Price* is, appropriately, the sum asked for *one* article; thus, when any one asks a flour dealer the *price* of flour, he is understood to ask what he must pay *for a single barrel*, not *fifty* barrels, nor *half a barrel*, nor *any quantity* except *one barrel*. Hence we distinguish between *price* and *cost*, or *price* and *value*.

9. Bought 356 bbls. of flour for \$3026; what was the price?

10. Bought a farm containing 125 acres for \$6843.75; what was the price per acre?

191. To find the quantity when the cost of the quantity and the price of one are given.

11. At \$6 per ton, how many tons of coal can I buy for \$24?

I can buy as many tons as \$6 is contained times in \$24, and $\$24 \div \$6 = 4$, $\therefore$ I can buy 4 tons. Hence,

RULE. *Divide the cost by the price of one.*

12. At \$3 per yard, how many yards of cloth can be bought for \$546? Ans. 182.

13. At \$22.50 per acre, how many acres of land can be bought for \$1822.50?

14. At 56 cents a pound, how many pounds of tea may be bought for \$25.20?

15. A drover bought oxen at \$62.50 each; how many oxen did he buy for \$1562.50?

192. To find the cost of articles sold by the 100 or by the 1000.

16. At \$4.50 per 100 feet, what will 342 feet of timber cost?

OPERATION.

```
   $4.50
    3.42
   -----
     900
    1800
   1350
 -------
$15.3900, Ans.
```

Had the price been \$4.50 per *foot*, the cost would have been $\$4.50 \times 342 = \$1539$; but since the price is \$4.50 per *hundred* feet, the true multiplier is *one hundredth part of* 342, viz. 3.42, and the true cost is $\$4.50 \times 3.42 = \$15.39$.

---

190. Meaning of *price?* Difference between price and cost, or price and value?
191. Rule for finding the number of things when the cost and price are known?
192. Explain Ex. 16.

Had the price been \$4.50 per *thousand* feet, the true multiplier would have been .342, and the cost would have been \$4.50 $\times$ .342 = \$1.539. Hence,

RULE. *First reduce the quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand, as the example may require; then multiply the price by the quantity, and point the product as in multiplication of decimals* (Art. 171).

NOTE 1. C is used to indicate *hundreds*, and M to indicate *thousands*.

17. What cost 1200 feet of boards, at \$2.10 per C?

Ans. \$25.20.

18. What cost 12514 feet of timber, at \$13.50 per M?

Ans. \$168.939.

NOTE 2. In business transactions the answer to Ex. 18 would be called \$168.94. In the remaining examples in U. S. Money, the mills in the answers will be omitted if less than 5, and one will be added to the cents if the mills are 5 or more.

19. What cost 20000 shaved pine shingles, at \$6 per M?

20. What cost 13725 bricks, at \$6.50 per M?

Ans. \$89.21.

(a) To find the cost of articles sold by the ton.

21. What cost 2440 lb. of hay, at \$18.50 per ton?

OPERATION.

$$\begin{array}{r} 2\,)\,\underline{2.440} \\ 1.22 \\ \underline{18\tfrac{1}{2}} \\ 61 \\ 976 \\ \underline{122} \\ \$\,22.57,\ \text{Ans.} \end{array}$$

First divide by 2000 (i. e. point off three decimal places and divide by 2), to reduce the weight to tons and decimals of a ton; then multiply by the price.

In multiplying, the 50 cents may be used *decimally*, or the *common* fraction, ½, may be used, as in the operation.

22. What cost 5848 lb. of coal, at \$6.25 per ton?

Ans. \$18.28.

---

192. Rule for finding the cost of articles sold by the 100 or 1000. For what is C used? M? What is Note 2? Mode of finding the cost of articles sold by the ton?

193. To find the cost or value of any number of articles when the price is an aliquot part of a dollar.

TABLE OF ALIQUOT PARTS OF A DOLLAR.

50 cents = $\frac{1}{2}$ of a dollar,	20 cents = $\frac{1}{5}$ of a dollar,
$33\frac{1}{3}$ cents = $\frac{1}{3}$ of a dollar,	$16\frac{2}{3}$ cents = $\frac{1}{6}$ of a dollar,
25 cents = $\frac{1}{4}$ of a dollar,	$12\frac{1}{2}$ cents = $\frac{1}{8}$ of a dollar.

23. What cost 64 yards of cloth, at $87\frac{1}{2}$ cents per yard?

OPERATION.

	\$64	= cost of 64yd. at	\$1.
	32	= cost of 64yd. at	50c., or $\frac{1}{2}$ of \$1.
	16	= cost of 64yd. at	25c., or $\frac{1}{2}$ of 50c.
	8	= cost of 64yd. at	$12\frac{1}{2}$c., or $\frac{1}{2}$ of 25c.
Ans.	\$56	= cost of 64yd. at	$87\frac{1}{2}$c.

The cost at \$1 is evidently as many dollars as there are yards; the cost at 50c. is half as much as at \$1; the cost at 25c., half as much as at 50c.; and the cost at $12\frac{1}{2}$c., half as much as at 25c. Then the cost at 50c., at 25c., and at $12\frac{1}{2}$c., added, gives the cost at $87\frac{1}{2}$c.

This process is usually called *Practice*, for which we have the following

RULE. *Take such aliquot parts* (Art. 119, Note) *of the number of articles as the price is of* \$1.

24. What cost 48 barrels of apples, at $\$3.37\frac{1}{2}$ per barrel?

OPERATION.

	\$48	= cost at	\$1.
	3		
	\$144	= cost at	\$3.
	12	= cost at	.25c., or $\frac{1}{4}$ of \$1.
	6	= cost at	$.12\frac{1}{2}$c., or $\frac{1}{2}$ of 25c.
Ans.	\$162	= cost at	$\$3.37\frac{1}{2}$c.

25. What cost 24 barrels of flour at $\$6.33\frac{1}{3}$ per barrel?

Ans. \$152.

---

193. Rule for finding the cost when the price is an aliquot part of a dollar? What is this process called? Name the most convenient aliquot parts of a dollar.

26. What cost 48 lb. of raisins, at $12\frac{1}{2}$c. per pound?
27. What cost 54 yd. of calico, at $16\frac{2}{3}$c. per yard?
28. What cost 75 bush. of apples, at $33\frac{1}{3}$c. per bushel?
29. What cost 40 pairs of gloves, at 50c. per pair?
30. What cost 36 sheep, at $5.66\frac{2}{3}$ each?

194. To find the cost when the number of articles is expressed by a compound or by a mixed number.

31. What cost 9a. 3r. 20rd. of land, at $40 per acre?

OPERATION.

$40,	price per acre.
9	
$360	= cost of 9a.
20	= cost of 2r., or $\frac{1}{2}$a.
10	= cost of 1r., or $\frac{1}{2}$ of 2r.
5	= cost of 20rd., or $\frac{1}{2}$r.
$395	= cost of 9a. 3r. 20rd., Ans.

32. What cost $8\frac{3}{4}$ shares of railroad stock, at $108.50 per share?

OPERATION.

$108.50,	price per share.
$8\frac{3}{4}$	
$868.00	= cost of 8 shares,
54.25	= cost of $\frac{1}{2}$ share,
27.13	= cost of $\frac{1}{4}$ share,
$949.38	= cost of $8\frac{3}{4}$ shares, Ans.

This process is also called *Practice*, and may be stated thus:

Multiply the price by the entire number of articles, and to this product add such aliquot parts of the price as the fractional part of the number is of a unit.

33. What cost 3t. 16cwt. 1qr. 20lb. of hay, at $16 per ton?
Ans. $61.16.
34. What cost 6c. 5c. ft. 8cu. ft. of wood, at $6 per cord?
35. What cost $24\frac{3}{8}$ acres of land, at $48.72 per acre?

---

194. Rule for finding the cost when the number of articles is expressed by a compound or by a mixed number?

195. To exchange goods.

36. How many pounds of butter, at 20c. per pound, shall be given in exchange for 4 yards of cloth, at \$2.37½ per yard?

SOLUTION. One yard costs \$2.37½, ∴ 4 yards cost 4 times \$2.37½ = \$9.50. Now since the price of the butter, 20c., is $\frac{1}{5}$ of a dollar, it will require five times as many pounds of butter as there are dollars in the cost of the cloth, and 5 times 9.5 = 47.5, or 47½, number of pounds of butter required, Ans.
Dividing \$9.50 by 20c. will give 47.5, or 47½, the same result as before.

This exchanging of goods is usually called *Barter*. The examples are solved by *Analysis*.

37. How many pounds of sugar, at 12½c. per pound, may be bought for 3 bushels of corn, at 87½c. per bushel? Ans. 21.

38. How many cords of wood, at \$5.50 per cord, shall be given in exchange for a barrel of flour, at \$7.50, and 5 yards of cloth, at \$2.35 per yard?

BILLS.

196. A BILL OF GOODS is a written statement of articles sold, giving the price of each article and the cost of the whole.

Find the cost of the several articles, and the amount or footing of each of the following bills.

(1.) *Boston, Jan.* 1, 1862.

Mr. ABEL SNOW,

Bought of JOHN ADAMS,

25 *lb. N. O. Sugar,*	*at*	9*c.*	
40 *lb. Maple Sugar,*	"	18¾*c.*	
6 *lb. Cheese,*	"	12½*c.*	
8 *lb. Butter,*	"	23*c.*	
4 *lb. Raisins,*	"	15*c.*	
2 *lb. Cream Tartar,*	"	45*c.*	
			\$13.84

Received Payment,

JOHN ADAMS.

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195. What is Barter? How are examples in barter solved? 196. What is a Bill of goods?

(2.) *New York, Jan.* 15, 1862.

Mr. CHARLES B. SMITH,

Bought of JAMES PHILLIPS,

8 *yd. Blue Broadcloth,*	*at*	$3.50	
10 *yd. Black Broadcloth,*	"	3.75	
7 *yd. Cassimere,*	"	1.25	
4 *yd. Black Satin,*	"	4.50	
			$92.25

Received Payment,

JAMES PHILLIPS,

By E. LOW.

(3.) *Philadelphia, Mar.* 1, 1862.

Mr. S. STEWART,

1861. *To* HOLT, WILDER & Co., *Dr.*

June 5.	*To*	6 *Webster's Dictionaries,*	*at*	$6.00	
Aug. 18.	"	12 *Day's Algebras,*	"	1.50	
Oct. 25.	"	36 *Testaments,*	"	.25	
Dec. 12.	"	9 *Folio Bibles,*	"	2.50	
					$85.50

Received Payment,

S. DANIELS,

For HOLT, WILDER & Co.

(4.) *Cincinnati, Mar.* 1, 1862.

Mr. A. P. JEWETT,

1861. *To* SAMUEL PALMER, *Dr.*

Apr. 8.	*To*	16750*ft. Boards,*	*at*	$12.50 *per M.*	$209.38
	"	1750*ft. Boards,*	"	24.00 *per M.*	
May 15.	"	3500*ft. Plank,*	"	25.00 *per M.*	
					$338.88

1861. *Cr.*

May 5.	*By*	3 *Tons Hay, at*	$15.50	$46.50	
July 18.	"	*Cash,*		50.00	
Sept. 12.	"	4 *Cords Wood, at*		6.00	
					$120.50
		Balance due S. P.			$218.38

Received Payment,

SAMUEL PALMER.

MISCELLANEOUS EXAMPLES IN U. S. MONEY.

1. What cost 3½ yards of ribbon, at 56c. per yard?

2. What cost 3 barrels of flour, at $7.62½ per barrel?

3. If 4 cords of wood cost $22.50, what is the price per cord?

4. If 15 yards of silk cost $16.87½, what is the price per yard?

5. If a merchant deposits $375.50 in a bank at one time, and $487.75 at another, how much will remain after he has withdrawn $176.37 and $346.83?

6. A merchant bought 75 barrels of flour for $650 and sold 25 barrels at $9.50 per barrel, and the remainder at $9.25 per barrel; did he gain or lose? How much? Ans. Gained $50.

7. What cost 87½ rods of wall, at 75c. per rod?

8. Reduce $28.756 to mills.

9. Reduce $6.18 to mills.

10. Reduce 54598 cents to dollars.

11. Reduce 47689 mills to dollars.

12. My farm cost $3725 and my house cost $1862.75; how much more did the farm cost than the house?

13. A gentleman bequeathed $750 to each of his 3 sons, and $500 to each of his 4 daughters; how much did he bequeath to his children?

14. Paid $16.50 for a coat, $4.25 for a vest, $5.75 for a pair of pants, $3.50 for a hat, $4.37½ for a pair of boots, and $12.62½ for other articles; what did I pay for all?

15. Divide $113.75 equally between 7 men.

16. Paid $68.75 for flour, at $6.25 per barrel; how many barrels did I buy?

17. How many yards of lace, at 62½c. per yard, may be bought for $3.75?

18. What cost 8725 feet of boards, at $12.50 per M?

19. What cost 8248 lb. of coal, at $6 per ton?

20. What cost 3a. 2r. 20rd. of land, at $48 per acre?

21. How many pounds of sugar, at 12½c. per pound, will pay for 12 dozen eggs at 16¾c. per dozen?

22. My real estate is worth $4756.75 and my personal estate $4562.75, I owe $2468.50; what am I worth?

23. At 25c. per mile for a horse and carriage, how far may I ride for $3.37½?

24. A drover bought sheep at $3.37½ per head and sold them at $3.87½ per head, and gained $37.50 by the transactions; how many sheep did he buy?

25. Bought 100 sheep at $3.375, and sold them again at $3.875; what was the gain per head and total?

26. Bought 20.5 tons of hay at $12.375 per ton; what was the cost of the whole?

27. What is the value of 67.75 acres of land at $62.50 per acre?

28. Paid $4234.375 for 67.75 acres of land; what was the price per acre?

29. Paid $4234.375 for a piece of land at $62.50 per acre; how many acres were bought?

30. Bought land at $62.50 per acre, and sold it again at $75 per acre, thereby making $846.875; how many acres were bought?

31. Bought 67.75 acres of land at $62.50 per acre, and sold the lot for $5081.25; was there a gain, or loss? how much total and per acre?

32. Bought 356.25lb. of wool at 37½c., which was manufactured into cloth at an expense of $62.50; for what sum must it be sold to gain $37.50?

33. Bought 14.75yd. of sheeting at 14 cents per yd.; what was the cost of the piece?

34. What would 7⅜ bales of cotton cost, each bale weighing 6.375cwt., at $11.75 per cwt.?

35. What cost 13yd. 2qr. 3na. of cloth at $4.67 per ell French, the ell French being 6qr.? Ans. $42.61⅜.

36. Bought 1bbl. flour at $12.50, 3bush. corn at 87½c., 24.5 lb. sugar at 8½c., 3gal. molasses at 37½c., 2lb. tea at 62½c., 6lb. coffee at 11c., 15lbs. rice at 4¼c. and 4lb. butter at 22c.; what was the cost of the whole? Ans. $21.76.

37. What cost 3t. 15cwt. 2qr. 12½lb. coal at $9.75 per ton?

38. What will be the expense of papering a room that is 20 feet long, 15 feet wide and 8.5 feet high, a roll of paper being 8 yards in length and $\frac{5}{8}$ of a yard in width, and costing 62½c. per roll?

39. Bought 133.5yd. of broadcloth at \$3.25, and sold 33yd. of it at \$3.33⅓, 50yd. at \$3.875, and the remainder at \$3.60; how much was gained by the transactions?

COMPOUND NUMBERS.

ADDITION.

197. A Compound Number is composed of two or more denominations (Art. 86) which do not usually increase decimally from right to left; consequently, in adding the different denominations, we do not carry one for *ten, but for the number it takes of the particular denomination added, to make a unit of the next higher denomination;* thus, in adding Sterling or English Money, we carry 1 for 4, 12, and 20, because 4qr. make 1d., 12d. make 1s., and 20s. make 1£.

Ex. 1. Add 6£ 7s. 9d. 3qr., 5£ 12s. 11d. 2qr., 27£ 18s. 10d. 3qr., and 19£ 14s. 8d. 1qr.

OPERATION.

	£	s.	d.	qr.
	6	7	9	3
	5	12	11	2
	27	18	10	3
	19	14	8	1
Sum,	59	14	4	1

Having arranged the numbers as in the margin, the *amount* of the right-hand column is 9qr. = 2d. and 1qr. Upon the same principle as in addition of simple numbers, the 1qr. is set under the column of farthings and the 2d. are added to the pence in the example, making 40d. = 3s. and 4d. Setting the 4d. under the the column of pence, add the 3s. to the shillings in the example, making 54s. = 2£ and 14s., and so proceed, until all the columns are added.

---

197. Do Compound Numbers increase decimally? Explain Ex. 1.

198. The *principle* of procedure is precisely the same as in addition of simple numbers. Hence,

To add compound numbers,

RULE. *Write the numbers so that each denomination shall occupy a separate column, the lowest denomination at the right, and the others towards the left in the order of their values. Add the numbers in the lowest denomination, divide the amount by the number it takes of this denomination to make one of the next higher, set the remainder under the column, and carry the quotient to the next column. So proceed until all the columns are added.*

199. PROOF. *The same as in Addition of Simple Numbers* (Art. 47).

2.

	£	s.	d.	qr.
	91	4	7	1
	48	9	0	3
	10	3	0	1
	36	8	4	3
	67	4	8	3
Sum,	253	9	9	3
Proof,	253	9	9	3

3.

£	s.	d.
36	14	9
18	12	11
64	8	4
56	13	6
42	12	10
219	2	4

4.

gal.	qt.	pt.
3	2	1
1	1	0
1	3	1
4	2	1
2	0	1
13	2	0

NOTE 1. In writing and adding the numbers of *a single denomination*, the rules of simple addition must be observed; thus, in writing the pounds, in Ex. 2, set units under units, and tens under tens, and then, having added the farthings, pence, and shillings, add the *units* of the pounds, and then the *tens*, as in addition of simple numbers.

5.

lb.	oz.	dwt.	gr.
8	4	18	22
3	6	4	8
5	11	12	18
6	8	14	12
2	10	16	23

6.

gal.	qt.	pt.	gi.
3	3	1	3
4	1	1	2
7	2	0	3
4	3	1	0
9	1	0	2

7.

a.	r.	rd.
4	1	25
6	3	16
1	2	38
2	0	14
6	2	24

198. Rule for Addition of Compound Numbers? Principle? **199.** Proof? Numbers of a single denomination, how written and added?

8.

lb.	oz.	dr.	sc.	gr.
8	4	6	2	18
6	9	2	1	4
2	1	0	2	16
8	8	3	2	6

9.

bush.	pk.	qt.	pt.
6	3	7	1
8	1	2	0
9	2	6	1
4	0	2	1

10.

c.	c. ft.	cu. ft.	c. in.
4	3	14	1600
2	4	8	128
3	6	10	864
7	7	4	900

11.

l. m.	wk.	d.	h.	m.	sec.
2	3	4	18	40	30
3	3	6	6	20	30
5	1	2	20	30	15
8	3	0	2	28	45

12.

t.	cwt.	qr.	lb.	oz.
6	14	2	20	8
4	6	2	10	8
3	18	3	10	12
4	6	3	18	6

13.

circ.	s.	°	′	″
2	8	20	40	50
1	4	12	18	20
6	6	25	50	7
4	9	29	49	59

14.

yd.	qr.	na.	in.
3	3	3	2
8	2	3	$1\frac{1}{2}$
6	3	1	0
7	1	2	2

15.

	fur.	rd.	yd.	ft.	in.	b. c.
	1	5	3	2	10	1
	2	4	4	2	4	2
	3	6	5	0	6	2
	1	3	4	2	7	0
	7	21	$1\frac{1}{2}$	2	4	2
or	7	21	2	0	10	2

16.

		yd.	ft.	in.
		4	2	4
		3	1	7
		5	0	6
	rd.	4	2	7
	3	$1\frac{1}{2}$	1	0
or	3	1	2	6

NOTE 2. A fraction occurring in the amount may sometimes be reduced to whole numbers of other denominations; thus, in Ex. 15, the half yard equals 1ft. and 6in.; the 6in. put with the 4in. make 10in. and the 1ft. put with the 2ft. make 3ft. or 1yd. 0ft., and, finally, the 1yd. put with the 1yd. in the original amount gives 2yd. The *answer*, when reduced, may contain a denomination higher or lower than any in the given example; higher, as in Ex. 16; lower, as in Ex. 17.

---

199. What may be done with a fraction in the amount? Explain Ex. 15. Ex. 16. Ex. 17. May the answer contain a higher or lower denomination than the example? How?

17.

	a.	r.	rd.	yd.	ft.	
	5	3	30	20	4	
	6	2	12	27	7	
	12	2	3	17¾	2	in.
or	12	2	3	17	8	108

18.

m.	fur.	rd.	ft.
4	7	39	16
3	6	8	12

19.

t.	cwt.	qr.	lb.	oz.	dr.
4	6	2	20	8	12
2	14	3	5	7	4
3	8	1	16	12	8
1	7	0	24	4	4
9	19	3	1	15	5
4	6	0	0	4	15

20.

bush.	pk.	qt.	pt.
1	3	7	1
4	2	4	0
5	0	6	1
3	3	3	1
6	1	0	0
5	2	5	1

21. Bought 4 pieces of cloth, measuring 6yd. 3qr. 1na. 2in., 8yd. 2qr. 3na. 1in., 25yd. 1qr. 2na. 2in., and 14yd. 3qr. 2na. 1in.; how much cloth did I buy?

22. A farmer raised in one field 21bush. 3pk. 7qt. 1pt. of wheat; in another, 48bush. 2pk. 1pt.; in another, 28bush 6qt.; and in another, 75bush. 1pk. 5qt. 1pt.; how much wheat did he raise in the 4 fields?

23. A planter sold cotton at various times, as follows: 2t. 18cwt. 2qr. 12½lb., 6t. 1cwt. 1qr. 6¼lb., 3t. 19cwt. 3qr. 18¾lb., 16t. 6cwt. 3qr. 12½lb., and 16t. 3qr. 18lb.; what did he sell in all?

24. What is the sum of 14a. 2r. 30rd. 25yd. 3ft. 72in., 37a. 3r. 39rd. 30yd. 6ft. 36in., 50a. 1r. 18rd. 25yd. 2ft. 108in., and 25a. 2r. 25rd. 25yd. 3ft. 72in.?

25. Add 3circ. 9s. 29° 59′ 59″, 2circ. 11s. 25° 20′ 30″, 5circ. 4s. 8° 25′ 55″, and 6circ. 10s. 10° 10′ 10″ together.

26. A horse traveled 35m. 6fur. 18rd. 5yd. in one day, 42m. 3fur. 25rd. 2yd. the next day, 37m. 5fur. 32rd. 4yd. the next, and 45m. 7fur. 24rd. 3yd. the next; how far did he travel in the 4 days?

27. A blacksmith bought 4t. 18cwt. 3qr. 20lb. of iron at one time, 6t. 15cwt. 3qr. 12lb. at another time, 3t. 6cwt. 1qr. 18lb. at another, and 8t. 3cwt. 2qr. 10lb. at another; how much did he buy in all?

14*

SUBTRACTION.

200. The principle is like that of subtraction of simple numbers. Hence,

To subtract compound numbers,

RULE. 1. *Write the less quantity under the greater, arranging the denominations as in addition.*

2. *Beginning at the right, take each denomination of the subtrahend from the number above it, and set the remainder beneath.*

3. *If any number of the subtrahend is greater than the number above it, add to the upper number as many as it takes of that denomination to make one of the next higher, and take the subtrahend from the* SUM; *set down the remainder, and, considering the number in the next denomination in the minuend* ONE LESS, *or that in the subtrahend* ONE GREATER, *proceed as before.*

201. PROOF. *As in subtraction of simple numbers* (Art. 53).

Ex. 1. From 8£ 6s. 9d. 3qr. take 2£ 4s. 5d. 1qr.

OPERATION.

	£	s.	d.	qr.
Min.,	8	6	9	3
Sub.,	2	4	5	1
Rem.,	6	2	4	2
Proof,	8	6	9	3

Only the 1st and 2d sections of the rule apply to this example.

2. From 9£ 6s. 10d. 1qr. take 2£ 17s. 2d. 3qr.

OPERATION.

	£	s.	d.	qr.
Min.,	9	6	10	1
Sub.,	2	17	2	3
Rem.,	6	9	7	2
Proof,	9	6	10	1

As 3qr. cannot be taken from 1qr., borrow one of the 10d., reduce it to farthings and add it to the 1qr., giving 5qr.; then say 3qr. from 5qr. leave 2qr. Now, as one of the 10d. has been employed, say 2d. from 9d., or, what is practically the same, 3d. from 10d. leave 7d., and so proceed through the example.

---

200. Rule for subtracting compound numbers? Principle? **201.** Proof? Explain Ex. 1. Ex. 2.

The *form* of the minuend may be changed and the work performed as follows (Art. 53, Ex. 28):

SECOND OPERATION.

	£	s.	d.	qr.		£	s.	d.	qr.
Min.,	9	6	10	1	=	8	26	9	5
Sub.,	2	17	2	3		2	17	2	3
Rem.,	6	9	7	2	=	6	9	7	2

3.

	t.	cwt.	qr.	lb.	oz.	dr.
From	12	8	3	22	6	15
Take	3	19	2	18	8	12
Rem.,	8	9	1	3	14	3
Proof,	12	8	3	22	6	15

4.

lb.	oz.	dr.	sc.	gr.
6	4	3	1	18
2	3	6	2	12
4	0	4	2	6

5.

	yd.	qr.	na.	in.
From	16	1	2	1
Take	6	3	1	2

6.

l.	m.	fur.	rd.	yd.	ft.	in.
6	2	4	27	5	1	8
2	2	2	35	2	2	5

7.

	a.	r.	rd.	yd.	ft.	in.
From	6	2	25	30	4	134
Take	1	3	39	5	8	140

8.

gal.	qt.	pt.	gi.
14	2	0	3
5	3	1	2

9.

	lb.	oz.	dwt.	gr.
Min.,	6	5	15	22
Sub.,	3	10	12	23
Rem.,	2	7	2	23
Proof,	6	5	15	22

10.

c.	c. ft.	cu. ft.	cu. in.
25	4	15	1727
4	7	5	169

11.

	bush.	pk.	qt.	pt.
Min.,	125	1	5	1
Sub.,	24	3	7	1

12.

wk.	d.	h.	m.	sec
3	4	23	45	30
1	6	16	30	45

201. Explain 2d operation in Ex. 2.

202. Sometimes, as in the following examples, it is necessary to borrow *two* of the higher denomination of the minuend instead of *one;* but in all such cases we must carry *two* to the next term of the subtrahend; i. e. *we must* PAY *as much as we* BORROW.

13.

	rd.	yd.	ft.	in.	b.c.		rd.	yd.	ft.	in.	b.c.
From	12	0	2	6	1	=	10	10	4	17	4
Take	3	5	2	8	2		3	5	2	8	2
Rem.,	7	5	2	9	2	=	7	5	2	9	2
Proof,	12	0	2	6	1	=	10	10	4	17	4

14.

	a.	r.	rd.	yd.	ft.	in.		a.	r.	rd.	yd.	ft.	in.
From	7	2	0	0	5	124	=	6	4	78	60	9	196
Take	1	3	39	30	8	143		1	3	39	30	8	143
Rem.,	5	1	39	29½	5	125	=	5	1	39	30	1	53
Proof,	7	2	0	0	5	124	=	6	4	78	60	9	196

15.

	m.	fur.	rd.	yd.	ft.
Min.,	6	3	7	0	1
Sub.,	2	5	5	5	2
Rem.,	3	6	0	5	2
Proof,	6	3	7	0	1

16.

circ.	deg.	m.	fur.	rd.	yd.	ft.
7	0	0	1	0	0	1
2	27	69	3	39	5	2

203. To find the time between two dates.

17. What is the difference of time between July 15, 1857, and Apr. 25, 1862? Ans. 4yr. 9m. 10d.

FIRST OPERATION. SECOND OPERATION.

	yr.	m.	d.		yr.	m.	d.
Min.,	1862	4	25	or	1861	3	24
Sub.,	1857	7	15		1856	6	14
Rem.,	4	9	10	=	4	9	10

202. What is said of borrowing *two?* Explain Ex. 13. **203.** How many modes of finding the time between two dates? What are they?

NOTE. In subtracting an earlier from a later date, it is customary to consider 30 days a month. In the first operation, the *number* of the year, month, and day of the month, is used; in the second, the number of years, months, and days that *have elapsed* since the commencement of the Christian era, is used. The two operations give the same result, but *the first is most convenient.*

18. How long from the battle of Waterloo, June 18, 1815, to the death of Napoleon, May 5, 1821? Ans. 5yr. 10m. 17d.

19. How long from the battle of Lexington, Apr. 19, 1775, to the surrender of Cornwallis, Oct. 19, 1781?

20. How long from the inauguration of Washington, Apr. 30, 1789, to the battle of New Orleans, Jan. 8, 1815?

21. How long from the Declaration of Independence, July 4, 1776, to the present time?

22. Daniel Webster was born Jan. 18, 1782, and died Oct. 24, 1852; at what age did he die?

23. A note given July 6, 1857, was paid Sept. 9, 1861; how long was it on interest?

24. Find the time from Apr. 4, 1857, to Dec. 12, 1862.

25. Find the time from Dec. 16, 1839, to Mar. 26, 1848.

26. Find the time from Nov. 13, 1816, to May 12, 1841.

27. Find the time from June 21, 1842, to Feb. 20, 1860.

EXAMPLES IN ADDITION AND SUBTRACTION.

1. A farmer raised 150bush. 3pk. 4qt. of oats. Having sold 50bush. 2pk. and used 27bush. 1pk. 4qt., how many has he remaining? Ans. 73bush.

2. Having a journey of 127m. 4fur. 10rd. to perform in 3 days, I travel 48m. 2fur. 6rd. the first day, and 54m. 4rd. the second; how far must I travel on the third day?

3. I have one piece of land containing 47a. 3r. 25rd. and another containing 25a. 2r. 15rd.; how much land shall I have after selling 37a. 3r.?

4. From the sum of 8bush. 3pk. 2qt. 1pt. and 10bush. 2pk. 7qt. 1pt., take the difference between 54bush. 1pk. 3qt. 1pt. and 49bush. 3pk. 2qt. 1pt.? Ans. 15bush. 1qt.

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203. Which is preferable? How many days are considered a month?

5. From the sum of 5rd. 1yd. 2ft. 4in. 1b.c. and 4rd. 2yd. 1ft. 9in. 2b.c., take the difference between 10rd. 5yd. 2ft. 7in. 2b.c. and 1rd. 1yd. 1ft. 6in. Ans. 1b.c.

6. From a piece of silk measuring 49yd. 1qr. 3na. 2in., there were cut 3 dresses, the first measuring 15yd. 3qr. 1na. 1in., the second 14yd. 3qr. 3na. 1in., and the third 14yd. 2qr. 3na. 2in.; what remnant remained?

7. B sold an ox which weighed 16cwt. 1qr. 15lb., and 2 cows that weighed 6cwt. 1qr. 10lb. and 5cwt. 3qr. 20lb.; also 2 swine that weighed 4cwt. 3qr. 18lb. and 3cwt. 3qr. 24lb. How much more beef than pork did he sell?

8. If from 2 casks of wine, containing 63gal. 3qt. 1pt. 3gi. and 56gal. 2qt. 2gi., there be taken 75gal. 2qt. 1pt. 3gi., how many gallons, quarts, etc., will remain?

9. From a mass of silver weighing 47lb. 8oz. 16dwt. 22gr., a silversmith made 48 spoons weighing 7lb. 8dwt. 14gr. and a cake-basket weighing 3lb. 6oz. 8dwt. 15gr.; how much silver remained in the mass?

MULTIPLICATION.

204. In the multiplication of both simple and compound numbers, the *multiplier* is always and necessarily a simple abstract number; for, to attempt to multiply by a concrete number, e. g. 4 *miles* times 10, is, in the highest degree, absurd, though it is perfectly proper to say 10 times 4 miles. The product is of the same kind as the multiplicand; for repeating a number does not change its nature.

205. The principle is the same as in multiplication of simple numbers. Hence,

To multiply a compound by a simple number,

RULE. *Multiply the lowest denomination in the multiplicand, divide the product by the number it takes of that denomination*

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204. What is the multiplier in all cases? What the product? **205.** Rule? Proof? Explain Ex. 1.

to make one of the next higher, set down the remainder, add the quotient to the product of the next denomination, and so proceed.

Ex. 1.

	£	s.	d.	qr.
Multiply	4	6	8	3
By				7
Product,	30	7	1	1

First, 7 times 3qr. = 21qr. = 5d. and 1qr.; write the 1qr. under the farthings, and then say 7 times 8d. = 56d., and 5d. added give 61d. = 5s. and 1d., etc.

NOTE. Multiplication and division prove each other. It is profitable to teach reverse operations simultaneously.

2.

	fur.	rd.	yd.	ft.	in.	b.c.
Multiplicand,		5	3	1	4	1
Multiplier,						8
Product,	1	4	5	1	10	2

3.

gal.	qt.	pt.	gi.
6	2	1	3
			7
47	0	0	1

4.

	lb.	oz.	dwt.	gr.
Multiplicand,	4	6	8	20
Multiplier,				4
Product,	18	1	15	8

5.

lb.	oz.	dr.	sc.	gr.
2	10	6	2	15
				6

6.

t.	cwt.	qr.	lb.	oz.	dr.
3	15	2	24	15	8
					8

7.

yd.	qr.	na.	in.
6	2	3	2
			9

8.

wk.	d.	h.	m.	sec.
1	2	4	45	59
				3

9.

circ.	s.	°	′	″
5	8	20	30	25
				10

10.

gal.	qt.	pt.	gi.
8	3	1	2
			12

11.

bush.	pk.	qt.	pt.
8	3	7	1
			11

12.

c.	c.ft.	cu.ft.	cu.in.
12	7	15	1725
			4

13.

a.	r.	rd.	yd.	ft.	in.
7	3	39	30	8	143
					4

14.

	a.	r.	rd.	yd.	ft.	
	3	2	24	25	8	
					12	
	43	3	18.	7½	6	in.
or,	43	3	18	8	1	72

The ½yd. in the product equals 4½ft., i. e. 4ft. 72in.; the 4ft. put with the 6ft. make 10ft., or 1yd. 1ft.; and, finally, putting the 1yd. with the 7yd. gives 8yd. and the whole product, ∴, is 43a. 3r. 18rd. 8yd. 1ft. 72in., Ans.

15.

m.	fur.	rd.	yd.	ft.	in.	b.c.
2	3	3	4	1	6	1
						7

16.

a.	r.	rd.	yd.
7	2	20	25
			9

17. Bought 5 loads of wood, each measuring 1c. 5c. ft. 8cu. ft., at \$6 per cord; what was the quantity bought and the cost of the whole? Ans. 8c. 3c. ft. 8cu. ft.; \$50.62½.

18. If a ship sail 2° 30′ 20″ per day, how far will she sail in 8 days?

19. Multiply 8m. 6fur. 12rd. 3yd. 2ft. 6in. 1b. c., by 6.

20. If a man travel 25m. 6fur. 25rd. per day, how far will he travel in 9 days?

21. If the crop of hay on 1 acre is 2t. 15cwt. 2qr. 12½lb., what will be the crop on 10 acres?

22. What cost 7 yards of cloth, at 15s. 6d. 3qr. per yard?

23. How much wine in 3 casks containing 28gal. 3qt. 1pt. 2gi. each?

24. Multiply 9m. 7fur. 8ch. 3rd. 15li. 6in. by 8.

25. Multiply 3circ. 5s. 25° 18′ 25″ by 9.

206. To multiply by a composite number:

RULE. *Multiply by the factors of the multiplier* (see Art. 61).

205. Explain Ex. 14. 206. Rule when the multiplier is composite?

26. Multiply 4lb. 8oz. 16dwt. 20gr. by 72.

	lb.	oz.	dwt.	gr.
Multiplicand,	4	8	16	20
1st Factor of Multiplier,				8
Partial Product,	37	10	14	16
2d Factor of Multiplier,				9
Product,	341	0	12	0

27. Multiply 7£ 6s. 8d. 2qr. by 54.
28. Multiply 8bush. 3pk. 6qt. 1pt. by 81.
29. Multiply 6℔ 4℥ 7ʒ 2℈ 6gr. by 49.

207. To multiply when the multiplier is large and not composite.

30. Multiply 3t. 4cwt. 2qr. 6lb. 8oz. 4dr. by 23.

FIRST OPERATION.

t.	cwt.	qr.	lb.	oz.	dr.	
3	4	2	6	8	4	Multiplicand.
					7	
22	11	3	20	9	12	= 7 times multiplicand.
					3	
67	15	3	11	13	4	= 21 times multiplicand.
6	9	0	13	0	8	= 2 times multiplicand.
74	4	3	24	13	12	= 23 times multiplicand, Ans.

First multiply by 21, i. e. by 7, and that product by 3; then *add* twice the multiplicand, and thus multiply by 23.

SECOND OPERATION.

t.	cwt.	qr.	lb.	oz.	dr.	
3	4	2	6	8	4	Multiplicand.
					6	
19	7	1	14	1	8	= 6 times multiplicand.
					4	
77	9	2	6	6	0	= 24 times multiplicand.
3	4	2	6	8	4	= 1 time multiplicand.
74	4	3	24	13	12	= 23 times multiplicand, Ans.

Here we multiply by 24, i. e. by 6 and 4; then *subtract* the multiplicand.

The foregoing plan may be indefinitely modified; hence this general direction:

Multiply by two or more numbers whose product is nearly the multiplier, and add to, or subtract from, the product such numbers as the case may require.

31. Multiply 15yd. 2qr. 1na. by 47.

32. Multiply 27gal. 1qt. 1pt. 2gi. by 43.

33. What is the cost of 753 acres of land, at 4£ 10s. 8d. 2qr. per acre?

OPERATION.

£	s.	d.	qr.	
4	10	8	2	
			10	
45	7	1	0	= cost of 10a.
			10	
453	10	10	0	= cost of 100a.
			7	
3174	15	10	0	= cost of 700a.
226	15	5	0	= cost of 50a.
13	12	1	2	= cost of 3a.
3415	3	4	2	= cost of 753a., Ans.

Multiply by 100, i. e. by 10 and 10; then multiply the cost of 100 acres by 7, the cost of 10 acres by 5, and the cost of 1 acre by 3, which will give the cost of 700, 50, and 3 acres, severally; finally, *add* the cost of 700, 50, and 3 acres together, and thus find the cost of 753 acres, the answer.

34. If 1 acre of land yield 54bush. 3pk. 6qt. 1pt. of corn, what will 643 acres yield?

35. If a man travel 33m. 6fur. 35rd. 5yd. 2ft. 11in. each day, how far will he travel in 313 days?

36. If a ship sail 2° 40′ 30″ each day, how far will she sail in 127 days?

37. How much wine in 157 casks if each cask contains 53gal. 3qt. 1pt. 2gi.?

---

207. Mode of multiplying when the multiplier is large and not composite?

208. To find the difference of the time of day in two places, at the same absolute moment of time, when the longitude of each place is known.

Since the sun appears to go from east to west round the earth, 360° (Art. 109), in 24 hours, it appears to go $\frac{1}{24}$ of 360°, viz. 15° in 1 hour, and, consequently, 1° in $\frac{1}{15}$ of 1 hour, viz. 4 minutes, and 1′ of distance in $\frac{1}{60}$ of 4 minutes, viz. 4 seconds. These facts give us the following

TABLE OF LONGITUDE AND TIME.

360°	of longitude	=	24 hours, or 1 day of time,
15°	of longitude	=	1 hour of time,
1°	of longitude	=	4 minutes of time,
1′	of longitude	=	4 seconds of time,
1″	of longitude	=	$\frac{4}{60}$ of a second of time.

38. When it is 12 o'clock, noon, at Washington, what time is it at London, Washington being 77° 2′ 48″ west of London?

OPERATION.

$$\begin{array}{rrr} ^\circ & ' & '' \\ 77 & 2 & 48 \\ & & 4 \\ \hline \end{array}$$

5h. 8m. 1 1⅕sec. Ans.

Since 1″ of longitude makes a difference of $\frac{4}{60}$ of a second of time, 48″ of longitude give $\frac{192}{60}=3\frac{1}{5}$sec. of time, and for a like reason 2′ of longitude give 8sec. of time, which added to the $3\frac{1}{5}$sec. previously obtained, give $11\frac{1}{5}$sec., and, finally, 77° of longitude give 4 times $77=308$m. $=5$h. 8m. of time; ∴ the difference in time between London and Washington is 5h. 8m. $11\frac{1}{5}$sec., and as London is farther east than Washington, the hour of the day is later in London than in Washington, i. e. it is 8m. $11\frac{1}{5}$sec. past 5 o'clock in the afternoon at London when it is noon at Washington. Hence,

RULE. *Multiply the difference of longitude, expressed in degrees, minutes, and seconds, by* 4, *and the product will be the difference in time, expressed in minutes, seconds, and* 60*ths of a second.*

---

208. How far does the sun appear to move in one hour? Which way? Give the table of longitude and time. Rule for finding difference in time of two places when the longitude of each is known?

NOTE. 1. The place most easterly, has its hour of the day, at a given moment, latest; i. e. the day begins first, noon comes first, and the day closes first at the place most easterly.

39. The longitude of Boston is 71° 4′ 9″ west, and that of Washington is 77° 2′ 48″ west; what is the difference in the time of the two places, and what time is it in Washington, at 3 o'clock, P. M., in Boston?

By subtraction, the difference of longitude is found to be 5° 58′ 39″, ∴ the difference in time is 23m. 54$\frac{3}{5}$sec., and at 3 in Boston it is 36m. and 5$\frac{2}{5}$sec. past 2 in Washington, Ans.

40. The longitude of Paris is 2° 20′ 15″ east, and that of New York, 74° 0′ 3″ west from Greenwich; what is the difference in time in the two places? Ans. 5h. 5m. 21$\frac{1}{5}$sec. Ans.

NOTE 2. Since Paris is in east longitude, and New York in west, their *difference* in longitude is found by *adding* 2° 20′ 15″ to 74° 0′ 3″.

41. What is the difference in time between Philadelphia, 75° 9′ west longitude, and Chicago, 87° 35′ west longitude?

42. What is the difference in time between New Orleans, 90° 7′ west, and St. Petersburg, 29° 48′ east longitude?

43. What is the difference in time for 90° in longitude?

DIVISION.

209. Here, as in the three preceding sections, the principle is the same as in the corresponding operation in simple numbers. Hence,

To divide a compound by a simple number,

RULE. *Divide the highest denomination of the dividend, and set down the quotient; if there is a remainder, reduce it to the next lower denomination; to the result add the given quantity of that denomination, and divide as before, setting down the quotient and reducing the remainder, etc.*

---

208. Which has the hour of the day latest, the most easterly or most westerly place? How is the difference in longitude found when one place is in east and the other in west longitude? **209.** Rule for dividing a Compound by a Simple Number? Principle?

Ex. 1. Divide 30£ 7s. 1d. 1qr. by 7.

OPERATION.

	£	s.	d.	qr.	
7)	30	7	1	1	
	4	6	8	3,	Ans.
				7	
	30	7	1	1,	Proof.

30£ ÷ 7 give a quotient of 4£ and a remainder of 2£; 2£ reduced to shillings and added to 7s. give 47s., which, divided by 7, give a quotient of 6s. and a remainder of 5s., etc.

2. Divide 1fur. 9rd. 2yd. 0ft. 9in. 1b.c. by 5.
Ans. 9rd. 4yd. 2ft. 6in. 2b.c.

3. Divide 20gal. 2qt. 0pt. 2gi. by 7.
Ans. 2gal. 3qt. 1pt. 2gi.

4. Divide 18lb. 1oz. 15dwt. 8gr. by 4.

5. Divide 17lb. 5℥ 1ʒ 1℈ 10gr. by 6.

6. Divide 30t. 5cwt. 3qr. 24lb. 12oz. by 8.

7. Divide 60yd. 2qr. 3na. by 9.

8. Divide 3wk. 6d. 14h. 17m. 57sec. by 3.

9. Divide 57circ. 2s. 25° 4′ 10″ by 10.

10. Divide 107gal. 1qt. by 12.

11. Divide 98bush. 3pk. 2qt. 1pt. by 11.

12. Divide 51c. 7c.ft. 15cu.ft. 1716cu.in. by 4.

13. Divide 16a. 1yd. 4ft. 70in. by 2.

14. Divide 37t. 12cwt. 3qr. 5lb. 10oz. 4dr. by 9.

15. Divide 71a. 3r. 14rd. 8yd. 1ft. 72in. by 6.

16. If 9 silver spoons weigh 1lb. 4oz. 17dwt. 12gr., what is the weight of each spoon? Ans. 1oz. 17dwt. 12gr.

17. If a family use 29gal. 3qt. 2gi. of molasses in 6 months, what is the average per month?

18. If 10t. 18cwt. 1qr. of hay is harvested from 5 acres, what is the crop on one acre?

19. If 8 boxes of sugar weigh 2t. 7cwt. 2qr. 10lb., what is the weight per box?

20. If 9 grain-bins contain 143bush. 2pk. 2qt. 1pt. of grain, what does one bin contain?

21. If a man travel 212m. 1fur. 26rd. 2yd. in 7 days, what distance does he travel per day?

22. Divide 96£ 5s. 7d. 2q. by 10.

210. To divide by a composite number, we may divide by its factors, as in division of simple numbers (Art. 79).

23. Divide 341lb. 0oz. 12dwt. by 72.

```
       lb.    oz. dwt.   gr.
  9 ) 341      0  12      0
     ---------------------
    8 ) 37    10  14     16
       --------------------
          4    8  16     20, Ans.
```

First divide by 9 and then the quotient by 8, and thus by 72.

24. Divide 396£ 2s. 3d. by 54.
25. Divide 725bush. 0pk. 6qt. 1pt. by 81.
26. Divide 397℔ 11℥ 7ʒ 1℈ 4gr. by 63.
27. Divide 958m. 5fur. 5ch. 12li. $5\frac{1}{2}\frac{2}{3}$in. by 48.

211. When the divisor is large and not composite, set down the work of dividing and reducing. There is no device for rendering the operation easier.

28. Divide 135bush. 3pk. 3qt. 1pt. by 47.

```
      bush.  pk. qt. pt.
  47 ) 135    3   3   1 ( 2bush. 3pk. 4qt. 1pt., Ans.
        94
       ---
        41 bush.
         4
       ---
       167 pk.
       141
       ---
        26 pk.
         8
       ---
       211 qt.
       188
       ---
        23 qt.
         2
       ---
        47 pt.
        47
       ---
         0
```

Having found that 47 is contained twice in 135, multiply 47 by 2, and subtract the product, 94, from 135, which leaves a remainder of 41 bushels; reduce the 41 bushels to pecks, and add the 3 pecks, making 167 pecks; then divide the 167 pecks by 47, and so continue the process till the work is done.

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210. Rule for dividing by a composite number? 211. Method of dividing when the divisor is large and not composite? Is there no easier mode?

29. If 587 yards of cloth cost 662£ 4s. 2d. 1qr., what is the price per yard?

30. Divide 1129gal. 1pt. 3gi. by 73.

31. A farmer raised 35334bush. 3pk. 3qt. 1pt. of corn on 643 acres of land; how much was the yield per acre?

32. Suppose a man should travel 10599m. 0fur. 14rd. 4yd. 2ft. 5in. in 313 days, what distance would he travel per day?

33. In 127 days a ship sails 11s. 9° 43′ 30″; what is the distance per day?

212. To find the difference in the longitude of two places, when the difference of time is known.

34. When it is 12 o'clock at Washington, it is 23m. 54$\frac{3}{5}$sec. past 12 at Boston; what is the difference in the longitude of the two places?

OPERATION.

$$\begin{array}{r|ll} & \text{m.} & \text{sec.} \\ 4\,) & 23 & 54\frac{3}{5} \\ \hline & 5^\circ\ 58' & 39''\ \text{Ans.} \end{array}$$

First divide the 23m. by 4, because 4m. of time make a difference of 1° of longitude. This gives 5° and a remainder of 3m. The 3m. and 54$\frac{3}{5}$sec. = 234$\frac{3}{5}$sec. The 234$\frac{3}{5}$sec. divided by 4, because 4sec. of time make a difference of 1′ of longitude, give 58′ and a remainder of 2$\frac{3}{5}$sec. Finally, reduce the 2$\frac{3}{5}$sec. to 60ths of a sec. and divide by 4, and the quotient is 39″; i. e. the difference in longitude between Boston and Washington, is 5° 58′ 39″, Ans. Hence,

RULE. *Divide the difference in time, expressed in minutes, seconds, and 60ths of a second, by* 4, *and the quotient is the difference in longitude, expressed in degrees, minutes, and seconds.*

35. Paris is 2° 20′ 15″ east of Greenwich; how many degrees west of Greenwich is New York, the difference in time between Paris and New York being 5h. 5m. 21$\frac{1}{5}$sec.? Ans. 74° 0′ 3″.

NOTE. The difference in longitude between Paris and New York is found to be 76° 20′ 18″ and this diminished by 2° 20′ 15″, the *east* longitude of Paris, gives 74° 0′ 3″ for the *west* longitude of New York.

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212. Rule for finding the difference in the longitude of two places, when the difference in time is known?

36. The difference in time between Philadelphia and Rome is 5h. 50m. 30⅔sec.; Philadelphia is 75° 9′ west; what is the longitude of Rome? Ans. 12° 28′ 40″ east.

37. A message telegraphed from St. Petersburg, 29° 48′ east, at 12 o'clock, noon, was instantly received at Paris at 10h. 10m. 9sec., A. M., of the same day; what is the longitude of Paris?

38. At sun-rise in Astoria, Oregon, the sun is about 3h. 49m. 16sec. high at Eastport in Maine; what is the difference in longitude?

39. What is the difference in longitude between the Cape of Good Hope and Cape Horn, if a meteor seen at midnight at Good Hope is so high as to be seen at the same moment at Cape Horn, the time at Cape Horn being 17 minutes past 6 in the evening? Ans. 85° 45′.

DUODECIMALS.

213. DUODECIMALS are compound numbers in which the scale is *uniformly* 12.

This measure is usually applied to feet and parts of a foot, and is used in determining distances, areas, and cubic contents. The denominations are feet (ft.), inches or primes (′), seconds (″), thirds (‴), fourths (⁗), etc. The accents, ′, ″, ‴, used to designate the denominations are called *indices.*

214. The foot being the unit, the denominations have the relations indicated by the following

TABLE.

$1' =$ $\frac{1}{12}$ of a foot.
$1'' = \frac{1}{12}$ of $1' = \frac{1}{12}$ of $\frac{1}{12}$ of 1 ft. $= \frac{1}{144}$ of a foot.
$1''' = \frac{1}{12}$ of $1'' = \frac{1}{12}$ of $\frac{1}{144}$ of 1 ft. $= \frac{1}{1728}$ of a foot.
$1'''' = \frac{1}{12}$ of $1''' = \frac{1}{12}$ of $\frac{1}{1728}$ of 1 ft. $= \frac{1}{20736}$ of a foot.
etc. etc.

Thus 12 of any lower denomination make 1 of the next higher; e. g.

$$12'''' = 1''',\ 12''' = 1'',\ 12'' = 1',\ 12' = 1\text{ft.}$$

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213. What are duodecimals? To what applied? For what used? The denominations? How designated? **214.** The unit, which denomination?

Addition and Subtraction.

215. Addition and Subtraction of duodecimals are performed as the like operations of other compound numbers.

Ex. 1. Add together 3ft. 6′ 8″ 4‴ 7⁗, 9ft. 7′ 8″ 2‴ 5⁗, and 4ft. 9′ 8″ 10‴ 8⁗.

OPERATION.

	3	6′	8″	4‴	7⁗
	9	7	8	2	5
	4	9	8	10	8
Sum,	18	0	1	5	8

Having arranged the numbers as in addition of compound numbers, we find the sum of the lowest denomination to be 20⁗ = 1‴ and 8⁗, ∴ set the 8⁗ in the column of fourths, and add the 1‴ to the thirds, and so proceed till all the columns are added, and so obtain 18ft. 0′ 1″ 5‴ 8⁗, Ans.

2. From 6ft. 8′ 7″ 9‴ 3⁗ take 1ft. 6′ 9″ 2‴ 8⁗.

OPERATION.

Min.,	6	8′	7″	9‴	3⁗
Sub.,	1	6	9	2	8
Rem.,	5	1	10	6	7
Proof,	6	8	7	9	3

As 8⁗ cannot be taken from 3⁗, add 12⁗ to the 3⁗, making 15⁗, and then take 8⁗ from the sum, giving a remainder of 7⁗; then take 3‴ from 9‴ or 2‴ from 8‴, giving 6‴ by either process, and so proceed.

3. Add 10ft. 6′ 4″, 12ft. 9′ 8″, and 7ft. 10′ 11″.

4. Subtract 3ft. 8′ 4″ 3‴ from 9ft. 4′ 6″ 1‴.

Multiplication.

216. Multiplication of duodecimals is like multiplication of other compound numbers, except that, when both factors are in the form of compound numbers, *it is required to find the denomination of the product.*

In this investigation, *for the sake of convenience,* we familiarly speak of multiplying feet by feet, feet by inches, inches by

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215. Addition and Subtraction, how performed? **216.** What in Multiplication is peculiar? What is the multiplier strictly? Why do we speak of multiplying feet by *feet*, feet by *inches*, etc.?

inches, etc., though here, as everywhere (Art. 204), the multiplier is strictly an *abstract* number; e. g., suppose a board is 10 feet long and 1 foot wide, it evidently contains 10 square feet, and if it is 10 feet long and 2 feet wide, it as evidently contains 2 times 10 square feet = 20 square feet (Art. 101), though it would be *nonsense* to affirm that it contains 2 *feet* times 10 feet; still, we are accustomed to say that the area of a board is equal to its *length multiplied* by its *breadth.* Again, if a board is 10 feet long and 1 inch wide, it contains $\frac{1}{12}$ as many *square feet* as it is *feet in length;* i. e. it contains $\frac{1}{12}$ of 10 square feet = $\frac{10}{12}$sq. ft. = $10'$; and if the board is 10ft. long and 2in. wide, it contains $\frac{2}{12}$ of 10sq. ft. = $\frac{20}{12}$ of a sq. ft. = $1\frac{8}{12}$sq. ft. = 1ft. and $8'$. This illustration can be carried to any extent.

217. Since $1' = \frac{1}{12}$ft., $1'' = \frac{1}{144}$ft., $1''' = \frac{1}{1728}$ft., etc., whether the measure is linear, square, or cubic, it follows that $1'$, in linear measure, is a *line*, $\frac{1}{12}$ of a foot in length; in square measure, $1'$ is an *area*, 1 foot long and 1 inch wide, and $1''$ is an *area* 1 inch square; in cubic measure $1'$ is a *solid*, 1 foot long, 1 foot wide, and 1 inch deep, $1''$ is a *solid*, 1 foot long, 1 inch wide, and 1 inch deep, and $1'''$ is a *cubic inch;* etc.

218. Let us now determine the denomination of the product obtained by multiplying any two denominations together.

PHILOSOPHICALLY.	FAMILIARLY.
2 units $\times$ 3 units $=$ 6 units,	i. e. 2ft. $\times$ 3ft. $=$ 6ft.
2 " $\times \frac{3}{12}$ unit $= \frac{6}{12}$ unit,	i. e. 2ft. $\times 3' = 6'$
2 " $\times \frac{3}{144}$ " $= \frac{6}{144}$ "	i. e. 2ft. $\times 3'' = 6''$
etc.	etc.
$\frac{2}{12}$ unit $\times \frac{3}{12}$ unit $= \frac{6}{144}$ unit,	i. e. $2' \times 3' = 6''$
$\frac{2}{12}$ " $\times \frac{3}{144}$ " $= \frac{6}{1728}$ "	i. e. $2' \times 3'' = 6'''$
$\frac{2}{12}$ " $\times \frac{3}{1728}$ " $= \frac{6}{20736}$ "	i. e. $2' \times 3''' = 6''''$
etc.	etc.
$\frac{2}{144}$ unit $\times \frac{3}{144}$ unit $= \frac{6}{20736}$ unit,	i. e. $2'' \times 3'' = 6''''$
$\frac{2}{144}$ " $\times \frac{3}{1728}$ " $= \frac{6}{248832}$ "	i. e. $2'' \times 3''' = 6'''''$
$\frac{2}{144}$ " $\times \frac{3}{20736}$ " $= \frac{6}{2985984}$ "	i. e. $2'' \times 3'''' = 6''''''$
etc.	etc.

217. What is $1'$ in linear measure? $1'$ in square measure? $1''$ in square measure? $1'$ in cubic measure? $1''$? $1'''$? $1''''$?

Hence, to determine the denomination of the product of two factors in duodecimals,

RULE. *Add the indices of the two factors together, and the sum will be the index of the product.*

Ex. 1. A board is 6ft. 7′ 9″ in length and 2ft. 7′ 5″ in breadth; what is its area?

OPERATION.

	6	7′	9″		
	2	7′	5″		
	13	3′	6″		
	3	10′	6″	3‴	
		2′	9″	2‴	9⁗
Ans.	17	4′	9″	5‴	9⁗

First, $9'' \times 2 = 18'' = 1'\,6''$; the 6″ we write under the seconds, and reserve the 1′ to add to the next product, thus, $7' \times 2 = 14'$, which increased by the 1′ previously obtained gives $15' = 1\text{ft. }3'$; the 3′ is written down, and the 1ft. is carried to the product of the feet, making 13ft. In like manner we multiply by the 7′ and then by the 5″, setting the partial products as in the margin. Finally, the *sum* of these partial products is the product sought. Hence,

219. To perform Multiplication of Duodecimals,

RULE. *By the rule for multiplication of compound numbers, multiply the multiplicand by each term in the multiplier, and write the terms of the several partial products in the order of their values, so that similar terms shall stand in a column together; the sum of the partial products will be the entire product.*

	2.					3.				
Multiplicand,	3	4′	6″			4	8′	9″		
Multiplier,	2	8′	5″			2	3′	7″		
	6	9′	0″			9	5′	6″		
	2	3′	0″	0‴		1	2′	2″	3‴	
		1′	4″	10‴	6⁗		2′	9″	1‴	3⁗
Product,	9	1′	4″	10‴	6⁗	10	10′	5″	4‴	3⁗

4. What quantity of boards will be required to lay a floor 12ft. 6′ 4″ long and 8ft. 3′ 6″ wide? Ans. 103ft. 10′ 6″ 2‴.

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218. Rule for determining the denomination of a product? Explain philosophically and familiarly. **219.** Rule for multiplication of duodecimals?

5. What are the contents of a granite block that is 6ft. 3′ long, 2ft. 4′ wide, and 1ft. 3′ thick?

Ans. 18ft. 2′ 9″. (See Art. 104).

6. How many feet of flag-stone in a walk 15ft. 6′ long and 3ft. 4′ wide?

7. How many solid feet of marble in a block that is 8ft. 3′ long, 3ft. 6′ wide, and 1ft. 4′ thick?

8. How many cubic feet of earth must be removed in digging a cellar 15ft. 6′ long, 12ft. 8′ wide, and 6ft. 8′ deep?

9. How many feet in a stock of 8 boards, that are 10ft. 8′ long and 10′ wide? Ans. 71ft. 1′ 4″.

10. How many feet of boards 1′ thick can be sawed from a a stick of timber that is 12ft. 8′ long, 10′ wide, and 8′ 4″ thick, provided no timber is destroyed by the saw-cut?

11. How many cords of wood in a pile that is 18ft. 6′ long, 6ft. 8′ high, and 4ft. wide?

12. How many square yards of carpeting will cover a room that is 18ft. long and 16ft. 6′ wide?

13. Multiply 3ft. 6′ 4″ by 8ft. 9′ 6″.

Division.

220. Division of duodecimals is like division of other compound numbers.

Ex. 1. Divide 24ft. 10′ 10″ 4‴ by 7. Also by 9.

OPERATION.

$$7\underline{)\,24\quad 10'\quad 10''\quad 4'''}$$
$$\text{Ans. }3\quad 6'\quad 8''\quad 4'''$$

OPERATION.

$$9\underline{)\,24\quad 10'\quad 10''\quad 4''\qquad\qquad\qquad}$$
$$2\quad 9'\quad 2''\quad 5'''\quad 9''''\quad 4'''''$$

2.

$$8\underline{)\,31\quad 6'\quad 8''\quad 8'''}$$

3.

$$6\underline{)\,45\quad 4'\quad 1''\quad 6'''}$$

NOTE. When both dividend and divisor are expressed as compound numbers, they may be reduced to the smallest denomination in either; after which divide, and the quotient will be units, i. e. feet; thus, 68ft. 10′ 8″ divided by 2ft. 8′ equals $9920'' \div 384'' = 25\frac{10}{12}$, i. e. 25ft. 10′, Ans.

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220. How is division of duodecimals performed? How when the divisor is compound?

4. The area of a floor is 197ft. 1′ 8″, and the length of the floor is 15ft. 8′; what is its width? Ans. 12ft. 7′.

5. The area of a garden walk is 89ft. 4′ and its width is 2ft. 8′; what is its length?

Miscellaneous Examples in Compound Numbers.

1. If 152bush. 3pk. 3qt. 1pt. of wheat grow on 9 acres of land, how many bushels grow on 7 acres?

2. A man having 207m. 4fur. 25rd. 1yd. to travel in 6 days, goes 30m. 3fur. 25rd. 5yd. on the first day, and 33m. 4fur. 20rd. 4yd. on the second day; how far per day must he travel to finish the journey in the remaining 4 days?

3. Multiply 3£ 15s. 6d. 1qr. by 857, and divide the product by 157.

4. I have a stock of 9 boards, which are 12ft. 8′ long and 10′ wide. With these boards I wish to lay a floor 15ft. in length; how wide can I make it?

5. If 1 cubic foot of water weighs 62lb. 8oz., and if a cubic foot of granite weighs $2\frac{1}{2}$ times as much, what is the weight of a block of granite 12ft. long, 1ft. 8′ wide, and 9′ thick?

6. From the sum of 3wk. 6d. 16h. 20m. 18sec. and 2wk. 3d. 18h. 50m. 40sec. take the difference between 6wk. 5d. 8h. 25m. 30sec. and 5wk. 2d. 22h. 18m. 15sec.

7. What is the difference in time between Amsterdam 4° 44′ east longitude, and Annapolis 76° 43′ west longitude?

8. When it is noon in Dublin, 6° 7′ 13″ west longitude, it is 10m. and $16\frac{1}{1}\frac{3}{5}$sec. past 8 o'clock in the evening in Peking; what is the longitude of Peking?

9. How many days, hours, etc., from 30m. 20sec. past 3 o'clock, P. M., Feb. 8, 1864, to 40m. 25sec. past 8 o'clock, A. M., July 4, 1864, reckoning each month at its actual length?

10. Bought 3cwt. 2qr. 18lb. of sugar at $8\frac{1}{2}$c. per pound, and sold $\frac{1}{2}$ of it at 8c. and the remainder at $9\frac{1}{4}$c. per pound; what was gained by the transactions?

11. What is the value in Avoirdupois Weight of 24lb. 6oz. 12dwt. 20gr. Troy Weight?

12. How long a time will be required for one of the heavenly bodies to move through a quadrant of a circle, if it moves at the rate of $1' \ 3''$ per minute?

13. The distance from Eastport, Maine, to San Francisco, California, is about 2760 miles. If a man, starting from Eastport, travel toward San Francisco for 75 days, at the rate of 24m. 3fur. 20rd. per day, how far will he then be from San Francisco?

14. A certain island is 75 miles in circumference. A and B, starting at the same time, and from the same point, and going in the same direction, travel round this island, A at the rate of 24m. 3fur. 10rd., and B at the rate of 15m. 6fur. 20rd. per day; how far apart are A and B at the end of five days?

15. A merchant bought 125 barrels of flour, at 1£ 15s. 6d. per barrel, and afterward exchanged the flour for 260 yards of broadcloth, which he sold at 18s. 9d. 3qr. per yard; did he gain or lose, and how much?

16. How many feet of boards will be required to make 12 boxes whose interior dimensions are 5ft. 6′, 4ft. 9′, and 3ft. 8′, the boards being 1′ in thickness?

17. How many feet less are required to make 12 boxes whose *exterior* dimensions are like the *interior* of those in Ex. 16, the boards being of the same thickness? Ans. 111ft. 4′.

18. What is the difference of the capacities of the two sets of boxes described in Ex. 16 and 17? Ans. 122ft. 10′.

19. How many times will a wheel 9ft. 8in. in circumference turn round in running from Boston to Worcester, a distance of 44m. 4fur.?

20. How many gallons, wine measure, in a water tank 4ft. 6in. long, 3ft. 8in. wide, and 3ft. 9in. deep?

21. If a teacher devote 5h. 30m. per day to 50 pupils, what is the average time for each pupil?

22. If a man, employed in counting money from a heap, count 75 silver dollars each minute, and continue at the work 12 hours each day, in how many days will he count a million of dollars?

23. How many pounds of iron in one scale of a balance, will poise 75 pounds of gold in the other scale?

PERCENTAGE.

221. PER CENT. is a contraction of *per centum*, a Latin phrase which means *by the hundred;* thus, *ten* per cent. of a bushel of corn means ten one-hundredths of it; i. e. ten parts out of every hundred parts; *six* per cent. of a sum of money, is six one-hundredths of the sum, i. e. \$6 out of every \$100; etc.

NOTE. Instead of the words *per cent.*, it is customary to use this sign, %; thus, 6 per cent. is written 6%; $4\frac{1}{2}$ per cent., $4\frac{1}{2}$%.

222. The RATE PER CENT. is the *number for each hundred;* thus, 6% is $\frac{6}{100}$, or .06, i. e. 6 parts for each hundred parts.

223. The PERCENTAGE is the sum computed on the given number; thus, the percentage on \$200 at 6 per cent. is \$12.

224. The BASE of percentage is the number on which the percentage is computed; thus, \$200 is the base on which the percentage is computed in Art. 223; a bushel of corn is the first base mentioned in Art. 221.

225. The rate per cent., being a certain number of hundredths, may be expressed either *decimally*, or by a *common fraction*, as in the following

TABLE.

		Decimals.		Common Fractions.		
1 per cent.	is	.01	=	$\frac{1}{100}$.		
2 per cent.		.02	=	$\frac{2}{100}$	=	$\frac{1}{50}$.
5 per cent.		.05	=	$\frac{5}{100}$	=	$\frac{1}{20}$.
6 per cent.		.06	=	$\frac{6}{100}$	=	$\frac{3}{50}$.
10 per cent.		.10	=	$\frac{10}{100}$	=	$\frac{1}{10}$.
50 per cent.		.50	=	$\frac{50}{100}$	=	$\frac{1}{2}$.
100 per cent.		1.00	=	$\frac{100}{100}$	=	1.
125 per cent.		1.25	=	$\frac{125}{100}$	=	$1\frac{1}{4}$.
$6\frac{1}{4}$ per cent.		.0625	=	$\frac{6\frac{1}{4}}{100}$	=	$\frac{1}{16}$.
$8\frac{1}{3}$ per cent.		$.08\frac{1}{3}$	=	$\frac{8\frac{1}{3}}{100}$	=	$\frac{1}{12}$.
$12\frac{1}{2}$ per cent.		.125	=	$\frac{12\frac{1}{2}}{100}$	=	$\frac{1}{8}$.
		etc.		etc.		

NOTE. When the per cent. is expressed by a decimal of more than 2 places, the figures after the second decimal place must be regarded as parts of 1 per cent.; thus, (in the last line of the foregoing table,) .125 is $12\frac{5}{10}$ or $12\frac{1}{2}$ per cent.

Ex. 1. Write the decimal for 4 per cent. Ans. .04.

2. Write the decimal for 8 per cent.; 12 per cent.; $16\frac{1}{2}$ per cent.; 25 per cent.; 72 per cent.

3. Write the common fraction for $16\frac{2}{3}$ per cent.; 20 per cent.; $33\frac{1}{3}$ per cent.; 75 per cent. 1st Ans. $\frac{1}{6}$.

PROBLEM 1.

226. To find the percentage, the base and rate per cent. being given.

Ex. 1. B had \$175, but lost 8 per cent. of it; how many dollars did he lose?

$$\begin{array}{r} \$175 \\ .08 \\ \hline \$14.00, \text{ Ans.} \end{array}$$

Since 8 per cent. is $.08 = \frac{2}{25}$, the loss is found by multiplying \$175 by .08 or by $\frac{2}{25}$. Hence,

RULE 1. *Multiply the base by the per cent., written decimally*; or,

RULE 2. *Find such part of the base as the rate is of* 100 (Art. 151).

2. A farmer having 48 sheep, lost 25 per cent. of them; how many did he lose?

By Rule 1.

$$\begin{array}{r} 48 \\ .25 \\ \hline 240 \\ 96 \\ \hline 12.00, \text{ Ans.} \end{array}$$

By Rule 2.

$.25 = \frac{25}{100} = \frac{1}{4}$.

$\frac{1}{4}$ of $48 = 12$, Ans.

Or, $48 \times \frac{1}{4} = 12$, Ans.

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221. Meaning of per cent.? 222. *Rate* per cent.? 223. Percentage? 224. *Base* of percentage? 225. In what ways may the rate be expressed? If expressed decimally by more than two figures, what are the figures after the second decimal place? 226. Rule for finding percentage when the base and rate are given? Second Rule?

3. What is 6 per cent. of $250? Ans. $15.

4. What is 8 per cent. of $250?

5. What is $12\frac{1}{2}$ per cent. of $500? Ans. $62.50.

6. What is $8\frac{1}{3}$ per cent. of 600bush. of wheat? Ans. 50bush.

7. What is $16\frac{2}{3}$ per cent. of 1200lb. of cheese? Ans. 200lb.

8. A farmer cultivates 25 acres of corn this year, and intends to cultivate 20 per cent. more next year; how many acres does he intend to cultivate next year? Ans. 30.

9. In an orchard of 900 trees, $33\frac{1}{3}$ per cent. are peach trees; how many peach trees are there in the orchard?

10. A teacher pronounced 56 words for his pupils to spell, but $14\frac{2}{7}$ per cent. were mis-spelled; how many words were mis-spelled?

11. Only $66\frac{2}{3}$ per cent. of a class of 27 pupils solved a problem given them for a lesson; how many of the class failed?

12. The population of a certain city is 18775, what will it be in one year from this time if it gains 8 per cent.?

13. The population of a certain State is 1376875, what will it be in one year if it loses 12 per cent.?

14. A and B commenced business, each with $8456. A gained 25 per cent. and B lost 12 per cent.; how much was A then worth more than B?

15. A speculator paid $56895 for a lot of flour, and lost 9 per cent.; for what sum did he sell the flour?

16. One acre of corn yields 80 bushels, and another acre 20 per cent. more; how many bushels does the second acre yield?

Problem 2.

227. To find the rate per cent. when the base and percentage are given.

Ex. 1. One dollar is what per cent. of $4?

$$4\,)\,\underline{1\,0\,0}$$
$$2\,5,\ \text{Ans.}$$

One dollar is $\frac{1}{4}$ of $4, and $\frac{1}{4}$ reduced to a decimal is .25; i. e. $1 is 25 per cent of $4. The same result is obtained by multiplying $1 by 100, and dividing the product by 4. Hence, RULE. *Multiply the percentage by* 100, *and divide the product by the base.*

NOTE. This rule is the converse of that in Art. 226; thus, 25 per cent. of \$4 is \$4 × .25 = \$1; and, conversely, \$1.00 ÷ \$4 = .25, i. e. 25 per cent.

2. What per cent. of \$150 is \$18?

1800 ÷ 150 = 12 per cent., Ans.

3. What per cent. of \$300 is \$19? Ans. 6⅓ per cent.

4. What per cent. of \$350 is \$43.75? Ans. 12½ per cent.

5. What per cent. of \$340 is \$34?

6. What per cent. of \$64 is \$16?

7. What per cent. of \$1000 is \$5? Ans. ½ of 1 per cent.

8. B inherited \$3500, and in 6 months spent \$875; what per cent. of his inheritance did he spend? What per cent. had he remaining? Ans. Spent 25 per cent., and had 75 per cent.

9. Out of a cask of wine containing 96 gallons, 32 gallons were drawn; what per cent. of the whole remained in the cask?

10. A merchant having \$1000, deposited \$650 in a bank; what per cent. of his money did he deposit?

11. A teacher having a salary of \$2400, spends \$2000 annually; what per cent. of his salary does he save?

PROBLEM 3.

228. To find the base when the percentage and the rate are given.

Ex. 1. \$6 is 3 per cent. of what sum?

If \$6 is 3 per cent., then 1 per cent. is ⅓ of \$6, which is \$2, and if \$2 is 1 per cent., then 100 per cent. is 100 times \$2, which is \$200; ∴ \$6 is 3 per cent. of \$200, Ans.

The same result is obtained by first multiplying \$6 by 100, and then dividing the product by 3; thus, \$600 ÷ 3 = \$200, Ans. Hence,

RULE. *Multiply the percentage by* 100, *and divide the product by the rate.*

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227. Rule for finding the rate when the base and percentage are known? What of this rule, and that in Art. 226? **228.** Rule for finding the base when the percentage and rate are known?

2. \$9 is 4 per cent. of what sum? Ans. \$225.

3. \$37.50 is 3 per cent. of what sum? Ans. \$1250.

4. \$12 is 7 per cent. of what sum? Ans. \$171.42$\frac{6}{7}$.

5. \$8 is 16 per cent. of what sum?

6. 12 is 3 per cent. of what number? Ans. 400.

7. 37$\frac{1}{2}$ is 6 per cent. of what number?

8. 33 is 1$\frac{3}{8}$ per cent. of what number?

9. A farmer bought a farm for \$2756, which was 25 per cent. of his property; what was his property? Ans. \$11024.

10. A man sold 56 geese, which was 28 per cent. of his flock; how many geese had he?

11. A merchant having a quantity of flour, bought 600 barrels more, when he found that the quantity bought was 75 per cent. of all he then had; how many barrels had he before he bought the last lot? Ans. 200.

12. A teacher saves \$400 annually, which is 16$\frac{2}{3}$ per cent. of his salary; what is his salary?

13. The population of a town was 769 greater in 1800 than in 1850, and this was an increase of 20 per cent. on the population of 1850; what was the population in 1850?

INTEREST.

229. Interest is money paid *for the use* of money.

The Principal is the sum for which interest is paid.

The Amount is the *sum* of the *principal* and *interest.*

230. An example in interest is only a question in *percentage.* The *principal* is the *base* of percentage (Art. 224), the *interest* is the *percentage* (Art. 223), and the interest on \$1 for a year is the *rate* written decimally (Art. 222).

231. The *rate* is usually *fixed by law,* and a higher rate than the law allows is *usury.*

In New England and most of the United States the *legal* or

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229. What is Interest? What the Principal? Amount? **230.** Interest, what relation to percentage? What is the *base?* The *percentage?* The *rate?* **231.** How is the rate fixed? What is usury? Name the legal rate in some of the States.

lawful rate is 6 per cent.; in New York, 7 per cent.; in most of the Western States, as high as 10 per cent. by agreement; in Texas, as high as 12 per cent.; in California, *any* rate by agreement, etc. On debts in favor of the United States, 6 per cent. In France and England, 5 per cent.

NOTE. In this treatise, 6 per cent. is understood when no per cent. is mentioned.

232. When the rate is 6 per cent., the interest of \$1 for a year is 6c.; for 2 years, 12c., etc.; for 1 month, $\frac{1}{12}$ of 6c. = 5 mills or $\frac{1}{2}$c.; for 2 months, 1c.; 3 months, $1\frac{1}{2}$c.; 6 months, 3c.; 9 months, $4\frac{1}{2}$c., etc.; for 1 day, $\frac{1}{30}$ of 5 mills = $\frac{1}{6}$ mill; for 2 days, $\frac{1}{3}$m.; 3 days, $\frac{1}{2}$m.; 4 days, $\frac{2}{3}$m.; 5 days, $\frac{5}{6}$m.; 6 days, 1m.; 7 days, $1\frac{1}{6}$m.; 9 days, $1\frac{1}{2}$m.; 12 days, 2m.; 24 days, 4m.; etc., etc. Hence,

To find the interest of \$1 at 6 per cent. for any time,

RULE. *Take* 6c. (= \$.06) *for each year,* 1c. *for each* 2 *months in the part of a year,* 5 *mills* (= \$.005) *for the odd month, if there be one, and* 1 *mill for each* 6 *days in the part of a month.*

Ex. 1. What is the interest of \$1 for 3yr. 9m. 18d.?

OPERATION.

\$.18	= interest	of	\$1	for	3	years.
.045 =	"	"	"	"	9	months.
.003 =	"	"	"	"	18	days.
\$.228 =	"	"	"	"		3yr. 9m. 18d., Ans.

2. What is the interest of \$1 for 2yr. 5m. 20d.?

OPERATION.

\$.12	= interest	of	\$1	for	2	years.
.025 =	"	"	"	"	5	months.
$.003\frac{1}{3}$ =	"	"	"	"	20	days.
$\$.148\frac{1}{3}$ =	"	"	"	"		2yr. 5m. 20d., Ans.

231. What will be understood when no rate is mentioned? **232.** Rule for finding the interest of \$1 at 6 per cent. for any given time?

NOTE. With very little practice the pupil will, without making a figure, *mentally* determine the interest of \$1 for any length of time. This habit is *very desirable*, as it will greatly facilitate the computation of interest.

3. What is the interest of \$1 for 3yr. 1m. 15d.?

Ans. \$.187$\frac{1}{2}$.

4. What is the interest of \$1 for 1yr. 3m. 29d.?

Ans \$.079$\frac{5}{6}$.

5. What is the interest of \$1 for 4yr. 2m. 4d.?

Ans. \$.250$\frac{2}{3}$.

6. What is the interest of \$1 for 4yr. 3m. 17d.?
7. What is the interest of \$1 for 4yr. 9m. 12d.?
8. What is the interest of \$1 for 10yr. 11m. 7d.?
9. What is the interest of \$1 for 2yr. 11m. 5d.?
10. What is the interest of \$1 for 1yr. 8m. 3d.?

233. To find the interest of any sum at 6 per cent. for any given time.

The interest of \$2 is evidently twice as much as the interest of \$1; so the interest of \$3, \$4, or \$7, is 3, 4, or 7 times the interest of \$1; and the interest of \$2.25 is 2.25 (i. e. 2 and 25 hundredths) times the interest of \$1; ∴ to find the interest of any number of dollars we have only *to find the interest of* \$1, *and then multiply the interest by the number of dollars in the principal.*

11. What is the interest of \$2 for 1yr. 5m. 9d.?

$$\begin{array}{r l} \$.086\tfrac{1}{2} & = \text{interest of \$1 for 1yr. 5m. 9d.} \\ 2 & \\ \hline \$.173 & = \text{interest of \$2 for 1yr. 5m. 9d., Ans.} \end{array}$$

12. What is the interest of \$6.50 for 3yr. 8m. 18d.?

$$\begin{array}{r l} \$.223 & = \text{interest of \$1 for 3yr. 8m. 18d.} \\ 6.50 & \\ \hline 11150 & \\ 1338 & \\ \hline \$1.44950 & = \text{interest of \$6.50 for 3yr. 8m. 18d., Ans.} \end{array}$$

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232. What is the Note? 233. Rule for finding the interest of any sum at 6 per cent., for any time? Reason?

13. What is the interest of \$300 for 2yr. 7m. 24d.?

$$\begin{array}{rl} \$.159 & = \text{interest of } \$1 \text{ for 2yr. 7m. 24d.} \\ 300 & \\ \hline \$47.700 & = \text{interest of } \$300 \text{ for 2yr. 7m. 24d., Ans.} \end{array}$$

14. What is the interest of \$700 for 1yr. 9m. 12d.?

Ans. \$74.90.

15. What is the interest of \$400 for 2yr. 6m. 15d.?

16. What is the interest of \$350 for 3yr. 8m. 24d.?

234. The mode of casting interest given in Art. 233 is perfectly simple, but the product is not changed when the multiplicand and multiplier change places (Art. 59, Note). Hence,

To cast interest at 6 per cent. per annum, on any sum, for any time,

RULE. *Multiply the principal by the decimal which represents the interest of* \$1 *for the given time.*

17. What is the interest of \$468 for 2yr. 6m. 11d.?

FIRST OPERATION.

$$\begin{array}{rl} \$468. & = \text{Principal.} \\ .151\tfrac{5}{6} & = \text{Int. of } \$1. \\ \hline 390 & \\ 468 & \\ 2340 & \\ 468 & \\ \hline \$71.058, & \text{Ans.} \end{array}$$

$\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$. Instead of multiplying by $\frac{5}{6}$, as in this example, it is usually easier to multiply by $\frac{1}{2}$ and $\frac{1}{3}$, i. e. divide by 2 and 3, as in the following operation:

SECOND OPERATION.

$$\begin{array}{r} \$468. \\ .151\tfrac{1}{2}\tfrac{1}{3} \\ \hline 234 \\ 156 \\ 468 \\ 2340 \\ 468 \\ \hline \$71.058, \text{ Ans.} \end{array}$$

In like manner, when the multiplier is $\frac{2}{3}$, we may divide by 3 and set down the quotient twice.

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234. Second rule? Reason? Easiest way of multiplying by five sixths? Why correct? Easiest way for two thirds?

18. What is the interest of $48.50 for 2yr. 7m. 21d.?

Ans. $7.68725.

19. What is the interest of $248 for 2yr. 3m. 18d.?

Ans. $34.224.

20. What is the interest of $965.188 for 2yr. 3m. 11d.?

Ans. $132.07—.

NOTE 1. In the 20th example the Ans. is $132.069891⅓, but this, in all ordinary business transactions, would be called $132.07. In the following examples in interest only 3 decimal places *in the product* will be preserved, but if the 4th decimal place is 5 or more, the third place will be increased by 1 thousandth.

21. What is the interest of $225.87 for 1yr. 3m. 15d.?

Ans. $17.505.

22. What is the interest of $35.40 for 2yr. 6m. 9d.?

Ans. $5.363.

23. What is the interest of $450.87 for 1yr. 7m. 9d.?

24. What is the interest of $375.50 for 2yr. 1m. 8d.?

25. What is the interest of $225.75 for 1yr. 5m. 12d.?

26. What is the interest of $84.82 for 2yr. 4m. 18d.?

27. What is the interest of $125.16 for 1yr. 11m. 25d.?

28. What is the interest of $658.25 for 1yr. 2m. 13d.?

29. What is the interest of $125 from June 7, 1851, to Feb. 11, 1854? Ans. $20.083.

NOTE 2. Ex. 29 differs from the preceding only in its being necessary to find the time (Art. 203).

30. Find the interest of $154.25 from April 18, 1852, to Jan. 26, 1855. Ans. $25.657.

31. Find the interest of $172 from Aug. 7, 1854, to Sept. 9, 1856.

32. Find the interest of $254 from Nov. 12, 1855, to Jan. 30, 1857.

33. What is the interest of $132.25 from Nov. 13, 1836, to May 2, 1841?

34. What is the interest of $100 from March 26, 1841, to June 21, 1842?

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234. What of decimal places after the *third* in the Ans.? Explain Ex. 29.

235. To find the interest when the principal is in pounds, shillings, pence, and farthings:

RULE. *Reduce the lower denominations to the decimal of a pound* (Art. 175), *then proceed as with dollars and cents, and finally reduce the decimal part of the interest back to shillings, pence, and farthings* (Art. 176).

NOTE. But 3 decimal places in the multiplicand are used.

35. What is the interest of 56£ 10s. 6d. 3qr. for 1 yr. 6m. 24d.? Ans. 5£ 6s. 3d. 1qr.

36. What is the interest of 246£ 18s. 9d. 1qr. for 2yr. 3m. 15d.?

37. What is the interest of 125£ 16s. 8d. 2qr. from Nov. 13, 1861, to March 26, 1863?

236. To find the interest of any sum for any time, at any other rate than 6 per cent.:

RULE. *First find the interest at* 6 *per cent.; then divide this interest by* 6, *which will give the interest at* 1 *per cent.; and, finally, multiply the interest at* 1 *per cent. by the given rate.*

38. What is the interest of $124.50 for 1yr. 4m. 12d., at 5 per cent.?

OPERATION.

```
     $124.50, Principal.
         .082 = Int. of $1 at 6 per cent. for 1yr. 4m. 12d.
       ------
       24900
      99600
     --------
6) $10.20900 = Int. of Principal at 6 per cent.
   ---------
    $1.7015  = Int. of Principal at 1 per cent.
          5
    -------
    $8.5075  = Int. of Principal at 5 per cent., Ans.
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39. What is the interest of $342.25 for 1yr. 9m. 18d., at 8 per cent.? Ans. $49.284.

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235. Rule for casting interest on pounds, shillings, etc.? How many decimal places in the multiplicand are used? 236. Rule for computing interest at any given rate?

40. What is the interest of $256.84 for 1yr. 3m. 15d., at 9 per cent.?

41. What is the interest of 24£ 6s. 8d. 1qr. for 2yr. 9m. 12d., at 5 per cent.? Ans. 3£ 7s. 8d. 3qr.

42. What is the interest of 150£ 10s. for 2yr. 4m. 6d., at $4\frac{1}{2}$ per cent.?

237. To find the *amount* of any sum at any rate for any time:

RULE. *First find the interest by the preceding rules, and to the interest add the principal.*

43. What is the amount of $325.75 for 1yr. 4m. 24d., at 6 per cent.?

OPERATION.

$325.75,		Principal.
.084	=	Int. of $1 for 1yr. 4m. 24d.
130300		
260600		
$27.36300	=	Int. of Principal.
$325.75	=	Principal.
$353.113	=	Amount, Ans.

44. What is the amount of $224.48 for 2yr. 6m. 15d.? Ans. $258.713.

45. What is the amount of $48.33 for 1yr. 6m.?

46. What is the amount of $365.25 for 1yr. 3m. 9d.?

47. What is the amount of $444 from July 18, 1861, to Sept. 4, 1862? Ans. 474.044.

48. What is the amount of $32.25 from Nov. 15, 1860, to July 25, 1862, at $7\frac{1}{2}$ per cent.?

49. What is the amount of $187.44 from May 25, 1859, to April 19, 1861, at $7\frac{3}{10}$ per cent.?

50. What is the amount of 82£ 12s. 6d. 3qr. from Feb. 12. 1860, to Dec. 24, 1862, at 5 per cent.?

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237. Rule for finding the amount of any sum for a given time and rate? 238. Rule for casting interest on notes when partial payments have been made? Exception? Explain Ex. 51.

238. To cast interest on Notes when Partial Payments have been made:

RULE. *Find the* AMOUNT *of the principal to the time of the first payment; from this amount subtract the first payment, and the* REMAINDER *is a* NEW PRINCIPAL, *with which proceed to the time of the second payment, and so on to the time of settlement.*

EXCEPTION. *If any payment is less than the interest due, cast the interest on the* SAME PRINCIPAL *up to the first time when the sum of the payments shall equal or exceed the interest due, then subtract the* SUM *of the payments from the* AMOUNT *of the principal, and the remainder is a new principal, with which proceed as before.*

51. $525. *Andover, Mass., June* 4, 1848.

For value received, I promise to pay John Davis, or order, five hundred and twenty-five dollars, on demand, with interest.

DANIEL TRUSTY.

On this note are the following indorsements: Sept. 9, 1849, $114.20; May 15, 1850, $78.285; Aug. 6, 1851, $244.375; what was due Feb. 9, 1853? Ans. $191.003.

OPERATION.

$525.	Principal.
39.813	Int. from June 4, '48, to Sept. 9, '49 ... 1yr. 3m. 5d.
564.813	Amount of Principal to Sept. 9, 1849.
114.20	1st Payment.
450.613	1st Remainder, forming the 2d Principal.
18.475	Int. from Sept. 9, '49, to May 15, '50 ... 8m. 6d.
469.088	Amount of 2d Principal to May 15, 1850.
78.285	2d Payment.
390.803	2d Remainder, forming the 3d Principal.
28.724	Int. from May 15, '50, to Aug. 6, '51 ... 1yr. 2m. 21d.
419.527	Amount of 3d Principal to Aug 6, 1851.
244.375	3d Payment.
175.152	3d Remainder, forming the 4th Principal.
15.851	Int. from Aug. 6, '51, to Feb. 9, '53 ... 1yr. 6m. 3d.
$191.003	Amount due Feb. 9, 1853, Ans.

NOTE 1. The pupil will observe that the *operation* is performed on the slate or elsewhere, only the *results* being here written. To do the work here would take up *too much space.*

52. $346.36. *Boston*, *Mar.* 26, 1860.

For value received, we promise to pay Stephen C. Jones, or bearer, three hundred forty-six and $\frac{36}{100}$ dollars, on demand, with interest. BRUCE & DAVIS.

INDORSEMENTS: July 20, 1860, $54.75; April 8, 1861, $10; Sept. 26, 1861, $5.50; Jan. 6, 1862, $150.46; what was due May 2, 1862?

OPERATION.

$346.36	Principal.
6.581	Int. from Mar 26, '60, to July 20, '60 . . . 3m. 24d.
352.941	Amount of Principal to July 20.
54.75	1st Payment.
298.191	1st Remainder, forming the 2d Principal.
26.141	Int. from July 20, '60, to Jan. 6, '62 . . . 1yr. 5m. 16d.
324.332	Amount of 2d Principal to Jan. 6, 1862.
165.96	Sum of 2d, 3d, and 4th Payments.
158.372	2d Remainder, forming the 3d Principal.
3.062	Int. from Jan. 6, '62, to May 2, '62 . . . 3m. 26d.
$161.434	Amount due May 2, 1862, Ans.

NOTE 2. Ex. 51 is solved by the rule, each payment being greater than the interest which had arisen on the principal at the time of the payment; but in Ex. 52 it is found by trial that the 2d and 3d payments were less than the interest due on the principal at the time of the payments, and ∴, in accordance with the exception in the rule, the interest is cast on the 2d principal, $298.191, from July 20, 1860, to Jan. 6, 1862, and then the sum of the 2d, 3d, and 4th payments is taken from the amount of the 2d principal.

53. $486.96. *Andover*, *May* 12, 1860.

For value received, we, jointly and severally, promise to pay Abel Stevens, or order, four hundred eighty-six dollars and ninety-six cents, on demand, with interest. JAMES CARTER,
JOHN DAVIS.

INDORSEMENTS: Jan. 24, 1861, $154.87; Dec. 6, 1861, $75.18; Aug. 18, 1862, $124.87; Dec. 6, 1862, $100; what is due April 24, 1863? Ans. $88.531.

54. $167.42. *Providence, April* 15, 1858.

For value received, I promise to pay A. B., or order, one hundred sixty-seven and $\frac{42}{100}$ dollars, in 6 months from date, with interest. C. D.

INDORSEMENTS: May 21, 1859, $42.18; July 17, 1860, $6.25; Sept. 9, 1860, $48.16; Jan. 27, 1861, $27.47; what was due April 15, 1862? Ans. $72.072.

55. $472.76. *New York, June* 4, 1860.

For value received of Walter Willis, I promise to pay him, or his order, four hundred seventy-two dollars and seventy-six cents, in six months from date, with interest at 7 per cent. afterwards.

SAMUEL JOHNSON.

INDORSEMENTS: April 10, 1861, $125.843; Nov. 28, 1861, $133.724; April 15, 1862, $223.081; what was due Nov. 13, 1862? Ans. $24.97.

56. $1500. *Andover, Aug.* 6, 1858.

For value received, I promise to pay to the Trustees of Phillips Academy, or their order, in Andover, the sum of fifteen hundred dollars, in one year from the first day of October, A. D. eighteen hundred and fifty-eight, with interest to be paid on the first day of April, A. D. eighteen hundred and fifty-nine, and thence afterward half yearly, at the office of the Treasurer of the said Trustees in Andover. J. S. PAYWELL.

In presence of

J. L. TRUMAN.

INDORSEMENTS: April 1, 1859, $58.75; October 1, 1859, $145; Nov. 1, 1859, $150; Feb. 1, 1860, $100; April 1, 1860, $137.614; what was due July 1, 1860? Ans. $1065.75.

239. The rule given in Art. 238 is the one adopted by the United States Courts and most of the State Courts; but, when settlement is made within a year after interest commences, it is customary to adopt the following

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238. Where is the work performed? Why not in the book? **239.** What rule is usually adopted when the time is a year or less?

RULE. 1. *Find the amount of the principal from the time when interest commenced to the time of settlement.*

2. *Find the interest of each payment from the time of payment to the time of settlement.*

3. *Subtract the sum of the payments with their interest from the amount of the principal.*

NOTE. The above rule is often used whatever may be the time; but for long periods it is manifestly unjust, for by it the debtor, *by merely paying interest annually* at 6 per cent., will in less than 24 years cancel his entire debt, and not only so, *the person who loans the money will actually become indebted to the one who borrows.*

57. \$387.75. *Boston, May* 15, 1861.

For value received, I promise to pay to Samuel Adams, on demand, three hundred eighty-seven and $\frac{75}{100}$ dollars, with interest from date. HENRY PHILLIPS.

INDORSEMENTS: July 21, 1861, \$75; Oct. 10, 1861, \$125; Feb. 24, 1862, \$50; what was due at the time of settlement, May 15, 1862?

SOLUTION.

Principal,		\$387.75
Interest of Principal for 1 year,		23.265
Amount of Principal,		\$411.015
1st Payment,	\$75.	
Int. of 1st Payment from July 21, 9m. 24d.,	3.675	
2d Payment,	125.	
Int. of 2d Payment from Oct. 10, 7m. 5d.,	4.479	
3d Payment,	50.	
Int. of 3d Payment from Feb. 24, 2m. 21d.,	0.675	
Sum of Payments, with their Interest,		258.829
Sum due May 15, 1862, Ans.,		\$152.186

58. A note of \$2500, dated June 4, 1861, has the following

INDORSEMENTS: Sept. 4, 1861, \$562.50; Dec. 24, 1861, \$846.37; Feb. 18, 1862, \$362.63; what was due May 12, 1862? Ans. \$821.539.

239. Is this rule just for long periods of time? Why not?

240. Many business men, in computing the interest on notes, adopt the following

RULE. *Find the interest of the principal for a year; also of each payment made during the year from the time of payment to the end of the year. Then subtract the sum of the payments, together with their interest, from the amount of the principal, and the remainder is a new principal, with which proceed for another year, and so on to the time of settlement.*

59. A note of $1500, dated July 25, 1859, has the following

INDORSEMENTS: Sept. 13, 1859, $100; Jan. 25, 1860, $300; Sept. 19, 1860, $250; Dec. 25, 1860, $225; Aug. 13, 1861, $300; what was due June 13, 1862?

SOLUTION.

Amount of Principal to July 25, '60, 1yr.,		$1590.
1st Payment,	$100.	
Int. of 1st Pay't to July 25,'60, 10m. 12d.,	5.20	
2d Payment,	300.	
Int. of 2d Payment to July 25, '60, 6m.,	9.	
Sum of 1st and 2d Pay'ts, with Int.,		414.20
1st Remainder or 2d Principal,		1175.80
Int. of 2d Principal to July 25,'61, 1yr.,		70.548
Amount of 2d Principal to July 25, '61,		1246.348
3d Payment,	$250.	
Int. of 3d Pay't to July 25,'61, 10m., 6d.,	12.75	
4th Payment,	225.	
Int. of 4th Pay't to July 25, '61, 7m.,	7.875	
Sum of 3d and 4th Pay'ts, with Int.,		495.625
2d Remainder or 3d Principal,		750.723
Int. of 3d Prin. to June 13, '62, 10m. 18d.,		39.788
Amount of 3d Prin. to June 13, 1862,		790.511
5th Payment,	$300.	
Int. of 5th Pay't to June 13, '62, 10m.,	15.	
5th Payment, with its Interest,		315.
Sum due at settlement, June 13, '62, Ans.,		$475.511

240. Third rule for computing interest on notes?

60. A note of $684, dated May 25, 1859, has the following

INDORSEMENTS: June 1, 1859, $100; July 7, 1860, $100; Oct. 13, 1860, $75; Dec. 19, 1860, $50; June 7, 1861, $100; Aug. 13, 1861, $40; what was due July 15, 1862?

Ans. $302.044.

NOTE. There is, perhaps, no other operation in Practical Arithmetic in which accountants differ so much as in the mode of computing interest. All the methods are based upon the principles developed in the preceding pages, and it is believed there is no plan, universally applicable, which is more brief and simple than the foregoing. The solution may usually, however, be much shortened, as in the following Articles.

The principal advantage arises from the best divisions of time. Facility in making the best divisions can be easily acquired by practice, and to one having frequent occasion to compute interest the attainment is of great importance.

241. The interest of $1 for 6 days, at 6 per cent., is 1 mill.
The interest of $1 for *ten times* 6d. = 60d. = 2m. is 1 cent.
The interest of $1 for *ten times* 2m. = 20m. = 1yr. 8m. is 1 dime.
The interest of $1 for *ten times* 20m. = 16yr. 8m. is $1.
So the interest of $2, $3, or $1000, for the same times, is 2, 3, or 1000 mills, cents, dimes, or dollars. Thus we see that any number of dollars expresses its own interest in mills, cents, dimes, or dollars for the above-mentioned times, and hence, to know the interest it is only necessary to determine the place of the decimal point.

61. What is the interest of $324 for 93 days?

OPERATION.

$3.24 = Int. for 60 d.
1.62 = Int. for 30 d.
.162 = Int. for 3 d.
$5.022 = Int. for 93 d., Ans.

All like examples can be solved in a similar manner. Hence,

242. To compute interest at 6 per cent. for months and days,

RULE. *Move the decimal point in the principal two places to-*

240. What of different modes of computing interest? What of the best division of time? **241.** Any sum of money expresses its own interest at six per cent. for what times?

ward the left, and the result will be the interest for TWO MONTHS *or* SIXTY DAYS. *Move the point three places toward the left, and the result will be the interest for* SIX DAYS. *Then take such multiples and aliquot parts of these results as the given time may require, and the sum of these will be the interest.*

PROOF. *Divide the computed interest by the interest of the principal for one month, and the quotient should be the number of months expressed in the example; or, divide by the interest for one day, and the quotient should be the number of days.*

NOTE 1. This is the most simple mode of proof, and applies to all rules for computing interest. The *Problems in Interest*, page 203, furnish other methods of proof.

NOTE 2. In computing interest it is customary to consider 30 days a month and 12 months a year, and ∴ the *computed* interest for 12 times 30 days, or 360 days (i.e. for $\frac{360}{365} = \frac{72}{73}$ of a year), is *truly the interest for a whole year.* Thus, the *computed* interest for *any number of days* is $\frac{1}{73}$ too large and it must ∴ be diminished by $\frac{1}{73}$ of itself to find the *true* interest. As interest is usually computed for *months* and days the difference is slight, and, in course of business, is seldom considered; but in England, and in dealing with the United States Government, it is customary to compute true interest.

62. What is the interest of \$720 for 7 months and 3 days?

\$7.20 = Int. for 2m.

21.60 = Int. for 6m. = 3 times 2m.

3.60 = Int. for 1m. = ½ of 2m.

.36 = Int. for 3d. = ½ of 6d.

\$25.56 = Int. for 7m. 3d., Ans.

PROOF. The interest of the principal for 1 month is \$3.60, and the Ans. to the example is \$25.56; ∴ the time in months is \$25.56 ÷ \$3.60 = 7.1m. = 7m. 3d., *the time given in the example.*

63. What is the interest of \$1260 for 75 days?

\$12.60 = Int. for 60 d.

3.15 = Int. for 15 d. = ¼ of 60 d.

\$15.75 = Int. for 75 d., Ans.

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242. Rule for computing interest for months and days, at 6 per cent.? Proof? Note 1? Note 2?

243. *Three days* is $\frac{1}{10}$ of a month, ∴ $\frac{1}{10}$ of the interest of \$1, *or any other sum*, for 1 month, is the interest of the *same* sum for 3 days. In like manner, $\frac{1}{10}$ of the interest of *any sum* for *any number of months* is the interest of the *same* sum for *three times* as many days.

64. What is the interest of \$765 for 2m. 6d.?

OPERATION.

\$ 7.6 5	= Int. for 2m., i. e. for		6 0d.
.7 6 5	= Int. for $\frac{1}{10}$ of 60d., i. e.		6d.
\$ 8.4 1 5	= Int. for		6 6d., Ans.

65. What is the interest of \$845 for 6 days?

845 mills = \$.845, Ans.

66. What is the interest of \$845 for 2 months?

845 cents = \$8.45, Ans.

67. What is the interest of \$845 for 1yr. 8m.?

Ten times 845 cents = \$84.50, Ans.

68. What is the interest of \$845 for 16⅔yr.?

Ten times \$84.50 = \$845, Ans.

NOTE. The pupil will observe that merely changing the position of the decimal point, as in the four preceding examples, gives the interest of any sum for 6 days, for 2 months, for 1 year and 8 months, or for 16⅔ years.

69. What is the interest of \$845 for 1yr. 10m. 6d.?

OPERATION.

\$ 8 4.5 0	= Int. for 1yr. 8m., i. e. for	2 0m.
8.4 5	= Int. for $\frac{1}{10}$ of 20m., i. e.	2m.
.8 4 5	= Int. for $\frac{1}{10}$ of 2m., i. e.	6d.
\$ 9 3.7 9 5	= Int. for	2 2m. 6d., Ans.

70. What is the interest of \$348 for 22 days?

3) \$ 3.4 8	= Int. for 6 0 days.
1.1 6	= Int. for 2 0 days.
.1 1 6	= Int. for 2 days.
\$ 1.2 7 6	= Int. for 2 2 days, Ans.

244. One tenth of the interest of any sum for any number of months, is the interest of the same sum for how many days? Rule for determining the interest of any sum for 6 days? For 2 months? For 1yr. 8m.? For 16yr. 8m.?

71. What is the interest of \$412 for 5m.? Ans. \$10.30.
72. What is the interest of \$42 for 2m. 22d.? Ans. \$.574.
73. What is the interest of \$54 for 22d.? Ans. \$.198.
74. What is the interest of \$2148 for 3m. 10d.?
75. What is the interest of \$75 for 1yr. 10m. 6d.?
76. What is the interest of \$173 for 1 yr. 8m.?

244. In some States interest is allowed on the annual interest of the principal which is due and unpaid, if the note is written "with interest annually." Such examples may be solved by *computing interest on the principal for the whole time and on each year's interest for the time it is due and unpaid;* but the following brief practical mode of computing "annual interest" will be of service to the business man.

RULE. *Find the interest on the principal for the given number of* ENTIRE YEARS; *on this interest find the interest for half of the years less one, and the months and days; and this latter interest is the* EXCESS OF ANNUAL OVER SIMPLE INTEREST *for the given time. To this excess add the interest on the principal for the whole time, and the sum is the annual interest for the given time.*

77. What is the annual interest of \$800 for 5 years?

\$800, Principal.	
.30	= Simple Int. of \$1 for 5 years.
240.00	= Simple Int. of \$800 for 5 years.
.12	= Simple Int. of \$1 for 2yr. i. e. for $\frac{5-1}{2}=2$yr.
28.80	= Excess of annual over simple Int. of \$800 for 5yr.
240	= Simple Int. of the principal, as above.
\$268.80	= Annual Int. of \$800 for 5yr., Ans.

78. What is the annual interest of \$600 for 6yr. 4m. 18d?

SOLUTION. The interest of \$600 for 6 years is \$216; the interest of \$216 for ½ of (6 — 1) yr., increased by the months and days, viz. 2½yr. 4m. 18d., or 2yr. 10m. 18d. is \$37.368, and this is the excess of the annual over the simple interest of \$600 for 6yr. 4m. 18d. To this add the interest of \$600 for 6yr. 4m. 18d., viz. \$229.80, and we have \$267.168, the annual int.

---

244. Rule for computing annual interest? Explain Ex. 77. Ex. 78.

79. What is the annual interest of $462.84 for 7yr. 8m. 6d.?
Ans. $256.33.

80. What is the excess of annual over simple interest of $250 for 5yr. 7m. 24d.? Ans. $11.925.

81. What is the *amount* of $325, at annual interest for 8yr. 6m. 15d.? Ans. $529.393.

82. What is the amount of $4692.80, at annual interest for 9yr. 4m. 24d.?

PROBLEMS IN INTEREST.

245. In every example in interest there are four elements or particulars which claim special attention, viz. *Principal*, *Rate*, *Time*, and *Interest*, any three of which being given, the other can be found.

To find the Interest when the Principal, Rate, and Time are given, has, thus far, been the object of our discussion.

The other branches of the subject give rise to the following problems:

246. PROBLEM 1. Principal, Interest, and Time given, to find the RATE.

Ex. 1. At what rate per cent. must $300 be put on interest to gain $18 in 2 years?

ANALYSIS. $300, at 1 per cent., will gain $6 in 2 years; ∴, to gain $18, the rate must be the quotient of $18 ÷ $6 = 3. Hence,

RULE. *Divide the given interest by the interest of the principal, for the given time, at* 1 *per cent., and the quotient will be the rate.*

2. At what rate per cent. must $142 be put on interest to gain $21.30 in 3 years? Ans. 5.

3. If $36 gain $7.56 in 3 years, what is the rate per cent.?

4. If $300 gain $43.80 in 2yr., what is the rate per cent.?

---

245. How many particulars claim attention in an example in interest? What are they? How many of them are given? **246.** Object of Prob. 1? Rule?

247. PROBLEM 2. Principal, Interest, and Rate given, to find the TIME.

Ex. 1. For what time must \$200 be on interest at 6 per cent. to gain \$36?

ANALYSIS. \$200 in 1 year, at 6 per cent., will gain \$12; ∴, to gain \$36, the time in years must be the quotient of \$36 ÷ \$12 = 3. Hence,

RULE. *Divide the given interest by the interest of the principal for one year at the given rate, and the quotient will be the time.*

2. How long must \$254 be on interest at 5 per cent. to gain \$44.45? Ans. 3.5yr. = 3yr. 6m.

3. How long must \$75 be on interest at 8 per cent. to gain \$15.80? Ans. 2.63⅓yr. = 2yr. 7m. 18d.

4. How long must \$200 be on interest at 6 per cent. to *amount* to \$236? Ans. 3 years.

5. For what time must \$72 be put to interest at 8½ per cent. to amount to \$87.30?

6. For what time must \$1000 be put to interest at 9 per cent. to gain \$247.50?

7. How long must \$100 be on interest at 5 per cent. to gain \$100? Ans. 20 years.

NOTE. \$100 in 1 year, at 5 per cent., will gain \$5; ∴, to gain \$100, the time in years must be the quotient of \$100 ÷ \$5 = 20; i. e.,

To find the time in which any sum will double itself, at any rate per cent., divide 100 *by the rate, and the quotient will be the time in years.*

8. In how many years will \$50 *amount* to \$100, it being on interest at 8 per cent.? Ans. 12yr. 6m.

9. How long will it take any sum of money to double itself on interest at 6 per cent.?

10. In what time will a sum of money triple itself on interest at 5 per cent.?

---

247. Prob. 2? Rule? Rule for finding the time in which any principal will double at any rate per cent.?

248. PROBLEM 3. Interest, Time, and Rate given, to find the PRINCIPAL.

Ex. 1. What principal, at 6 per cent., will gain $18 in 1 yr. 6m.?

ANALYSIS. $1, in 1yr. 6m., at 6 per cent., will gain 9cts., i. e. $.09; ∴ the principal must be the quotient of $\$18 \div .09 = \$200$. Hence,

RULE. *Divide the given interest by the interest of* $1 *for the given rate and time, and the quotient will be the principal.*

2. What principal, at 6 per cent., will gain $13 in 8 months?
Ans. $325.

3. What principal, on interest at 8 per cent. per annum, will gain $150 semi-annually?

4. B endowed a professorship with a salary of $2000 per annum; what sum did he invest at 6 per cent.?

(a) To the preceding we may add

PROBLEM 4. Amount, Rate, and Time given, to find the PRINCIPAL.

Ex. 1. What principal, at 5 per cent., will amount to $110 in 2 years?

ANALYSIS. $1 in 2 years, at 5 per cent., amounts to $1.10; ∴ the principal must be the quotient of $\$110 \div 1.10 = \$100$. Hence,

RULE. *Divide the given amount by the amount of* $1 *for the given rate and time, and the quotient will be the principal.*

2. What principal, at 6 per cent., will amount to $130.39 in 8 months? Ans. $125.375.

3. What principal, at 8 per cent., for 3 years, will amount to $74.40?

4. What is the interest of that sum for 2yr. 6m., at 8 per cent., which will, at the given rate and time, amount to $240?

---

248. Object of Prob. 3? Rule? Prob. 4? Rule?

COMPOUND INTEREST.

249. COMPOUND INTEREST is interest on both *principal and interest*, the latter not being paid when it becomes due.

The principal may be increased by adding the interest to it annually, semi-annually, quarterly, etc., according to agreement, and the creditor may *receive* compound interest without being liable to the charge of usury (Art. 231), though he cannot *legally collect* it if the debtor refuses to pay.

250. To calculate Compound Interest:

RULE. *Make the* AMOUNT *for the* FIRST *year or specified time, the* PRINCIPAL *for the* SECOND; *the amount for the* SECOND *the principal for the* THIRD; *and so on. From the* LAST AMOUNT *subtract the* FIRST PRINCIPAL, *and the* REMAINDER *is the compound interest.*

Ex. 1. What is the compound interest on $100 for 3yr. 3m., at 6 per cent. per annum?

OPERATION.

		$100.	1st Principal.
$100	× .06 =	6.	Interest for 1st year.
		106.	1st Am't or 2d Prin.
$106	× .06 =	6.36	Interest for 2d year.
		112.36	2d Am't or 3d Prin.
$112.36	× .06 =	6.7416	Interest for 3d year.
		119.1016	3d Am't or 4th Prin.
$119.1016	× .015 =	1.786524	Interest for 3 months.
		120.888124	4th or last Amount.
		100.	1st Principal.
		$ 20.888124	Com. Int. for 3yr. 3m.

NOTE 1. Find the *amount* for the *years* as though there were no months in the given time, and this amount *is the principal for the remaining months.*

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249. Compound Interest, what is it? How often may the interest be compounded? May the creditor *receive* compound interest if the debtor chooses to pay? Can he *collect* it if the debtor refuses to pay? 250. Rule for computing compound interest? Rule when there are months and days in the given time?

2. What is the compound interest on \$200 for 2yr. 8m., at 4 per cent. per annum. Ans. \$22.089.

3. What is the compound interest on \$500 for 3 years, at 7 per cent.? Ans. \$112.5215.

4. What is the *amount* of \$5000 at compound interest, for 4yr. 10m. 12d.? Ans. \$6640.629.

5. What is the amount of \$3000 at compound interest for 2yr. 6m. 18d ? Ans. \$3482.036.

6. What is the compound interest of \$10000 for 2yr. 6m. 18d., at 6 per cent.? Ans. \$1606.788.

7. What is the compound interest of \$10000 for 2yr. 6m. 18d., at 4 per cent.? Ans. \$1053.952.

8. What is the compound interest of \$10000 for 2yr. 6m. 18d., at 8 per cent.? Ans. \$2177.216.

NOTE 2. *Four* per cent of any number is $\frac{2}{3}$, and 8 per cent is $\frac{4}{3}$ of 6 per cent. of the same number, but the compound interest of any sum of money at 4 per cent. is *less* than $\frac{2}{3}$ of the compound interest of the same sum for the same time at 6 per cent., and the interest at 8 per cent. is *more* than $\frac{4}{3}$ of the interest at 6 per cent., as may be seen by examples 6, 7, and 8.

The compound interest at 4 per cent. is less than half the compound interest of the same sum at 8 per cent., because the *base* of percentage, (i. e. the principal,) after the 1st year, is *less* in computing interest at 4 per cent. than in computing it at 8 per cent.; thus, in computing interest at 4 and 8 per cent. the 1st year the base is the *same*, and *one interest is just half of the other*; but the 2d year one base is \$104 and the other \$108; ∴ the interest at 4 per cent. *is less than half of that at* 8 *per cent.*

9. What is the amount of \$250 for 2yr. 6m., at 3 per cent. for each 6m., compounding the interest semi-annually?

Ans. \$289.818.

10. What is the interest of \$36 for 1yr. 9m., at 2 per cent. per quarter, compounding the interest quarterly?

Ans. \$5.352.

11. What is the compound interest of \$864.75 for 3yr. 8m. 15d., at 6 per cent.? Ans. \$208.953.

12. What is the compound interest of \$327.54 for 4yr. 4m. 8d.?

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250. Is compound interest at 4 per cent. half as much as at 8 per cent.? Why?

251. Compound interest may be calculated more expeditiously by means of the following

TABLE,

Showing the Amount of $1, £1, *etc., interest compounded annually at* 4, 5, 6, 7, *and* 8 *per cent., from* 1 *to* 20 *years.*

Yr.	4 per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.	Yr.
1	1.040000	1.050000	1.060000	1.070000	1.080000	1
2	1.081600	1.102500	1.123600	1.144900	1.166400	2
3	1.124864	1.157625	1.191016	1.225043	1.259712	3
4	1.169859−	1.215506+	1.262477−	1.310796+	1.360489−	4
5	1.216653−	1.276282−	1.338226−	1.402552−	1.469328+	5
6	1.265319+	1.340096−	1.418519+	1.500730+	1.586874+	6
7	1.315932−	1.407100+	1.503630+	1.605781+	1.713824+	7
8	1.368569+	1.477455+	1 593848+	1.718186+	1.850930+	8
9	1.423312−	1.551328+	1.689479−	1.838459+	1.999005−	9
10	1.480244+	1.628895−	1.790848−	1.967151+	2.158925−	10
11	1.539454+	1.710339+	1.898299−	2.104852−	2.331639−	11
12	1.601032+	1.795856+	2.012196+	2.252192−	2.518170+	12
13	1.665074−	1.885649+	2.132928+	2.409845+	2.719624−	13
14	1.731676+	1.979932−	2.260904−	2.578534+	2.937194−	14
15	1.800944−	2.078928+	2.396558+	2.759032−	3.172169+	15
16	1.872981+	2.182875−	2.540352−	2.952164−	3.425943−	16
17	1.947900+	2.292018+	2.692773−	3.158815+	3.700018+	17
18	2.025817−	2.406619+	2.854339+	3.379932+	3.996019+	18
19	2.106849+	2.526950+	3.025600−	3.616528−	4.315701+	19
20	2.191123+	2.653298−	3.207135+	3.869684+	4.660957+	20

NOTE. The *interest* is $1, £1, etc., *less* than the *amount* in the above table.

13. What is the compound interest on $600 for 20yr.?

```
$2.207135    = Int. of $1 for 20yr. taken from the Table.
       600
-----------
$1324.281000 = Int. of $600 for 20yr., Ans.
```

14. What is the compound interest on $30 for 5yr. 6m.?

```
 $1.338226   = Amount of $1 for 5yr.
       .03   = Int. of $1 for 6m.
 ----------
  .04014678
  .338226    = Int. of $1 for 5yr.
 ----------
$.37837278   = Int. of $1 for 5yr. 6m.
         30
 ----------
$11.35118340 = Int. of $30 for 5yr. 6m., Ans.
```

15. What is the amount of \$50, at 7 per cent. per annum, for 15yr. at compound interest?

$$
\begin{array}{rl}
\$2.759032 & = \text{Amount of } \$1 \text{ for } 15\text{yr.} \\
50 & \\
\hline
\$137.951600 & = \text{Amount of } \$50 \text{ for } 15\text{yr., Ans.}
\end{array}
$$

16. What is the amount of \$350.50, at 8 per cent. compound interest, for 18 years?

17. What is the compound interest of \$75 for 20 years, at 8 per cent.?

18. What is the interest of \$500 for 9yr. 6m., at 4 per cent. for each 6 months, compounding the interest semi-annually?
Ans. \$553.425.

19. What is the amount of \$100 at compound interest for 40 years, at 7 per cent. per annum? Ans. \$1497.445.

20. What is the amount of \$100 at compound interest for 30 years, at 6 per cent. per annum?

DISCOUNT.

252. Discount is an abatement or deduction made for the payment of a debt before it is due.

The present worth of a debt, payable at a future time without interest, is, evidently, a sum which, put at legal interest, will amount to the debt at the time of its becoming due.

The *debt*, then, is an *amount*, the *present worth* is the *principal*, and the *discount* is the *interest* of this principal. Hence,

253. The *rule* for finding the present worth is that given in Prob. 4, Art. 248, viz.:

Divide the given sum by the AMOUNT *of* \$1 *for the given rate and time.*

The DISCOUNT *is found by subtracting the present worth from the face of the debt.*

---

252. What is Discount? Present Worth? The debt is the same as what in Art. 248? Present Worth? Discount? 253. Rule for finding present worth? Discount? Explain Ex. 1.

Ex. 1. What is the present worth of $37.44, due in 8 months? What the discount?

OPERATION.

Amount of $1 for 8m.,	1.04) 37.44 (36, Present worth.
	312
$37.44, Given sum,	624
36.00, Present worth.	624
$1.44, Discount.	0

2. What is the present worth of a debt of $100, payable in one year, without interest? What the discount?

Ans. Present worth, $94.339+; discount, $5.661—.

3. What is the present worth of $1319.29, due in 2yr. 11m.? Ans. $1122.80.

4. What is the present worth of $141.50, due in 1yr. 3m. 15d.? Ans. $131.32+.

5. What is the present worth of $346.87, due in 2yr. 4m. 12d.? Ans. $303.74—.

6. What is the *discount* on $456.25, due in 9m. 12d.? Ans. $20.48.

7. What is the present worth of $490.50, due in 1yr. 6m.? What the discount?

8. What is the discount on $315, due in 1 year, at 5 per cent.?

9. I have a note for $1000, payable May 1, 1863; what discount shall I make for payment to-day, Aug. 19, 1862, money bearing interest at 10 per cent. per annum? Ans. $65.42.

NOTE. The interest on the present worth equals the discount on the debt.

10. What is the interest for 6 months on the present worth of a note for $350, due 6 months hence? Ans. $10.19.

11. What is the interest for a year on the present worth of a note for $756, due 1 year hence?

12. I have a note for $436, payable June 21, 1863; what is the worth of the note to-day, May 12, 1863, money being worth 8 per cent. per annum?

13. What is the discount on $896, due in 1yr. 8m.?

14. What is the present worth of $475, due in 2yr. 4m. 12d.?

BANKING AND BANK DISCOUNT.

253 a. A Bank is an institution, incorporated by law, for the safe keeping and loaning of money, dealing in exchange, furnishing a currency for circulation, etc.

The charter incorporating a bank, defines its privileges and limits its powers.

The Capital Stock of a bank is the money, paid into the bank in specie by the stockholders, as a basis of business.

Note 1. Banks are of three kinds, viz.: Banks of *Deposit*, Banks of *Discount*, and Banks of *Circulation*.

A *Bank* of *Deposit* receives and takes care of money, subject to the order of the depositor.

A *Bank* of *Discount* loans money upon notes, drafts, and other securities.

A *Bank* of *Circulation* issues its own *bills* or *notes*, which are usually redeemable in coin at the bank which issues them, and, because redeemable in coin, *they pass as money* in business transactions.

Banks in this country usually combine the threefold office of deposit, discount, and circulation.

Note 2. The affairs of a bank are controlled by a *Board* of *Directors*, chosen annually by the stockholders from among themselves.

The *President* and *Cashier*, appointed by the Directors, superintend the business of a bank and sign all bills which it issues.

A Bank Check is an order for money, drawn on the bank.

The *face* of a note is the sum for which it is written.

The *maturity* of a note is the day when it becomes due.

In most of the states a note is not *legally* due until *three days* after the time which the note specifies for its payment. These three days are called *days of grace.* A note matures upon the last day of grace.

Note 3. When a note becomes due on Sunday or a legal holiday, it is legally payable on the preceding day.

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253 a. What is a Bank? What of its privileges and powers? What is the Capital Stock of a Bank? Banks are of how many kinds? What? The office of each? What of banks in this country? Directors, how chosen? Duties of President and Cashier? A Bank Check, what? The face of a note? The maturity? What of days of grace? When does a note mature? What of Sundays and holidays?

NOTE 4. A note made payable in a *certain number of days* is not due until that number of days and grace expire; thus, a *thirty days note*, dated Jan. 31, becomes due Mar. 5 (or, in leap-year, Mar. 4), but a note made payable in a *certain number of months, nominally* matures on the same day of the month that it is dated, if there are so many days in the month when it matures; or, if there are not so many days in the month, it matures on the *last day of the month*; thus, a *one month note*, dated on the 28th of February, *nominally* matures Mar. 28, and *legally* matures Mar. 31; but a *one month note*, dated on Jan. 28 (except in leap-year) or on Jan. 29, Jan. 30, or Jan. 31, nominally matures Feb. 28, and legally Mar. 3.

253 b. Interest on money borrowed at a bank is paid *when the money is borrowed.* The interest deducted in advance from the face of a note, and retained by the bank as compensation for the money borrowed, is called *Bank Discount.* The money received by the borrower is called the *Proceeds* or *Avails* of the note, and is equal to the face of the note, less the interest. The note is said to be *discounted.*

To find the bank discount and the proceeds of a note, payable at a specified future time, without interest,

RULE. 1. *Find the interest on the face of the note, at the given rate, from the time of discounting to the maturity, and the result will be the discount.*

2. *Subtract the discount from the face of the note, and the remainder will be the proceeds or avails.*

Ex. 1. What is the bank discount on a 90 days note for \$368? What are the proceeds?

$$
\begin{aligned}
\$3.68 &= \text{Interest for } 60 \text{ days.} \\
1.84 &= \text{Interest for } 30 \text{ days.} \\
\underline{.184} &= \text{Interest for } \underline{\ \ 3} \text{ days.} \\
\$5.704 &= \text{Interest for } 93 \text{ days, 1st Ans.}
\end{aligned}
$$

$$\$368 - \$5.704 = \$362.296, \text{ proceeds, 2d Ans.}$$

2. I have a 6 months note for \$768, dated May 12; what will be the avails if I get it discounted Sept. 3?

---

253 a. A note payable in a number of days, when due? In a number of months, when due? 253 b. Interest paid at bank, when? Money received, called what? Rule for finding bank discount? For finding the proceeds of a note?

$$\begin{array}{rl} \$7.68 & = \text{Interest for } 2\text{m.} \\ 1.536 & = \text{Interest for } 12\text{d.} \\ \hline \$9.216 & = \text{Discount.} \end{array}$$

$768 — $9.216 = $758.784, proceeds, Ans.

Six months and grace from May 12 expire Nov. 15. From Sept. 3 to Nov. 15 is 2m. 12d., the time for which the note is discounted.

3. What will be the bank discount and what the proceeds on a 4 months note for $8646?

4. On a 90 days note for $1842, at 7 per cent.?

5. On a 6 months note for $489, at 5 per cent.?

6. A 4 months note for $629, dated Feb. 27, was discounted Apr. 12; what were the proceeds?

7. What is the difference between bank discount and true discount (Art. 252) on an 8 months note for $4600?

NOTE. 1. When a note bearing interest is discounted before its maturity, the *amount* of the note *at maturity*, rather than its *face*, is the base for discounting.

8. What are the proceeds of a note for $10000, payable in 6 months and bearing interest, if discounted 2 months before its maturity?

The amount of $10000 for 6m. 3d. is $10305, and the interest of $10305 for 2m. is $103.05, which taken from $10305, leaves $10201.95, Ans.

9. What are the proceeds of a note for $6844, payable in 4 months and bearing interest, if discounted 1 month after date?

NOTE 2. Business men often deduct more than the legal rate of interest for present payment of a bill having a term of credit.

10. What shall I pay on a 6 months bill of $75, if 5 per cent. be deducted for cash?

11. What on a bill of $250, if 8 per cent. is deducted?

253 c. To find the sum for which a note must be written that the proceeds may be a specified sum.

Ex. 1. For what sum must a 45 days note be written, that the proceeds may be $240?

---

253 b. What is Note 1? Note 2?

OPERATION.

$$\begin{array}{lr} & \$1.000 \\ \text{Interest of \$1 for 48 days,} & \underline{.008} \\ \text{Proceeds of \$1,} & .992 \end{array}$$

$\$240 \div .992 = \$241.935$, Ans.]

The proceeds of $1 for 45 days and grace, are $0.992, and ∴ the face of the note must be as many dollars as $0.992 is contained times in $240, viz. $241.935. Hence,

RULE. *Divide the required proceeds by the proceeds of* $1 *for the given rate and time, and the quotient will be the number of dollars in the face of the required note.*

2. For what sum must a 3 months note be given, that the proceeds may be $300?

3. A farmer sold produce for which he received a 60 days note, which he immediately had discounted at the bank. The proceeds of the note were $593.70; what was its face?

4. A merchant wishes to borrow $1200 at a bank, for 90 days; what shall be the face of the note, the rate of interest being 7 per cent.?

INSURANCE.

254. INSURANCE is security against loss from the damage or destruction of property by fire, shipwreck, or other specified casualty; or from loss of life or health by disease or accident.

255. The PREMIUM is the sum paid for the insurance, and is usually computed at a certain per cent. on the sum insured. The per cent. varies according to the nature, locality, etc., of the property, or the age, place of residence, etc., of the person insured; also according to the length of time for which the security is given.

NOTE. Some property is so hazardous, that insurance companies decline taking the risk at *any* per cent.

256. The POLICY is the writing or record of the contract, given by the insurer to the insured. The policy specifies the *nature* of the risk, and names the *hour* when it *begins* and *ends*.

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253 c. To find the face of a note such that the proceeds shall be a specified sum, Rule? **254.** What is Insurance? **255.** Premium? How computed? Does the per cent. vary? Why? **256.** What is the Policy? What does it specify?

257. If property is fully insured the owner is tempted to destroy the property, and secure its value from the insurance company. To prevent such fraud, companies will usually insure the property for only about $\frac{2}{3}$ or $\frac{3}{4}$ its value, requiring the owner to risk the remainder. The same property may be insured at several different offices, by consent of the companies insuring it, but not so that the whole sum insured at the different offices shall exceed that per cent. of its value which a single company is accustomed to insure.

258. To calculate the premium on a given sum:

RULE. *Multiply the sum insured by the rate per cent., written decimally.*

NOTE. The insured usually pays a given sum, say, \$1.25, *for the policy*, in addition to the premium of a certain per cent. on the sum insured.

Ex. 1. What is the cost of insuring \$2500 on my house for 1 year at 2 per cent., the policy being \$1.25?

OPERATION.

$$\begin{array}{rl} \$2500 \times .02 = \$50.00, & \text{Premium.} \\ \underline{1.25,} & \text{Policy.} \\ \$51.25, & \text{Ans.} \end{array}$$

2. What is the annual premium for insuring a manufacturing establishment in the sum of \$75000, at 3 per cent.?

Ans. \$2250.

3. In a certain house, the furniture, worth \$2400, is insured for $\frac{2}{3}$ its value at $1\frac{3}{4}$ per cent.; what is the premium?

4. The Merrimac Mutual Fire Insurance Company have insured \$2000 on my house for a period of 5 years, at $\frac{3}{5}$ of 1 per cent.; what is the cost, the policy being \$1.25?

5. I buy a house for \$8000, and get it insured for $\frac{3}{4}$ of its value at $\frac{2}{3}$ of 1 per cent.; the house being burned, what is my loss? What the loss of the insurers?

Ans. My loss, \$2040; loss of Co., \$5960.

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257. Is property usually insured for its full value? Why not? May it be insured at more than one office? On what conditions? 258 Rule for computing premium? Cost of policy?

6. What is the premium, at $1\frac{1}{3}$ per cent., for insuring \$75000 on a steamboat and cargo from Boston to Havre?

7. A cotton factory worth \$25000, and the machinery and stock worth \$35000, are insured for $\frac{1}{2}$ their value at 3 per cent.; what is the premium?

8. What is the annual premium for insuring \$6000 for 7 years on the life of a man 25 years of age, the rate being .97 of 1 per cent. annually? Ans. \$58.20.

9. What will be the annual premium for insuring \$8500 for 10 years on the life of a man 30 years of age, the premium being 1.09 per cent.?

STOCKS.

259. The CAPITAL or STOCK of a Bank, Railroad, Insurance, Mining, or Manufacturing Company, or other Corporation, is the money or other property employed in transacting the business of the Company. City, State, and Government Bonds are also called *Stocks*.

260. The capital or stock of a company, is usually divided into a number of equal parts, called *shares*, and the owners of the shares are called *stockholders*.

261. Shares of stock are bought and sold like any other property. The *nominal* or *par value* of a share of stock is a *fixed sum* (in most companies \$100, though in some companies *more*, and in some, *less*), but the *market value* varies, according to circumstances; as, e. g., if a company is prosperous, and its prospects are good, its stock rises in price; but if the company has been unfortunate, and its prospects are bad, its stock declines.

The abundance or scarcity of money also affects the price of stocks. The price of government stocks also depends upon the state of the country as to peace or war, the prospects of the stability or instability of the government, etc., etc.

NOTE. In this work, \$100 is considered the par value of a share of stock, unless some other sum is named.

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259. What is the Capital or Stock of a Company? 260. How divided? 261. What is the *par* value of stock? The *market* value, how does it vary?

262. If a share of stock sells for its nominal value, it is said to be *at par*; if it sells for *more*, it is *at a premium*, *in advance*, or *above par*; if it sells for *less*, it is *at a discount*, or *below par*.

263. The interest paid on government stocks, and the profits from the business of companies, distributed from time to time among the stockholders, are called *Dividends*.

The sums of money occasionally required of the stockholders, to meet the losses or expenses of the company, are called *Assessments*.

264. Assessments, dividends, discounts, and premiums are percentages on the par value of the stock as a base. Hence,

PROBLEM 1. To find an assessment, a dividend, discount, or premium:

RULE. *Multiply the par value of the stock by the rate per cent., written decimally.*

Ex. 1. The directors of a manufacturing company, wishing to enlarge their works, call for an assessment of 5 per cent. on the capital of the company; what will be the assessment on $15000 worth of the stock?

OPERATION.

$$\begin{array}{r} \$15000 \\ .05 \\ \hline \$750.00, \text{ Ans.} \end{array}$$

The operation is the same as for computing interest for 1 year, at any given rate.

2. The Boston and Maine Railroad Company paid a dividend of 4 per cent., Jan. 1, 1861; what was paid on 25 shares of its stock?

OPERATION.

$$\begin{array}{r} \$100 \\ 25 \\ \hline \$2500 \\ .04 \\ \hline \$100.00, \text{ Ans.} \end{array}$$

First find the value of 25 shares, and then compute the dividend.

262. When is stock at par? Above par? Below par? **263.** What are dividends? Assessments? **264.** Rule for computing dividends, assessments, etc.?

3. What is the discount on $1400 worth of stock which sells at 30 per cent. below par? Ans. $420.

4. Suppose the New England Glass Co. Stock sells at an advance of 10 per cent., what is the premium on 5 shares at $500 per share?

265. PROBLEM 2. To find the market value of stock when sold at a premium, or at a discount.

Ex. 1. What is the market value of $5000 worth of stock, at a discount of 5 per cent.?

```
  $5000
    .95
  -----
  25000
 45000
 ------
$4750.00, Ans.
```

Since the stock sells at a discount of 5 per cent., $1 of the stock sells for 95 cents, i. e. the market value is .95 of the par value.

2. What is the market value of 6 shares of Fitchburg Railroad Stock, at an advance of 2 per cent.?

OPERATION.

```
  $100
     6
  ----
  $600
  1.02
  ----
  1200
 600
 -----
$612.00, Ans.
```

First find the par value of 6 shares, and then increase the par value by the 2 per cent. premium, i. e. multiply the par value by 1.02.

Similar reasoning holds in all cases. Hence the

RULE. *Multiply the par value of the stock by the number which represents the market value of* $1 *of the stock.*

3. What shall I receive for 12 shares of the Andover Bank Stock at 9 per cent. premium? Ans. $1308.

4. What is the market value of 75 shares of Railroad Stock at a discount of 85 per cent.?

5. What is the premium on 15 Shares of the Western Railroad Stock, at 18 per cent. advance?

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265. Rule for finding the market value of stocks?

266. PROBLEM 3. To find how many shares of stock may be bought for a given sum.

Ex. 1. How many shares of Railroad Stock may be bought for \$870, when the market price is 13 per cent. below par?

OPERATION.

$\$870 \div .87 = \$1000.$

$\$1000 \div \$100 = 10$, Ans.

\$1 of stock is worth only 87 cents, ∴ the quotient of \$870 ÷ .87, viz. \$1000, is the nominal value of the stock bought. Again \$1000 divided by \$100, the nominal value of 1 share, gives 10 shares, Ans.

2. How many shares of the Western Railroad stock may be purchased for \$575, when it is worth 15 per cent. premium?

OPERATION.

$\$575 \div 1.15 = \$500.$

$\$500 \div \$100 = 5$, Ans.

\$1 of stock is worth \$1.15, ∴ \$575 ÷ 1.15 = \$500, is the nominal value of the purchase. Again, \$500 ÷ \$100 = 5, the number of shares purchased. Hence,

RULE. 1. *Divide the sum expended by the number representing the market value of* \$1 *of the stock, and the quotient is the nominal value of the stock bought.*

2. *Divide the nominal value of the purchase by the nominal value of* 1 *share, and the quotient is the number of shares bought.*

3. How many shares of the Exchange Bank Stock, at 25 per cent. premium, can be bought for \$1000? Ans. 8.

4. How many shares of Mining Stock, at 12 per cent. discount, may be bought for \$2200?

COMMISSION AND BROKERAGE.

267. COMMISSION or BROKERAGE *is the compensation received by an agent for transacting certain kinds of business,* such, e. g. as collecting and loaning money, or buying and selling goods, notes, stocks, etc.

The agent is variously styled as *factor, broker, collector, correspondent, commission merchant,* etc.

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266. Rule for finding how many shares of stock may be bought for a given sum? **267.** What is Commission or Brokerage? What is the agent styled?

268. Commission or Brokerage is a certain percentage on the money collected or expended. Hence,

PROBLEM 1. To compute Commission or Brokerage on a given sum:

RULE. *Multiply the given sum by the rate per cent., written decimally, and the product will be the commission.*

Ex. 1. What shall I pay my agent for selling $4786 worth of goods, his commission being 4 per cent.?

$$\$4786 \times .04 = \$191.44, \text{ Ans.}$$

2. A commission merchant sells farm produce to the amount of $1892; what is his commission at 2 per cent.?

3. The taxes in the town of B for 1862, are $15000; what is the cost of collecting them at ½ of 1 per cent.? Ans. $75.

4. My agent has lent for me $2124. His commission is ¼ of 1 per cent.; what shall I pay him?

5. My correspondent in Paris has bought for me 6 bales of French calico, each bale comtaining 50 pieces of 30 yds. each, at 25c. per yd.; what is his commission at ¾ per cent.?

6. My agent in New Orleans has sold for me 400 pairs of boots at $1.50, 400 pairs of shoes at 75c., and 500 pairs do. at $1; what is his commission at 3 per cent., and what shall he remit to me? 2d Ans. $1358.

269. PROBLEM 2. To find the commission or brokerage, when the agent is to take his pay from the sum remitted and invest the balance

Ex. 1. Sent my agent in London $5100, out of which he is to take a commission, and invest the balance in goods. What sum will he invest, his commission being two per cent. on the purchase, and what is his commission?

$$\$5100 \div 1.02 = \$5000, \text{ Investment.}$$
$$\$5100 - \$5000 = \$100, \text{ Commission.}$$

Since the commission is 2 per cent. on the sum expended, the agent must have $1.02 for every dollar he pays for goods; ∴ he

268. Rule for computing commission?

can invest as many dollars as $1.02 is contained times in $5100, viz. $5000, and this subtracted from the $5100 gives $100 for the commission. Hence,

RULE. 1. *Divide the given sum by* 1 *increased by the decimal expressing the rate per cent. of commission, and the quotient will be the sum to be invested.*

2. *The sum invested subtracted from the given sum will leave the commission.*

2. I intrust $10000 to my factor in New Orleans for the purchase of cotton. What sum shall he invest after deducting ½ per cent. commission for the purchase, and what are his fees?

Ans. $9950.25—, Investment; $49.75+, Commission.

3. Sent $40100 to a Boston broker for the purchase of bank stock. The brokerage is ¼ per cent. on the purchase; what does he pay for stock, and what is the brokerage?

4. Sold a quantity of merchandise for my employer for $5000. Also purchased goods for him to a certain amount, and, having calculated my commission at 5 per cent. on the sale and 3 per cent. on the purchase, our accounts balanced; what did I pay for the goods bought? What was my commission on the sale? On the purchase?

TAXES.

270. A TAX is a sum of money assessed upon the person, the property, or the income of individuals by the authorities of a town, county, state, or other section of a country, or by the national government, to defray the expenses of government, to construct public works of common utility, etc.

271. A tax on *property* is assessed at a certain per cent. on the estimated value of the property.

The tax on the *person*, called the *capitation* or *poll tax*, is assessed *equally* upon all individuals liable to pay a poll tax. A person so taxed is called a *poll*.

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269. Rule when the commission is to be taken from the sum remitted? **270.** What is a tax? By whom assessed? For what? **271.** How is the tax on property assessed? The tax upon the person, called what? What is a poll?

272. Property is of two kinds, viz. *real* and *personal estate.*

REAL ESTATE consists in *immovable* property; e.g. *lands, houses, mills,* etc.

PERSONAL ESTATE consists in *movable* property, as *money, notes, cattle, tools, bank stocks, railroad stocks, ships, etc.*

273. An INVENTORY is a list of articles of property, with their estimated value.

274. The method of assessing taxes is not the same in all its details in the different States, but the essential principles are.

In some of the States the tax bill is so made as to show the amount of tax upon the real estate and personal property *separately;* in other States no such distinction is made.

In Vermont, each taxable poll is reckoned as so much property, say $200, and no separate poll tax is calculated. This shortens the operation of making out a tax list.

In Connecticut, personal property is taxed just twice as high as real estate; thus, if A pays $30 on a farm worth $4000, then B would pay $60 on $4000 at interest.

275. In Massachusetts, the assessors are required to assess upon the polls about one sixth part of the tax to be raised, provided the poll tax of one individual for town, county, and state purposes, except highway taxes, shall not exceed $2.00 for one year. The remainder of the sum to be raised is apportioned upon the taxable property of the town, county, or state. Hence,

To Assess Taxes,

RULE. *Ascertain the number of polls liable to taxation, and take an inventory of the taxable property. Multiply the sum assessed upon one poll by the number of taxable polls, and subtract the product from the sum to be raised. Divide the remainder by the taxable property, and the quotient will be the tax upon* $1. *Multiply the taxable property of an individual by the number expressing the tax upon* $1, *to the product add his poll tax, and the sum will be his total tax.*

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272. How many kinds of property? What is Real Estate? Personal Estate? **273.** What is an Inventory? **274.** Are the details of taxation the same in all the States? What peculiarity in Vermont? In Connecticut? **275.** The rule in Massachusetts?

Ex. 1. The town of A is to be taxed \$5999. The real estate of the town is valued at \$500000 and the personal at \$300000. There are 666 taxable polls, each of which is assessed \$1.50. What is the tax of B, whose real estate is valued at \$4000 and his personal property at \$8000, and who pays 1 poll tax?

\$1.50 × 666 = \$999, sum assessed on the polls.
\$5999 — 999 = \$5000, sum to be assessed on the property.
\$500000 + 300000 = \$800000, amount of taxable property.
\$5000 ÷ 800000 = 6¼ mills, tax on \$1.
\$4000 + \$8000 = \$12000, B's taxable property.
\$12000 × .006¼ = \$75, tax on B's property.
\$75 + \$1.50 = \$76.50, B's entire tax, Ans.

NOTE. To save labor, (by using smaller numbers,) assessors frequently take 6 per cent. of the inventory instead of the entire valuation; but the labor may be lessened still more by taking 10 per cent., as in Ex. 2.

2. The town of F, whose valuation is \$356400, has 6 taxable inhabitants, A, B, C, D, E, and F, who wish to raise a tax of \$1800. The taxes of the several inhabitants are for the number of polls and the property, as in the following

INVENTORY.

Names.	Number of Polls.	Real Estate.	Personal Estate.	Total.	10 per Cent.
		\$	\$	\$	\$
A	3	24875	70405	95280	9528
B	2		38460	38460	3846
C		19462	47628	67090	6709
D	1	28424	56486	84910	8491
E	3	15860		15860	1586
F	3	19933	34867	54800	5480
Totals,	12	108554	247846	356400	35640

The tax upon each poll being \$1.50, what per cent. is levied on the property, and what is the tax of A, B, C, D, E, and F?

273. Explain Ex. 1. What is often done by assessors to save labor? What improvement is suggested? What is the object of the *Table?* Explain Ex. 2.

In calculating a tax list it is most convenient to form a *table* showing the tax upon \$1, \$2, \$3, etc. in the percentage column, and then calculate the taxes of the several inhabitants from the table; thus, in solving Ex. 2, first find the tax raised on all the polls (\$1.50 × 12 = \$18), and, having deducted this from the total tax, (\$1800 — \$18 = \$1782), divide the remainder by the assumed percentage of the taxable property in town (\$1782 ÷ 35640 = \$.05), to find the tax on \$1 in the percentage column. Then form the

TABLE.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
\$	\$	\$	\$	\$	\$	\$	\$
1	0.05	10	0.50	100	5.00	1000	50.00
2	0.10	20	1.00	200	10.00	2000	100.00
3	0.15	30	1.50	300	15.00	3000	150.00
4	0.20	40	2.00	400	20.00	4000	200.00
5	0.25	50	2.50	500	25.00	5000	250.00
6	0.30	60	3.00	600	30.00	6000	300.00
7	0.35	70	3.50	700	35.00	7000	350.00
8	0.40	80	4.00	800	40.00	8000	400.00
9	0.45	90	4.50	900	45.00	9000	450.00

Now to find A's tax from this table:

OPERATION.

Tax on	\$9000 =	\$450.
" "	500 =	25.
" "	20 =	1.
" "	8 =	.40
" "	3 polls =	4.50
	A's total tax =	\$480.90

In the same manner the tax of B, C, etc., may be found. By the above reasoning the tax is found to be 5 per cent. on the percentage column, or ½ per cent. on the entire taxable property.

CUSTOM-HOUSE BUSINESS.

276. Customs or Duties are *taxes* levied by the General Government on imported or exported goods, to support the government and to protect home industry.

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276. What are Customs or Duties?

277. All goods brought into the United States from foreign countries, must be landed at certain places called *ports of entry.*

At each port of entry *a custom-house* is established by government, with officers to compute and collect the duties.

All duties are regulated by government, and are different at different times and in different countries.

NOTE. To bring in merchandise secretly and without paying duties is called *smuggling,* and persons so engaged are liable to punishment if detected.

278. TONNAGE is a tax upon the *vessel,* without reference to its cargo, for the privilege of coming into a port of entry. The *amount* of tonnage depends upon the *size of the vessel.*

The income from duties and tonnage is the *revenue* of the government. Occasionally, when the revenue from duties and tonnage is insufficient to defray the expenses of government, *direct taxes* are levied, by authority of our national congress, upon the person, the property, and the incomes of the inhabitants.

279. Duties are either *ad valorem* or *specific.*

An AD VALOREM DUTY is a certain percentage computed on the market value of the goods in the country from which they are imported.

A SPECIFIC DUTY is a certain sum per ton, gallon, yard, etc., without regard to the cost of the article.

280. An INVOICE is a list of the articles sent to a purchaser or agent, with the prices annexed.

AD VALOREM DUTIES.

281. PROBLEM 1. To compute ad valorem duties:

RULE. *Multiply the cost of the goods by the given per cent.*

Ex. 1. What is the duty, at 40 per cent., on 25 cases of French broadcloths, invoiced at \$30000?

$$\$30000 \times .40 = \$12000.00, \text{ Ans.}$$

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277. Imported goods, where landed? A custom-house, what? Smuggling, what? **278.** Tonnage? Government revenue, how obtained? Direct taxes, when levied? **279.** How many kinds of duties? What? Ad valorem Duties, what? Specific? **280.** An Invoice? **281.** Rule for computing ad valorem duties?

2. What is the duty, at 25 per cent., on 4796 lb. of Russia iron, worth 10 c. per lb.? Ans. $119.90.

3. What is the duty, at 36 per cent., on an invoice of silks, which cost $5765 in Italy?

4. At $33\frac{1}{3}$ per cent., what is the duty on an invoice of Irish linen, amounting to $13248?

SPECIFIC DUTIES.

282. Specific duties are computed only on the actual weight or measure of merchandise; hence certain allowances are made before calculating the duties.

LEAKAGE is an allowance of a certain per cent. on liquors in casks, paying duty by the gallon.

BREAKAGE is an allowance of a certain per cent. on liquors in bottles.

DRAFT or TRET is an allowance made in the weight of goods, because of waste or refuse matter.

TARE is an allowance on account of the weight of the box, cask, bag, etc., which contains the goods.

GROSS WEIGHT is the weight of the article before any of these allowances are made.

NET WEIGHT is the weight of the merchandise after all the allowances are made. Duties are computed on *net weight.*

NOTE. The rates of draft, tare, leakage, etc., are regulated by law, and are different on different articles and at different times.

283. PROBLEM 2. To compute specific duties.

Ex. 1. What is the duty on 10 casks of molasses, containing 65 gallons each, at 5 cents per gallon, allowing 2 per cent. for leakage?

OPERATION.

$65 \times 10 = 650$, No. gal. in 10 casks.
$650 \times .02 = \underline{13}$, Allowance for leakage.
637, No. gal. net.
$637 \times .05 = 31.85$; $\therefore$ duty $= \$31.85$, Ans. Hence,

282. Specific duties, computed on what? What is Leakage? Breakage? Draft or Tret? Tare? Gross Weight? Net Weight?

RULE. *Deduct the legal draft, tare, leakage, etc., from the given quantity of merchandise; then multiply the remainder by the duty on each gallon, pound, yard, etc., and the product will be the duty.*

2. What is the duty, at 4c. per lb., on 500 bags of coffee, weighing 200 lb. each, tare 2 per cent.?

3. What is the duty, at 6c. per lb., on 300 boxes of figs, weighing 112 lb. each, allowing 1 lb. draft and 15 lb. tare on each box? Ans. $1728.

4. What is the duty, at 15c. per lb., on 48 chests of tea, each weighing 66 lb., draft being 1 lb. per box and tare 4 per cent. on the remainder?

5. What is the duty, at 5c. per lb., on 800 bags of coffee, weighing 56 lb. each, draft being 1 lb. for each 112 lb. and tare 5 per cent. on the remainder?

EXCHANGE.

284. EXCHANGE, in commerce, is a mode of paying debts due in distant places by means of *drafts* or *bills of exchange*, without the cost or risk of transporting specie.

285. A DRAFT or BILL OF EXCHANGE is a written order or request to one person to pay to another a certain sum of money, and charge the same to the account of the person who makes the request.

286. The MAKER or DRAWER of a draft or bill of exchange is the person who requests another to pay; the DRAWEE is the person who is requested to pay; and the PAYEE is the person to whom the drawee is requested to pay the money.

287. To explain the operation of exchange and show its benefits, let us suppose an example: A of Boston owes B of London $1000, and C of London owes D of Boston $1000. Now A and C can each pay his debt by sending $1000 in gold or silver and paying the cost of shipment and insurance; but

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283. Rule for computing specific duties? **284.** What is Exchange? **285.** A Draft or Bill of Exchange? **286.** The Maker or Drawer? Drawee? Payee?

exchange furnishes a better way. Thus, D of Boston writes a request (bill of exchange) to C of London that he would pay A of Boston, or his order, $1000. A buys this bill of exchange of D and pays him for it in Boston money, endorses the bill and sends it to B of London, who presents it to C, and C pays B the $1000 in London money; thus A and C have paid their debts and B and D have received their dues without the trouble, cost, or risk of sending a dollar in money or merchandise across the Atlantic; and besides, there is the same amount of money in both London and Boston as there would be if A and C had paid their respective debts by remitting gold.

288. Some bills of exchange are made payable *at sight;* i. e. as soon as they are presented to the drawee; others are made payable on a given day or in a specified time, say 30, 60, or 90 days after sight. Usually 3 days of grace (Art. 253 a) are added to the time specified in the bill, but this custom is not uniform in all places.

289. The payee, instead of receiving the money from the drawee, may sell the bill to another, and he in turn may sell it again, and so on indefinitely. Any person who buys the bill is called the BUYER or REMITTER.

The person who owns the bill at any given time is the HOLDER or POSSESSOR.

The payee and the several buyers, by writing their names across the back of the bill, become INDORSERS, and responsible to the holder for the payment of the bill at *maturity*, i. e. at the time when the bill becomes due.

290. Bills payable in a given time after sight are presented to the drawee, and if he agrees to pay, he writes the word "Accepted" and his name across the face or on some other part of the bill, and returns it to the holder. The drawee is then the ACCEPTER, and responsible for the payment of the bill when due.

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287. Explain the operations of Exchange. 288. When are some bills payable? Others? 289. What may the payee do with a bill? What is the buyer called? The owner? How does the seller of a bill become responsible for the payment of it? What is the maturity of a bill? 290. What is it to accept a bill?

291. If the drawee declines to pay or accept the bill, the holder employs an officer called a *Public Notary* to give notice of the refusal to the drawer and each indorser. This notice is called a PROTEST.

292. A bill should be presented for payment during the regular business hours of the day on which it matures, and, if the accepter fails to make payment, the holder should *protest it for non-payment* by giving the proper notice to the drawer and the several indorsers. If this notice is not given in due time the indorsers cease to be holden for the payment.

293. The United States annually export to and import from Europe, goods to the value of hundreds of millions of dollars. Sometimes the exports exceed the imports, and sometimes the reverse. When our exports to a given country, England, e. g., exceeed our imports from England, the balance of trade is in our favor; England owes us more than we owe England, and hence more merchants here wish to sell bills drawn on England, for the purpose of collecting their dues in England, than wish to buy for the purpose of paying their debts there, and consequently, the supply being greater than the demand, bills on England will sell at a discount. When the balance of trade is in favor of England, our indebtedness is greater than that of England, and bills on England will sell at a premium. This change in the price of bills is called the COURSE OF EXCHANGE. The variation in the price of bills can never be very great, for merchants will not pay more for premium than the cost of freight and insurance to transport specie.

294. Bills of exchange, payable after sight, like promissory notes, are subject to a discount for the term of credit, the discount being computed on the face of the bill.

295. In the United States the exchange value of the pound

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291. For what is a bill *protested?* By whom? How? **292.** When should a bill be presented for payment? What is necessary to hold the endorsers? **293.** When is the balance of trade in our favor? When against us? How does this affect the price of bills of exchange? What is the Course of Exchange? Why cannot the variation be great? **294.** Are time bills subject to discount? **295.** What is the exchange value of the £? What the commercial value?

sterling is 4.44\frac{4}{9}$ and bills of exchange are drawn upon this basis, but the intrinsic and commercial value is about 9 per cent. more than the exchange value; thus,

Exchange value of 1£ = 4.44\frac{4}{9}$
Add 9 per cent. = .40
Average commercial value of 1£ = 4.84\frac{4}{9}$;

∴ when exchange on England sells at a premium of 9 per cent. it is at true or commercial par.

296. Problem 1. To find the cost of a draft or bill of exchange.

Ex. 1. $1000. *Boston, June* 4, 1862.

At sight, pay John Jones, or order, one thousand dollars, value received, and charge the same to my account. A. Tyler.

To Messrs. Smith & Dana,
Merchants, Chicago.

What is the cost of the above draft at 2 per cent. discount?

$1000 × .98 = $980, Ans. Since exchange is at 2 per cent. discount, each dollar costs 98 cents, i.e. the bill costs .98 (98 hundredths) of its face.

2. $320. *Pittsburg, Aug.* 6, 1862.

Sixty days after sight, pay to S. Day, or bearer, three hundred and twenty dollars, value received, and charge the same to the account of T. Fox & Co.

To Alfred Stearns,
New York.

What is the cost of this draft at 3 per cent. premium?

OPERATION.

$320
9.60 = premium on $320 at 3 per cent.
329.60
3.36 = discount on $320 for 60 days and grace.
$326.24 = cost of draft, Ans.

3. What is the cost of a draft on St. Louis for $8325, at 2 per cent. discount?

4. What is the cost of a draft on New York for $7850, at 1 per cent. premium?

NOTE 1. An order payable in the same country where it is drawn, is called a *draft* or an *inland bill of exchange.* An order drawn in one country and payable in another, is called a *foreign bill of exchange.* In making foreign bills it is customary to draw a set of two or more bills of the same tenor and date, each containing a clause, in parenthesis, which renders all the bills in the set worthless except the one first presented to the drawee.

These bills are sent in different vessels so that, if one or more of the set is delayed or lost on the passage, there may be no unnecessary delay in obtaining the money.

5. 2000£. *Boston, May* 12, 1862.

At sight of this first of exchange (second and third unpaid), pay to the order of John Flint, in London, two thousand pounds sterling, value received, and charge the same to my account.

DAVID FAY.

To George Peabody & Co.,
Bankers, London.

What is the cost of this bill in United States money, at $9\frac{1}{2}$ per cent. premium?

OPERATION.

$\$4.44\frac{4}{9} \times 2000 = \$8888.88\frac{8}{9} = 2000£.$

$844.44\frac{4}{9} =$ premium at $9\frac{1}{2}$ per cent.

$\$9733.33\frac{1}{3} =$ cost of bill, Ans. Hence,

RULE. *First, if necessary, find the value of the bill, at par, in United States money; then increase or diminish this value as the rate of exchange and the term of credit may require.*

6. Stuart, Field, & Co., of New York, bought of J. & P. Smith, a set of exchange, payable at sight for 800£, on Bates, Baring, & Co., London, at $8\frac{3}{4}$ per cent. premium. What was the cost in U. S. money? Ans. $\$3866.66\frac{2}{3}$.

NOTE 2. An English coin worth 1£ is called a sovereign.

7. I wish to pay a debt of 1200£ in Liverpool. Which can I best afford, to buy sovereigns at \$4.85 and pay 2 per cent. for freight and insurance, or buy a set of exchange at $9\frac{1}{4}$ per cent. premium? Ans. I save $\$109.73\frac{1}{3}$ by buying the bills.

296. Rule for finding the cost of a bill? What is an inland bill? A foreign bill? A sovereign?

297. PROBLEM 2. To find the face of a bill which a given sum in United States money will buy.

Ex. 1. When exchange is at $9\frac{3}{4}$ per cent. premium, what is the face of a bill on London which I can buy for \$4390?

1 £ = \$ $4.44\frac{4}{9}$; \$ $4.44\frac{4}{9} + 9\frac{3}{4}$ per cent. = \$ $4.87\frac{7}{9}$, cost of 1£;
\$ 4390 ÷ \$ $4.87\frac{7}{9}$ = 900, No. pounds in face of bill, Ans.

2. My agent in Chicago, bought a draft on New York, at 2 per cent. premium, for \$8160; what was the face of the draft?

\$ 1 + 2 per cent. = \$ 1.02, cost of \$ 1.
\$ 8160 ÷ 1.02 = \$ 8000, Ans. Hence,

RULE. *Divide the cost of the bill by the cost of a bill for* \$1, 1£, *etc., and the quotient will be the face of the bill in dollars, pounds, etc.*

3. A Boston merchant bought a draft on Chicago, at 3 per cent. discount, for \$5820; what was the face of the draft?

Ans. \$6000.

4. Bought a set of exchange on London, at $9\frac{1}{2}$ per cent. premium, for \$4168.30; what debt in London may be paid by this sum? Ans. 856.5£ = 856£ 10s.

EQUATION OF PAYMENTS.

298. EQUATION OF PAYMENTS is the method of determining when several debts due from one person to another, payable at *different* times, may be paid at *one* time, so that neither party may suffer loss. The *equated time* is the *date* of payment.

The time to elapse before a debt becomes due is called the *term of credit*. The *average term of credit* is the time to elapse before the *equated time.*

299. PROBLEM 1. To find the equated time when all the terms of credit begin at the same date.

Ex. 1. On the 1st of Jan. A owes B \$2, payable in 4 months

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297. Rule for finding the face of a bill? **298.** What is Equation of Payments? What the equated time? Term of credit? Average term of credit?

and \$6, payable in 8 months; what is the average term of credit and the equated time of payment?

FIRST METHOD.

$$\begin{array}{r} 4 \times 2 = 8 \\ 8 \times \underline{6} = \underline{48} \\ 8) \;\; \underline{56} \\ 7\text{m., 1st Ans.} \end{array}$$

Jan. 1 + 7m. = Aug. 1, 2d Ans.

The privilege of keeping \$2 for 4m. is the same as the privilege of keeping \$1 for 8m.; so \$6 for 8m. is the same as \$1 for 48m.; ∴, for the two debts, A might keep \$1 for 56m., but as he has \$8 to keep, he may retain it only $\frac{1}{8}$ of 56m., viz. 7m., and 7m. from Jan 1, extend to Aug. 1, the equated time. Hence,

RULE 1. *Multiply each debt by the number expressing the time to elapse before it becomes due, then divide the sum of the products by the sum of the debts, and the quotient is the average term of credit. Add the average term of credit to the date of the debts, and the result is the equated time.*

REMARK. *Express each time in months, or else each in days.*

SECOND METHOD.

The interest of	\$2 for 4m. =	4 c.
" "	\$6 " 8m. =	24 c.
Sum of debts =	\$8	28 c. = total interest.

Now the question is, in what time will the interest on the *sum of the debts* be the same as the *sum of the interests on the several debts?* This may be found by dividing the total interest by the interest on the sum of the debts for 1 month; thus, interest of \$8 for 1m. = 4c., and 28c. ÷ 4c. = 7, number of months in the average term of credit, as by the 1st method. Hence,

RULE 2. *Find the interest on each debt for its term of credit, then divide the sum of these interests by the interest on the sum of the debts for one month, and the quotient will be the average term of credit in months.*

Find the equated time as in Rule 1.

NOTE 1. To find the interest of the sum of the debts for a month, it is

---

299. Rule for finding average term of credit? Equated time? Second method? Explain Ex. 1 by each method. Second Rule? What is Note 1?

only necessary to move the decimal point two places to the left and divide by 2 (Art. 241), for the interest of $1 is just half a cent a month.

NOTE 2. It is the custom of business men to consider 30 days a month; also, in computing interest, to neglect the cents in the principal if they are less than 50, and to add 1 to the number of dollars in the principal if the cents are 50 or more.

So the fraction of a day, in equating accounts, is neglected if less than ½, and it is counted as 1 if it is ½ or more.

NOTE 3. Each method above given is much used by accountants in averaging accounts, but the second is thought to be the shorter and better method. The second only is given in the following problems, but the pupil will practice upon either or both, as his teacher may direct.

2. July 6, 1861, I owe to John Smith $4550, payable in 4m., $5075 in 8m., and $3500 in 12m.; what is the average term of credit and the equated time?

1st Ans. Average term, 7.68 m. = 7m. 20d.
2d Ans. Equated time, Feb. 26, 1862.

NOTE 4. The decimal of a month may be reduced to days by multiplying by 30 (Art. 176), or more conveniently by taking 3 days for each *tenth* and 1 day for each 3⅓ *hundredths* in the decimal.

3. $1500, $2100 and $2400 are due in 4, 8, and 12 months, respectively; what is the average term of credit?

300. PROBLEM 2. To find the equated time when all the terms of credit are of equal length, but begin at different times.

In solving examples where the *terms of credit are equal*, it is only necessary to find the average *date* of the debts, and then to this date *add the term of credit.*

In finding the *average date*, interest may be computed from the date of the first bill, or *from any other date;* but it is *most convenient* to compute the interest from the *first of the month in which the first bill is bought*, because the *time* for which interest is to be computed on the several bills is thereby most easily determined, as will be seen by the following examples.

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299. What is Note 2? Note 3? Note 4? **300.** In finding the average *date* of debts, interest may be reckoned from what time? Most convenient time? Why?

The date from which interest on the several bills is computed, is called the *Focal Date*, or *Date of Reference*.

Ex. 1. Required the equated time of paying the following bills of goods, each bought on a credit of 6 months.

0m.	Mar. 12,	\$300	\$0.60, Int. for 12d.
0m.	" 18,	200	.60, " " 18d.
1m.	Apr. 6,	600	3.60, " " 1m. 6d.
4m.	July 24,	100	2.40, " " 4m. 24d.
	2) 100)	\$1200	\$7.20, total interest.

Int. on *sum* of bills for 1m., \$6

$7.20 \div 6.00 = 1.2\text{m.} = 1\text{m. } 6\text{d.}$

This gives the *average date of purchase* 1 month and 6 days from Mar. 1, viz. Apr. 6. To this add the term of credit, 6m., and we have Oct. 6 for the *equated time of payment*, Ans.

NOTE. Since the time for which interest is computed *includes* both the 1st day of the month and the day of purchase, so 1m. and 6d. from Mar. 1 is considered as ending on the 6th of Apr. and not on the 7th. The same principle holds in the following examples.

EXPLANATION. The *time* for interest on the *first* bill is 0 months and 12 days, *the number of days being determined by the* DATE OF THE BILL. So the time of the *second* bill is 0m. 18d.; of the *third*, 1m. 6d.; and of the *fourth*, 4m. 24d. The number of *months* may be obtained by *counting* from the focal date (e. g. for the *fourth* bill above, April, May, June, July, i. e. 1, 2, 3, 4) and, for convenient use, the number of months is set at the *left of the date* of the bills, severally.

The *interest* of each bill is computed for its own time and written at the *right*. The *aggregate* or *total* interest on the bills (in this example, \$7.20) is then divided by the interest of the sum of the bills for 1 month (\$6), as in Ex. 1, Art. 299, 2d method, to obtain the average date of purchase. Hence,

RULE. *Find the interest on each bill from the first of the month in which the first bill was bought to the time of the purchase of the bills, severally; divide the sum of these interests by*

---

300. Focal date, what is it? What is the Note? Explain Ex. 1. Number of months, how found? Where set? Rule for finding average date? Equated time?

the interest on the sum of the bills for one month, and the quotient will be the number of months from the focal date to the average date of purchase. To this average date of purchase add the term of credit, and the equated time of payment is found.

2. Required the equated time of paying the following bills, each bought on 8 months' credit?

0m.	June 9, 1862,	$180	$0.27,	Int. for 9d.
1m.	July 15, "	84	.63	" " 1m. 15d.
3m.	Sept. 14, "	240	4.16	" " 3m. 14d.
4m.	Oct. 10, "	96	2.08	" " 4m. 10d.
	2)100)	$600	$7.14,	total interest.

3) 7.14

2.38m. = 2m. 11d.

∴ Average date of purchase, Aug. 11, 1862.

Equated time of payment, Apr. 11, 1863, Ans.

3. Bought the following bills on 6 months' credit:

May 12,	1862,	$400
June 4,	"	150
Aug. 6,	"	80
Nov. 24,	"	170

What is the average date of purchase and equated time of payment?

1st Ans. July 5, 1862;
2d Ans. Jan. 5, 1863.

4. Bought the following bills on 4 months:

Feb. 17,	1862,	$1200
Mar. 25,	"	472
" 30,	"	468
July 21,	"	500

What is the average date of purchase and equated time of payment?

1st Ans. Apr. 1, 1862;
2d Ans. Aug. 1, 1862.

5. Bought the following bills on 6 months:

Jan. 8,	$12
" 24,	20
Apr. 18,	1200
June 6,	4000

What is the average date of purchase and the equated time? 1st Ans. May 24;
2d Ans. Nov. 24.

6. Bought the following bills on 6 months:

Jan. 8,	$4000
" 24,	1200
Apr. 18,	20
June 6,	12

What is the average date of purchase and the equated time? 1st Ans. Jan. 12;
2d Ans. July 12.

REMARK. The two foregoing examples, consisting of the same bills, with the order of purchase reversed, show very clearly that the average date of purchase (and consequently the equated time of payment) is greatly changed by buying the smaller bills at the earlier or at the later dates.

301. PROBLEM 3. To find the equated time when the terms of credit are unequal and begin at different times.

The *maturity* of a note or bill is the *time when it becomes due.*

The process for finding the *equated time of payment* in this Problem is the same as for finding the *average date of purchase* in Problem 2, except that the interest is computed to the *maturity* of the bills severally, rather than to the *time of purchase.* Hence no new rule is needed.

Ex. 1. Required the equated time of paying the following bills of goods?

1862.	Cr.	Bills.	Int.	
0m. Feb. 12,	4m.	$200	$ 4.40	for 4m. 12d.
2m. Apr. 15,	6m.	400	17.00	" 8m. 15d.
4m. June 8,	2m.	300	9.40	" 6m. 8d.
		2)100)$900	$30.80,	total interest.

Int. on sum of bills for 1m. = $4.50

30.80 ÷ 4.50 = 6.84m. = 6m. 25d., the time from Feb. 1 to the *average date of maturity*, i. e. to the *equated time.* Now 6m. 25d. from Feb. 1, 1862, gives Aug. 25, 1862, Ans.

EXPLANATION. The maturity of the 1st bill is 4 months and 12 days from Feb. 1; the maturity of the 2d bill (found by adding its term of credit, 6m., to the 2m. 15d. from the focal date, Feb. 1, to the time of purchase, Apr. 15) is 8m. 15d.; the maturity of the 3d bill, found in like manner, is 6m. 8d.

2. Required the average maturity of the following bills?

Jan. 18,	8m.	$2000
Feb. 21,	6m.	3000
June 6,	2m.	600

300. What is the Remark? 301. What is the maturity of a note or bill? How does Problem 3 differ from Problem 2? Explain Ex. 1.

302. PROBLEM 4. To find the equated time for paying the *balance* of an account which has both *debit* and *credit* entries.

Ex. 1. From the accounts of A and B it appears that

A owes B	And that B owes A
$254, due July 18,	$500, due Aug. 15,
475, " Sept. 6,	288, " " 30,
425, " " 18,	612, " Oct. 3,
46, " Oct. 9,	400, " " 21,

When shall B pay the balance of $600?

OPERATION.

		A's Debts.	*Int.*	
0m.	July 18,	$254	$0.762	for 18d.
2m.	Sept. 6,	475	5.225	" 2m. 6d.
2m.	" 18,	425	5.525	" 2m. 18d.
3m.	Oct. 9,	46	.759	" 3m. 9d.

Sum of A's debts = $1200 $12.271, Total interest on A's debts from the *focal date*, July 1, to *maturity*, i. e. the interest that B would gain if A paid the sum of his debts, $1200, on the 1st of July.

		B's Debts.	*Int.*	
1m.	Aug. 15,	$500	$3.75	for 1m. 15d.
1m.	" 30,	288	2.88	" 2m.
3m.	Oct. 3,	612	9.486	" 3m. 3d.
3m.	" 21,	400	7.40	" 3m. 21d.

Sum of B's debts = $1800 $23.516, Total interest on B's debts from the *focal date*, July 1, to *maturity*, i. e. the interest A would gain if B paid the sum of his debts, July 1.

From the above it appears that if each party paid his debts July 1, A would gain $23.516, and B would gain $12.271; ∴ A's net gain and B's net loss would be $23.516 — $12.271 = $11.245. Now as it is proposed to settle by B's paying the *balance* of the account, viz. $600, it is plain he may keep the $600 after July 1, until its interest shall equal $11.245, the loss he would sustain by paying July 1. The interest of $600 for 1m. is $3, and $11.245 ÷ $3 = 3.748, the time in months. Now 3.748m. = 3m. 22d.; ∴ the time of payment is 3m. 22d. after July 1, viz. Oct. 22, Ans.

2. The accounts of A and B show that

A owes B			And that B owes A		
$624,	due	Jan. 12,	$346,	due	Feb. 9,
896,	"	Mar. 6,	960,	"	Apr. 9,
734,	"	May 12,	454,	"	July 18,
146,	"	June 3,	240,	"	Aug. 18,

When shall A pay the balance of $400?

OPERATION.

A's Debts.		*Int.*	*B's Debts.*		*Int.*
0m. Jan. 12,	$624,	$1.248	1m. Feb. 9,	$346,	$2.249
2m. Mar. 6,	896,	9.856	3m. Apr. 9,	960,	15.84
4m. May 12,	734,	16.148	6m. July 18,	454,	14.982
5m. June 3,	146,	3.723	7m. Aug. 18,	240,	9.12
	$2400,	$30.975		$2000,	$42.191

$2400 | $42.191
2000 | 30.975

2)100)$400, Bal. of account. $11.216, Bal. of interest.

Int. for 1m. $2.00)$11.216

Time in m. = 5.608 = 5m. 18d., which, reckoned *back* from Jan. 1, gives July 13 of the preceding year for the time of settlement, Ans.

EXPLANATION. By a process like that in Ex. 1, it is shown that if A and B each paid his debts, i. e. if A paid the *balance* of $400, at the focal date, Jan. 1, A would gain and B would lose $42.191 — $30.975 = $11.216; ∴, evidently, A should pay the $400 long enough *before* Jan. 1, so that its interest shall equal $11.216, the gain he would have by paying Jan. 1. This time is found to be 5m. 18d., which, reckoned *back* from Jan. 1, gives July 13 of the preceding year for the equated time of settlement. Hence,

To equate accounts,

RULE. *Compute the interest of each item of the account from the focal date to its maturity; find the sum of the interests on the debit items, also the sum on the credit items, and subtract the less sum from the greater; divide this difference by the interest*

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303. Explain Ex. 1. Explain Ex. 2. Rule for equating accounts which have both debit and credit items?

on the BALANCE OF THE ACCOUNT *for one month, and the quotient will be the time in months between the focal date and the equated time of settlement, the time to be reckoned* FORWARD *when the greater interest arises on the greater side of the account, and* BACKWARD, *excluding the focal date, when the greater interest arises on the smaller side.*

NOTE 1. When the larger interest arises on the smaller side of the account, as in Ex. 2, the rule may require the settlement to be made before some of the transactions have occurred, *a result which is obviously impracticable,* and *usually* some other time of settlement is *more convenient* than the equated time. If the settlement is made before the equated time, a discount should be made; if after, the interest should be added.

Ex. 3. When ought A to pay the balance of the following account, and for what sum may he settle June 6, 1863?

Dr. *A in account with B.* *Cr.*

1862.		$	1862.		$
April 24	To Mdse., 6m.	356	Feb. 6	By Mdse., 4m.	530
June 18	" Mdse., 4m.	875	May 27	" Mdse., 6m.	652
July 3	" Mdse., 6m.	433	July 15	" Cash,	300

1st Ans. June 6, 1864; 2d Ans. $171.08. (See Art. 253b.).

NOTE 2. In Ex. 3, Feb. 1 is the most convenient focal date, the earliest entry being made Feb. 6. The meaning of the account is, that A has, at three different times, bought merchandise of B to the amount of $356, $875, and $433, severally, the 1st and 3d bills on a credit of 6m., and the 2d on 4m.; also, that on the 6th of Feb. A sold B merchandise worth $530 on a credit of 4m., on the 27th of May merchandise worth $652 on 6m., and on the 15th of July he paid B $300 in cash.

4. Required the equated time of settling the following account, and the sum due Oct 4, 1862?

Dr. *A in account with B.* *Cr.*

1862.		$	1862.		$
Mar. 14	To Mdse., 4m.	452	April 13	By Cash,	500
May 8	" Cash,	1224	May 21	" Note, 4m.	1000
" 20	" Mdse., 8m.	150	Aug. 18	" Cash,	192
" 27	" Mdse., 6m.	2496	Sept. 11	" Cash,	5420
June 19	" Mdse., 3m.	5724			
July 30	" Mdse., 6m.	88			

1st Ans. Nov. 4, 1862; 2d Ans. $3006.89.

NOTE 3. Not unfrequently a business man, in full or partial payment of a debt, gives his note, payable in a given time without interest. The holder of the note may indorse it and get it discounted (See Art. 253 b.), thus obtaining money for his own use before the note matures; or he may pass it to his creditor in payment of his own debts. Such a note may be entered in an account, as in Ex. 4, and treated in the same way as merchandise bought or sold on credit.

5. When was the equated time of settling the following account, and what was due Nov. 13, 1862?

Dr. *A in account with B.* *Cr.*

1861.		$	1861.		$
Nov. 18	To Mdse., 4m.	800	Sept. 27	By Mdse.,	1200
1862.			Dec. 12	" Mdse., 4m.	800
April 6	" Mdse., 2m.	350	1862.		
" 30	" Cash,	125	May 15	" Mdse., 4m.	850
May 15	" Note, 4m.	1200	July 18	" Mdse.,	625
Oct. 12	" Mdse., 2m.	200			

1st Ans. Apr. 25, 1861; 2d Ans. $874.40.

6. When is the equated time of settling the following account, each item being due at date, and what shall A pay on the 27th of July, 1862?

Dr. *A in account with B.* *Cr.*

1861.		$	Int.	1861.		$	Int.
0m. June 20	To Mdse.,	986	3.287	1m. July 4	By Mdse.,	158	0.895
5m. Nov. 16	" Mdse.,	152	4.205	6m. Dec. 18	" Note,	228	7.524
1862.				1862.			
8m. Feb. 26	" Mdse.,	110	4.877	9m. Mar. 5	" Mdse.,	450	20.625
		1248	12.369			836	29.044

$$\begin{array}{lr} & \$1248 \\ & 836 \\ \hline 2\,)\,100\,) & 412, \text{ Balance of acct.} \end{array} \qquad \begin{array}{r} \$29.044 \\ 12.369 \\ \hline 16.675, \text{ Balance of int.} \end{array}$$

2.06) 16.675 (8.09 m. = 8m. 3d.

June 1, 1861 — 8m. 3d. = Sept. 27, 1860, 1st Ans.

$412 + $45.32 (Int. for 1yr. 10m.) = $457.32, 2d Ans.

7. What would be the equated time of settlement in Ex. 6, if each item were on a credit of 6 months?

302. What is Note 1? Note 2? Note 3?

PROOF. *Some of the debts are due before the equated time, and some after. The sum of the interests on the former, from their several maturities to the equated time, will be equal to the sum of the interests on the latter from the equated time to their several maturities. When the account has both debit and credit items, equate each side of the account, and the interest on the two sides for the time between the respective average dates, and the equated time will be the same, or nearly the same* (Art. 299, Note 2).

PROFIT AND LOSS.

303. "PROFIT AND LOSS," as a commercial term, signifies the gain *or* loss in business transactions. The rule may refer to the *absolute* gain or loss, or to the *percentage* of gain or loss, on the *purchase price* of the property considered.

304. PROBLEM 1. To find the absolute gain or loss on a quantity of goods sold at retail, the purchase price of the whole quantity being given:

RULE. *Find the whole sum received for the goods, and the difference between this and the purchase price will be the gain or loss.*

Ex. 1. Bought 16 bbl. of flour for $100 and sold it at $7 per bbl.; did I gain or lose? How much, total and per bbl.?

2. Bought 24 bbl. of flour for $168 and sold ⅓ of it at $6.75 and the remainder at $7.50 per bbl.; did I gain or loss? How much? Ans. Gained $6.

3. Bought 3cwt. 2qr. 18 lb. of sugar for $36.80 and sold it at 8¼c. per lb.; did I gain or lose? How much, total and per lb.?

4. Bought 164yd. of broadcloth and 287yd. of cassimere for $1107; sold the broadcloth at $3 and the cassimere at $2.25 per yd.; did I gain or lose? How much?

305. PROBLEM 2. To find the per cent. of gain or loss when the cost and selling price are given:

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302. Proof of rule for equation of payments? **303.** What is Profit and Loss? To what may it refer? **304.** Rule for finding absolute gain or loss?

Ex. 1. Bought 4 bbl. of flour for \$32 and sold it at \$9.50 per bbl.; did I gain or lose? How much per cent.?

$$
\begin{array}{rl}
\$\,9.50, & \text{selling price.} \\
4 & \\
\hline
\$\,38.00, & \text{whole sum rec'd.} \\
\$\,32. & \text{cost.} \\
\hline
\$\,6, & \text{whole gain.}
\end{array}
$$

$\frac{6}{32} = \frac{3}{16} = .18\frac{3}{4}$, Ans.

The gain, \$6, is $\frac{6}{32} = \frac{3}{16}$ of the whole cost, and $\frac{3}{16}$ reduced to a decimal (Art. 173), gives $.18\frac{3}{4}$; i. e. the gain is $18\frac{3}{4}$ per cent of the cost. Hence,

Rule. *Having found the total gain or loss by Problem* 1, *make a common fraction by writing the gain or loss for the numerator and the cost of the article for the denominator, and then reduce this fraction to a decimal.*

2. Bought 50 lb. of wool for \$20 and sold it at 34c. per lb.; did I gain or lose? How much per cent.?

Ans. Lost 15 per cent.

3. Bought a case of boots at \$4 per pair and sold them at \$5; what per cent. was gained?

4. Bought boots at \$5 per pair and sold them at \$4; what per cent. was lost?

5. Bought goods for \$2000, and, in one year, sold the same for \$2155, out of which paid \$95 for storage, etc.; how much per cent. on the first cost was lost?

306. Problem 3. To find the selling price, the cost and gain or loss per cent. being given.

Ex. 1. Bought goods for \$400; how must the same be sold so as to gain 25 per cent.

$$
\begin{array}{rl}
\$\,400 & \\
.25 & \\
\hline
2000 & \\
800 & \\
\hline
\$\,100.00 & = \text{gain.} \\
\$\,400. & \\
\hline
\$\,500. & \text{Ans.}
\end{array}
$$

This is the same as finding the *amount* of a sum of money on interest for 1 year at 25 per cent. (Art. 237).

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305. Rule for finding the per cent. of gain or loss?

2. Bought a horse for \$150, but it being injured, I am willing to lose 6 per cent.; for what shall I sell him?

$$\begin{array}{r} \$150 \\ .06 \\ \hline \$9.00 = \text{loss.} \end{array}$$

$\$150 - \$9 = \$141$, Ans.

This is the same as finding the present worth of a sum due a year hence, discounting interest (Art. 253b.). Hence,

RULE. *Multiply the purchase price by the per cent. to be gained or lost, written decimally, and add the product to, or subtract it from, the purchase price.*

3. Bought a farm for \$4848; for what shall I sell the same to gain 5 per cent.? Ans. \$5090.40.

4. Bought 3cwt. of sugar at 12c.; how shall the same be sold per lb. so as to gain 10 per cent.?

5. Bought a house for \$3500, expended \$750 in repairing it, and then sold it so as to lose 15 per cent. on the whole cost; what did I receive for it?

307. PROBLEM 4. To find the first cost of an article, the selling price and gain or loss per cent. being given.

Ex. 1. Sold wheat at \$1.50 per bushel, and thereby gained 25 per cent. on the cost; what was the purchase price?

$\frac{100}{125} = \frac{4}{5}$

$\frac{4}{5}$ of $\$1.50 = \$1.20$, Ans.

That which cost 100c. was sold for 125c., ∴ the cost was $\frac{100}{125} = \frac{4}{5}$ of the selling price; hence the cost was $\frac{4}{5}$ of $\$1.50 = \$1.20$.

2. Sold apples at \$1.80 per barrel, and thereby lost 10 per cent. on the cost; what was the cost?

$\frac{100}{90} = \frac{10}{9}$

$\frac{10}{9}$ of $\$1.80 = \$2$, Ans.

The cost was $\frac{100}{90} = \frac{10}{9}$ of the selling price, ∴ the cost was $\frac{10}{9}$ of $\$1.80 = \$2$. Hence,

RULE. *Make a fraction by writing* 100 *for a numerator, and* 100+ *the gain per cent., or* 100— *the loss per cent., for a denominator; then multiply the selling price by this fraction.*

---

306. Rule for finding the selling price, the cost and gain or loss per cent. being given? **307.** Rule for finding the first cost, the selling price and gain or loss per cent. being given?

3. Sold 6 yards of cloth for \$26.88, and gained 12 per cent. on the cost; what was the purchase price per yard? Ans. \$4.

4. Sold 10 shares of the Fitchburg R. R. Stock for \$1090, gaining 9 per cent. on the cost; what did I pay per share?

5. By selling 25 lb. of sugar for \$2, I lose 20 per cent. on the cost; what was the cost per lb.?

308. PROBLEM 5. The selling price of goods, and the gain or loss per cent. being given, to find what would be gained or lost per cent. if sold at some other price.

Ex. 1. Sold a pair of oxen for \$175 and gained 5 per cent.; what per cent. should I have gained if I had sold them for \$200?

$\frac{200}{175} = \frac{8}{7}$
$\frac{8}{7}$ of $105 = 120$
$120 - 100 = 20$, Ans.

The proposed price is $\frac{200}{175} = \frac{8}{7}$ of the actual selling price, but the actual selling price is 105 per cent. of the cost, ∴ the proposed price is $\frac{8}{7}$ of 105 per cent. $= 120$ per cent. of the cost; hence 120 per cent. — 100 per cent. $= 20$ per cent. would be the gain per cent. if the oxen were sold for \$200.

2. Sold a farm for \$5000, and thereby made 25 per cent.; should I have gained or lost, and how much per cent., if I had sold it for \$3500?

$\frac{3500}{5000} = \frac{35}{50} = \frac{7}{10}$; $\frac{7}{10}$ of $125 = 87\frac{1}{2}$; $100 - 87\frac{1}{2} = 12\frac{1}{2} =$ loss per cent., Ans.

The proposed price is found to be $87\frac{1}{2}$ per cent. of the cost, ∴ there would be a *loss* of $12\frac{1}{2}$ per cent. if the farm were sold for \$3500.

RULE. *Make a fraction by writing the proposed piece for the numerator, and the actual price for the denominator, then multiply the per cent. at which the article is sold by this fraction, and the product will be the per cent. at the proposed price. The difference between the product and* 100 *is the gain or loss per cent. at the proposed price.*

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308. Rule for finding loss or gain per cent. when goods are sold at a proposed price?

3. Sold flour at \$7 per bbl. and thereby gained 12 per cent.; what per cent. should I have gained if I had sold it at \$7.25?

Ans. 16 per cent.

4. Sold beef at \$6 per cwt., and thereby lost 4 per cent.; should I have gained or lost, and how much per cent., had I sold it at \$6.50?

5. Sold a watch for \$21, and gained 5 per cent. on the cost; had I sold it for \$18 should I have gained or lost, and how much per cent.?

309. PROBLEM 6. To mark goods so that the merchant may fall a certain per cent. on the marked price and yet sell the goods at cost, or at a certain per cent. above or below cost.

(a) To sell at cost.

Ex. 1. How shall I mark a coat that cost me \$18 so that I may fall 10 per cent. from the marked price and yet sell the coat at cost? $\frac{100}{90} = \frac{10}{9}$; $\frac{10}{9}$ of \$18 = \$20, Ans.

Since I am to fall 10 per cent., it follows that the cost, \$18, is only $\frac{90}{100} = \frac{9}{10}$ of the marked price, and if \$18 is $\frac{9}{10}$ then $\frac{1}{10}$ will be $\frac{1}{9}$ of \$18 = \$2, and $\frac{10}{10}$ will be 10 times \$2 = \$20; i. e. the marked price will be $\frac{10}{9}$ of \$18 = \$20, Ans.

PROOF. 10 per cent. of \$20 = \$2, which taken from \$20 leaves \$18, the cost. Hence,

RULE. *Make a fraction by writing* 100 *for the numerator, and* 100 *diminished by the per cent. to be abated for the denominator; multiply the cost by this fraction, and the product will be the marked price.*

2. Bought a case of watches at \$23.50; at what price shall I mark them to enable me to abate 6 per cent., and yet sell them at cost? Ans. \$25.

(b) To sell at a certain per cent. above or below cost:

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309. Rule for *marking* goods so as to fall a certain per cent. and yet sell at cost? To sell at a given per cent. above or below cost?

RULE. *First find the selling price by Problem 3; then find the marking price by Problem 6, (a.)*

3. Bought a piece of broadcloth at \$5 per yard, but it being damaged, I am willing to lose 20 per cent. on the cost; how shall I mark it so that I may fall 25 per cent. from the marked price?

\$5 = cost.	\$5 — \$1 = \$4, selling price.
.20	
\$1.00 = loss.	$\frac{100}{75} \times \$4 = \$5.33\frac{1}{3}$, marked price.

4. Paid \$4 a pair for a case of boots; how shall I mark the same so that I may fall 10 per cent. from the marked price and yet make $12\frac{1}{2}$ per cent. on the cost?

5. Paid \$8 each for a case of bonnets; how shall I mark the same so that I may fall 16 per cent. from the marked price and yet make 5 per cent. on the cost?

MISCELLANEOUS EXAMPLES IN PROFIT AND LOSS.

1. Bought 75 pounds of tea for \$37.50 and sold $\frac{1}{2}$ of it at 48 cents per pound and the remainder at 56 cents; did I gain or lose? How much?

2. What per cent. do I gain if I buy boots at \$3 per pair and sell them at $\$3.37\frac{1}{2}$?

3. Sold flour at \$7.50 per barrel and lost $6\frac{1}{4}$ per cent. on the cost; for what should it be sold to gain $12\frac{1}{2}$ per cent.?

4. Paid \$3 per yard for a piece of lace; how shall I mark the same to enable me to fall 10 per cent. from the marked price and yet gain 20 per cent. on the cost?

5. Bought hats at \$3 per hat and sold them at \$2.50; what per cent. on the cost was lost?

6. Sold a watch for \$42 and lost $12\frac{1}{2}$ per cent. on the cost; what was the cost?

7. Sold cloth at \$2 per yard and lost 10 per cent.; should I have gained or lost, and how much per cent., if I had received $\$2.12\frac{1}{2}$?

8. Bought a horse for \$87.50 and sold him so as to gain 12 per cent.; what did I receive for him?

PARTNERSHIP.

310. PARTNERSHIP is the association of two or more persons in business.

The company thus formed is called a *firm* or *house.*

The money or other property invested is called the *capital* or *stock* of the company.

The profits and losses of the firm are divided among the partners in accordance with their interest in the business.

311. PROBLEM 1. To find each partner's share of gain or loss when their capital is employed equal times.

Ex. 1 A and B trade in company; A furnishes \$400 and B \$800. They gain \$300; how shall they share the gain?

A furnishes $\frac{400}{1200} = \frac{1}{3}$ of the stock, ∴ he is entitled to $\frac{1}{3}$ of the gain, viz. \$100. For a like reason B's gain is $\frac{2}{3}$ of \$300 = \$200.

Or we may solve the question as follows:

\$300 ÷ \$1200 = .25; i. e. the profits = 25 per cent. of the stock; ∴ A's share of profits = \$400 × .25 = \$100
B's share of profits = \$800 × .25 = \$200
Entire profits, \$300 Hence,

RULE 1. *Multiply the total gain or loss by each partner's fractional part of the stock, and the products will be the respective shares of gain or loss;* or,

RULE 2. *Find what per cent. the total gain or loss is of the whole stock, and then multiply each partner's stock by this per cent. written decimally.*

312. PROOF. *The sum of the shares of gain or loss must equal the total gain or loss.*

2. A, B, and C form a partnership; A furnishes \$4000, B \$5000, and C \$6000. They gain \$3000; how shall the gain be divided? Ans. A's, \$800; B's, \$1000; C's, \$1200.

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310. What is Partnership? What is the company called? What is the capital or stock? How are the profits and losses divided among the partners? **311.** Rule for finding the shares of gain or loss? Second rule? **312.** Proof?

3. Had the firm in Ex. 2 lost $750, what part of the loss should each partner sustain? How many dollars?

1st Ans. A, $\frac{4}{15}$; B, $\frac{1}{3}$; C, $\frac{2}{5}$.

4. A, B, and C engage in trade. A puts in $6000, B $10000, and C $8000. They gain $4000; what is each partner's share?

NOTE. These rules are equally applicable to distributing the property of a bankrupt, and many other similar problems.

5. A bankrupt whose property is worth $5000 owes A $3000. B $1500, and C $3500; to what fractional part of the property is each creditor entitled? To how many dollars?

6. Divide $1500 between A, B, and C so that A shall receive $2 as often as B receives $3 and C $5.

7. A, B, and C hire a pasture, for which they pay $90; A pastures 3 cows, B 5, and C 7; what part of the rent shall each pay? How many dollars?

8. A and B hire a pasture for $12; A's horse was in the pasture $4\frac{2}{3}$ weeks and B's $7\frac{1}{3}$ weeks; what rent shall each pay?

9. A, B, C, and D freight a ship to Canton; A furnishes $3000 worth of the cargo, B $5000, C $7000, and D $11000. They gain $5200; what is each one's share of the gain?

10. A and B form a partnership with a joint capital of $1200, of which A furnishes $\frac{2}{3}$ in cash, and B, for his share, furnishes 160 yards of broadcloth. They lose $300; how shall the loss be divided? What is the price of B's cloth per yard?

313. PROBLEM 2. To find each partner's share of gain or loss when their capital is employed unequal times.

Ex. 1. A and B trade in company; A puts in $300 for 8 months, and B $400 for 9 months. They gain $800; what part of the gain belongs to each? How many dollars?

$$\begin{array}{l} \text{A's \$300 for 8m.} = \$2400 \text{ for 1m.} \\ \text{B's \$400 for 9m.} = \underline{\$3600} \text{ for 1m.} \\ \qquad\qquad\qquad\qquad\quad \$6000 \text{ for 1m.} \end{array}$$

It is, ∴, as though the joint stock were $6000 for 1 month,

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312. What is the Note? 313. Explain Ex. 1.

of which A put in \$2400, and B \$3600; hence A is entitled to $\frac{2400}{6000}=\frac{2}{5}$ of the gain, and B to $\frac{3600}{6000}=\frac{3}{5}$; i. e. A is entitled to $\frac{2}{5}$ of \$800 = \$320, and B to $\frac{3}{5}$ of \$800 = \$480, Ans. Hence,

RULE. *Multiply each man's stock by the time it is continued in trade, and, regarding the products as the respective shares of stock, and the sum of the products as the total stock, proceed as in Problem* 1.

2. A and B engage in trade; A furnishes \$4000 for 12 months, and B \$6000 for 11 months. They lose \$570; what is the loss of each? Ans. A's loss, \$240; B's, \$330.

3. A, B, and C engage in partnership; A furnishing \$600 for 9m., B \$800 for 8m., and C \$1000 for 12m. They gain \$1071; what is each one's share of the gain?

4. A, B, and C hire a pasture for \$48. A pastures 3 horses for 8 weeks, B 5 horses for 6 weeks, and C 6 horses for 7 weeks; what part of the rent shall each pay?

5. B, T, and C enter into partnership, doing business in the name and signature of B, T, and C. Jan. 1, B puts in \$3000, T \$4000, and C \$2000. May 1, B puts in \$2000 more, C \$1000, and T takes out \$1000. Sept. 1, B takes out \$3000, T puts in \$2000, and C \$2000. At the end of the year they settle, having gained \$6400; what is each partner's share of the gain?

Ans. B's \$2000, T's \$2400, C's \$2000.

6. Jan. 1, 1860, B commenced business with a capital of \$3000. Sept. 1, 1860, wishing to enlarge his business, he took in H as a partner, with a capital of \$4000. July 1, 1861, they admit L into the partnership, with a capital of \$2500. On the 1st of Jan. 1862, they dissolve partnership, having gained \$7550; what is each one's share of the gain?

7. A, B, and C hire a pasture for \$92. A pastures 6 horses for 8 weeks, B 12 oxen for 10 weeks, and C 50 cows for 12 weeks. Now if 5 cows are reckoned as 3 oxen, and 3 oxen as 2 horses, what part of the rent shall each pay? How many dollars?

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313. Rule for finding the shares of gain or loss when the capital is in for unequal times?

8. A, B, and C hire a pasture for \$300. A puts in 10 oxen for 20 weeks, 15 cows for 14 weeks, and 99 sheep for 26 weeks; B puts in 7 oxen for 24 weeks, 12 cows for 20 weeks, and 66 sheep for 25 weeks; C puts in 25 oxen for 8 weeks, 12 cows for 12 weeks, and 33 sheep for 15 weeks. Now, if 11 sheep are reckoned as 1 cow, and 3 cows as 2 oxen, what is the cost per week for a sheep? a cow? an ox? How many dollars does each man pay for sheep? cows? oxen? What part of the rent does each man pay? How many dollars?

Ans. Cost per week for a sheep, $1\frac{5}{11}$c; a cow, 16c.; an ox, 24c. A pays for sheep, \$37.44; for cows, \$33.60; for oxen, \$48. B pays for sheep, \$24; for cows, \$38.40; for oxen, \$40.32. C pays for sheep, \$7.20; for cows, \$23.04; for oxen; \$48. A pays $\frac{248}{625}$ = \$119.04; B, $\frac{214}{625}$ = \$102.72; C, $\frac{163}{625}$ = \$78.24.

9. J. Fox and S. Low enter into partnership. January 1, Fox puts in \$5000, but Low puts in nothing until May 1; what shall he then put in that the partners may be entitled to equal shares of the profits at the close of the year?

10. Jan. 1, 1853, A, B, and C form a partnership for 1 year, and each furnishes \$3000; Mar. 1, A furnishes \$1000 more; June 1, B withdraws \$500, and C adds \$500; Sept. 1, A withdraws \$2000 and C \$500, and B adds \$1500. Having gained \$1000, at the close of the year the partnership is dissolved. What is each partner's share of the gain?

11. A, B, and C traded in company. A at first put in \$1000, B \$1200, and C \$1800; in three months A put in \$500 more and B \$300, and C took out \$400; in 7 months from the commencement of business, A withdrew all his stock but \$700, B put in as much as he at first put in, and C withdrew $\frac{1}{3}$ as much as A at any time had in the firm. At the end of a year they found they had gained 10 per cent. on the largest total stock at any one time in trade. What is the total gain? What fractional part shall each have? How many dollars?

Ans. Total gain, \$140.	A's part, $\frac{125}{511}$	= \$107.63$\frac{107}{511}$.
	B's part,	= \$
	C's part,	= \$
	Proof,	= \$

EXAMPLES IN ANALYSIS.

313 a. 1. If 6 barrels of flour cost $42, what will 11 barrels cost?

2. If $\frac{5}{8}$ of a cask of wine cost $35, what will 7 casks cost?

3. Twenty is $\frac{5}{9}$ of what number?

4. Fifty-one is $\frac{17}{173}$ of what number?

5. Ninety-five is $\frac{19}{23}$ of what number?

6. If $\frac{19}{23}$ of a ton of hay cost 95 shillings, what will a ton cost?

7. If $\frac{37}{80}$ of a cask of oil is worth $74, what is the value of 5 casks?

8 Sixty-four is $\frac{8}{9}$ of how many times 12?

9. Seventy-two is $\frac{8}{9}$ of how many times 4?

10. A man sold a watch for $63, which was $\frac{9}{7}$ of its cost; what was its cost?

11. A pole is $\frac{3}{8}$ in the mud, $\frac{3}{7}$ in the water, and 6 feet above water; what is the length of the pole?

12. A ship's crew have provisions sufficient to last 12 men 7 months; how long would they last 24 men?

13. A can build 35 rods of wall in 33 days, but B can build 9 rods while A builds 7; how many rods can B build in 44 days?

14. $\frac{3}{7}$ of 28 is $\frac{4}{11}$ of how many fifths of 55?

15. $\frac{5}{11}$ of 44 is $\frac{4}{5}$ of how many thirds of 15?

16. $\frac{7}{3}$ of 27 is $\frac{21}{5}$ of how many twelfths of 60?

17. A fox has 39 rods the start of a hound, but the hound runs 27 rods while the fox runs 24; how many rods must the hound run to overtake the fox? Ans. 351.

18. A hare has 32 rods the start of a hound, but the hound runs 12 rods while the hare runs 8; how many rods will the hare run before the hound overtakes him?

19. A man being asked how many sheep he had, replied that if he had as many more, $\frac{1}{2}$ as many more, and $2\frac{1}{2}$ sheep he should have 100; how many had he?

20. A detachment of 2000 soldiers was supplied with bread sufficient for 12 weeks, allowing each man 14 ounces a day, but

finding 105 barrels, containing 200lb. each, wholly spoiled, how many ounces may each man eat daily, that the remainder may last them 12 weeks?

21. A detachment of 2000 soldiers, having $\frac{1}{4}$ of their bread spoiled, were put upon an allowance of 12 oz. each per day for 12 weeks; what was the whole weight of their bread, good and bad, and how much was spoiled?

22. A detachment of 2000 soldiers having lost 105 barrels of bread, weighing 200 lb. each, were allowed but 12oz. each per day for 12 weeks; but if none had been lost, they might have had 14 oz. daily; what was the weight, including that which was lost, and how much was left to subsist on?

23. A detachment of 2000 soldiers, having lost $\frac{1}{4}$ of their bread, had each 12oz. per day for 12 weeks; what was the weight of their bread, including the part lost, and how much per day might each man have had, had none been lost?

24. A gentleman left his son an estate, $\frac{1}{4}$ of which he spent in 7 months, and $\frac{1}{3}$ of the remainder in 3 months more, when he had only \$5000 remaining; what was the value of the estate?

25. The quick-step in marching being 2 paces of 28 inches each per second, what is the rate per hour? and in what time will a detachment of soldiers reach a place 60 miles distant, allowing a halt of $1\frac{1}{2}$ hours?

26. Two men and a boy engage to reap a field of rye; one of the men can reap it in 10 days, the other in 12, and the boy in 15 days. In how many days can the three together reap it?

27. A merchant bought a number of bales of hops, each bale containing $246\frac{18}{133}$ lb., at the rate of \$3 for 11 lb., and sold them at the rate of \$5 for 12 lb., and gained \$248; how many bales did he buy? Ans. 7.

28. Suppose I pay $3\frac{3}{4}\frac{1}{8}$ cents per bushel for carting my wheat to mill, the miller takes $\frac{1}{18}$ for grinding, it takes $4\frac{1}{2}$ bushels of wheat to make a barrel of flour, I pay 25 cents each for barrels and $\$1\frac{1}{4}$ per barrel for carrying the flour to market, where my agent sells 60 barrels for $\$367\frac{1}{2}$, out of which he takes 25 cents per barrel for his services; what do I receive per bushel for my wheat? Ans. $87\frac{1}{2}$ cents.

RATIO.

314. RATIO *is the relation of one quantity to another of the same kind;* or, it is the quotient which arises from dividing one quantity by another of the same kind.

315. Ratio is usually indicated by two dots; thus, 8 : 4 expresses the ratio of 8 to 4.

The two quantities compared are the *terms* of the ratio; the first term being the *antecedent*, the second the *consequent*, and the two terms, collectively, a *couplet.*

316. Most mathematicians consider the *antecedent* a *dividend*, and the *consequent* a *divisor;*

$$\text{thus,}\quad 8:4=8\div4=\tfrac{8}{4}=2,$$
$$\text{and}\quad 3:12=3\div12=\tfrac{3}{12}=\tfrac{1}{4};$$

but others take the *antecedent* for the *divisor*, and the *consequent* for the *dividend;*

$$\text{thus,}\quad 8:4=4\div8=\tfrac{4}{8}=\tfrac{1}{2},$$
$$\text{and}\quad 3:12=12\div3=\tfrac{12}{3}=4.$$

NOTE 1. The first method is often called the *English* method, and the other the *French;* but there appears to be no good reason for such a distinction.

NOTE 2. The first is a *direct* ratio; the second is an *inverse* or *reciprocal* ratio. The first being considered the more simple and natural, is adopted in this work.

317. The antecedent and consequent being a dividend and divisor, it follows that any *change in the* ANTECEDENT *causes a* LIKE *change in the value of the ratio, and any change in the* CONSEQUENT *causes an* OPPOSITE *change in the value of the ratio* (Art. 84, 85, and 131). Hence,

1*st. Multiplying the antecedent multiplies the ratio; and dividing the antecedent divides the ratio* (Art. 83, a and b).

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314. What is Ratio? **315.** How indicated? What are the terms? The 1st? The 2d? The two collectively? **316.** Which term is divisor? Is the custom uniform? Which method is here taken? Why? What is a direct ratio? An inverse ratio? **317.** Explain and illustrate Art. 317 fully.

2d. *Multiplying the consequent divides the ratio; and dividing the consequent multiplies the ratio* (Art. 83, c and d).

3d. *Multiplying both antecedent and consequent by the same number, or dividing both by the same number, does not affect the ratio* (Art. 84, a and b).

318. The antecedent, consequent, and ratio are so related to each other, that, if either *two* of them be given, the other may be found; thus, in $12 : 3 = 4$, we have

antecedent $\div$ consequent $=$ ratio,
antecedent $\div$ ratio $=$ consequent, and
consequent $\times$ ratio $=$ antecedent.

319. When there is but one antecedent and one consequent the ratio is said to be *simple;* thus, $15 : 5 = 3$, is a simple ratio.

320. When the corresponding terms of two or more simple ratios are multiplied together the resulting ratio is said to be *compound;* thus, by multiplying together the corresponding terms of the simple ratios,

$$\left\{\begin{matrix} 6:2=3 \\ 8:2=4 \end{matrix}\right\} \text{ or } \left\{\begin{matrix} 6:3 = 2 \\ 8:2 = 4 \\ 10:2 = 5 \end{matrix}\right\}, \text{ we have the com-}$$

pound ratio, $48:4=12$ or $480:12=40$.

A compound ratio is always equal to the product of the simple ratios of which it is compounded.

NOTE. A compound ratio is not different in its *nature* from a simple ratio, but it is called *compound* merely *to denote its origin.*

Ex. 1. What is the ratio of 20 to 4? Ans. $20 : 4 = 5$.

2. What is the ratio of 2 to 9? Ans. $2 : 9 = \frac{2}{9}$.

3. What is the *inverse* ratio of 20 to 4? Ans. $\frac{4}{20} = \frac{1}{5}$.

4. What is the inverse ratio of 2 to 9?

5. What is the ratio compounded of 8 to 6 and 9 to 2?

6. Which is the greater, the ratio of 9 to 7 or of 19 to 14?

7. Which is the greater, the ratio of 5 to 4 or of 15 to 13?

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318. What of antecedent, consequent, and ratio? **319.** What is simple ratio? **320.** Compound ratio? Its value? Its nature? Why called *compound?*

PROPORTION.

321. PROPORTION *is an equality of ratios.*

Two ratios, and ∴ 4 terms, are required to form a proportion.

322. Proportion is indicated by means of dots; thus,

$$8 : 4 :: 6 : 3,$$

which is read, 8 is to 4 as 6 is to 3; or, as 8 is to 4 so is 6 to 3; or it may be indicated thus,

$$8 : 4 = 6 : 3,$$

which is read, the ratio of 8 to 4 equals the ratio of 6 to 3.

Any 4 numbers are in proportion, and may be written and read in like manner, if the quotient of the 1st divided by the 2d is equal to the quotient of the 3d divided by the 4th.

323. The 1st and 4th terms are called *extremes*, and the 2d and 3d, *means*. The 1st and 3d are the antecedents of the two ratios, and the 2d and 4th are the consequents. The product of the extremes is always equal to the product of the means; thus, in the proportion $8 : 4 :: 6 : 3$, we have $8 \times 3 = 4 \times 6$.

324. Since the product of the extremes is equal to the product of the means, any one term may be found when the other three are given; for the product of the extremes divided by either mean will give the other mean, and the product of the means divided by either extreme will give the other extreme.

Fill the blank in each of the following proportions:

1. $8 : 2 :: \quad : 3.$ Ans. $\frac{8 \times 3}{2} = 12.$
2. $6 : 9 :: 8 : \quad .$ Ans. $\frac{9 \times 8}{6} = 12.$
3. $4 : \quad :: 2 : 9.$
4. $\quad : 16 :: 7 : 14.$

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321. What is Proportion? **322.** How indicated? Proportion, how read? When are four numbers in proportion? **323.** What are the 1st and 4th terms called? 2d and 3d? 1st and 3d? 2d and 4th? The product of the extremes equals what? **324.** How many terms must be given? How can the other be found?

325. It follows from Art. 317, that if the 1st and 2d, or 3d and 4th, or 1st and 3d, or 2d and 4th, or all four terms of a proportion are multiplied or divided by the same number, the resulting numbers will be in proportion.

326. If 4 numbers are proportional they will be in proportion in 8 different orders; thus,

(1) Given	8 : 4 : : 6 : 3
(2) Alternating (1)	8 : 6 : : 4 : 3
(3) Inverting (1)	4 : 8 : : 3 : 6
(4) Alternating (3)	4 : 3 : : 8 : 6
(5) Inverting (1) and transposing couplets	3 : 6 : : 4 : 8
(6) Alternating (5)	3 : 4 : : 6 : 8
(7) Inverting (5)	6 : 3 : : 8 : 4
(8) Alternating (7)	6 : 8 : : 3 : 4

NOTE. These 4 numbers may be written in 16 other orders, but none of them will be in proportion.

327. When the means of a proportion are alike, the term repeated is a *mean proportional* between the other two, and the last term is a *third proportional* to the 1st and 2d; thus, in 4 : 6 : : 6 : 9, 6 is a mean proportional between 4 and 9, and 9 is a third proportional to 4 and 6.

328. A mean proportional between two numbers may be found by multiplying the two given numbers together, and then resolving the product into *two equal factors;* thus, the mean proportional between 2 and 8 is 4, for $2 \times 8 = 16 = 4 \times 4$; $\therefore$ 2 : 4 : : 4 : 8.

329. A third proportional to two numbers may be found by *dividing the square of the 2d by the 1st.* The third proportional to 5 and 10 is 20; for $10^2 \div 5 = 20$; $\therefore$ 5 : 10 : : 10 : 20.

SIMPLE PROPORTION.

330. In all examples in *Simple Proportion* there are three

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325. What terms may be multiplied without destroying the proportion? What divided? **326.** In how many orders may four proportional numbers be in proportion? In how many not in proportion? **327.** What is a mean proportional? A third proportional? **328.** How is a mean proportional found? **329.** A third proportional?

numbers given to find a fourth; ∴ Proportion is often called the *Rule of Three.*

Two of the three given numbers must be of the same kind, and the other is of the same kind as the answer.

Ex. 1. If 3 men build 6 rods of wall in a day, how many rods will 5 men build?

This example may be analyzed as follows: If 3 men build 6 rods, 1 man will build $\frac{1}{3}$ of 6 rods, i. e. 2 rods; and if one man build 2 rods, 5 men will build 5 times 2 rods, i. e. 10 rods, Ans.; but to solve it by proportion, we say, that 3 men have to 5 men the same ratio that the given number of rods has to the required number of rods; thus,

3 men : 5 men : : 6 rods : required number of rods.

Now, since the means and 1st extreme are given, we find the 2d extreme by dividing the product of the means by the given extreme (Art. 324); thus,

$6 \times 5 = 30$ and $30 \div 3 = 10$, Ans. as before. Hence,

331. To solve an example in Simple Proportion,

RULE. *Write that given number which is of the same kind as the required answer for the third term; consider whether the nature of the question requires the answer to be greater or less than the third term; if greater, write the greater of the two remaining numbers for the second term and the less for the first; but if less, write the less for the second and the greater for the first; in either case, divide the product of the second and third terms by the first, and the quotient will be the term sought.*

NOTE 1. If the first and second terms are in different denominations, they should be reduced to the same before stating the question.

REMARK. Every one who intelligently solves an example by proportion, does, in effect, solve it by analysis; but the teacher should use much care on this point, since the scholar learns much faster when he analyzes a question than when he merely follows

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330. Of what kind must two of the three given numbers be? What the other? **331.** Rule for solving an example in proportion? Note 1? Remark?

a rule. Let the following examples be solved by analysis and by proportion.

2. If a man earn $24 in 2 months how much will he earn in 9 months?

$$\begin{array}{r} 2:9::24:\text{4th term.} \\ 9 \\ 2\,)\overline{216} \\ \$108,\text{ Ans.} \end{array}$$

Since we are seeking for dollars, we make $24 the 3d term, and then, as a man will earn more in 9 months than he will in 2 months, we make 9 the 2d term and 2 the 1st. To analyze the above, we say, If a man earn $24 in 2 months, then in 1 month he will earn ½ of $24, i. e. $12; and if he earn $12 in 1 month, then in 9 months he will earn 9 times $12, i. e. $108, Ans.

3. If 15 bush. of wheat make 3 bbl. of flour, how many bushels of wheat will be required to make 7 bbl. of flour? Ans. 35.

4. If 40 bush. of wheat make 8 bbl. of flour, how many barrels of flour will 75 bush. of wheat make? Ans. 15.

5. If a man can walk 75 miles in 3 days, how far can he walk in 8 days? Ans. 200 miles.

6. If a man travel 64 miles in 2 days, how long will it take him to travel 160 miles? Ans. 5 days.

7. If a locomotive run 39000 miles in 13 weeks, how far, at that rate, would it run in 52 weeks?

BY PROPORTION.

$13:52::39000:$ 4th term.
$39000 \times 52 = 2028000$;
$2028000 \div 13 = 156000$, Ans.

BY CANCELING.

$$\frac{39000 \times \overset{4}{\cancel{52}}}{\cancel{13}} = 156000, \text{ Ans.}$$

8. If 20 men perform a piece of work in 8 days, in how many days will 4 men perform the same? Ans. 40.

9. If 24 cords of wood cost $60, what will 18 cords cost.

10. If $30 pay for 5 cords of wood, how many dollars will pay for 12 cords? Ans. 72.

11. If 4 cords of wood cost $20, how many cords may be bought for $45? Ans. 9.

12. If 6 horses eat 42 bushels of oats in 5 weeks, how many bushels will 11 horses eat in the same time?

13. What cost 7 tons of coal when 4 tons cost $24?

14. In how many days can 6 men build a house, if 10 men can build it in 72 days?

15. If 72 lb. of cheese are worth as much as 30 lb. of butter, how many pounds of cheese will pay for 20 lb. of butter?

16. How many tons of coal can be bought for $84, when 3 tons cost $18? Ans. 14.

17. If 9 horses eat a ton of hay in 20 days, how many horses will eat a ton in 30 days? Ans. 6.

18. How many tons of hay will 6 horses eat in 25 weeks, if 8 horses eat 20 tons in the same time?

19. If I pay 2s. 8d. per week for pasturing 2 cows, what shall I pay for pasturing 11 cows?

2 : 11 :: 2s. 8d. :	or,	2 : 11 :: 32d :
11		11
2) 29s. 4d.		2) 352d. :
Ans. 14s. 8d.		Ans. 176d. = 14s. 8d.

20. If I pay 2s. 8d. for pasturing 2 cows, how many cows can be pastured the same time for 14s. 8d.?

21. If 8 acres of land cost 75£ 6s. 4d., how many acres may be bought for 131£ 16s. 1d.?

22. If 14 acres of land cost 131£ 16s. 1d., what will 8 acres cost?

23. If $\frac{5}{8}$ of a ship cost $9875, what are $\frac{3}{8}$ of her worth?

24. If $\frac{3}{5}$ of a barrel of flour cost $3.20, what will 6 bbl. cost?

25. If a man walk 192 miles in 6 days of 8 hours each, in how many days of 12 hours each will he walk 192 miles?

26. Lent a friend $400 for 6 months; afterwards he lent me $300. How long may I keep it to balance the favor?

27. How many yards of cloth $\frac{5}{8}$ of a yard wide are equal to 20 yards $1\frac{1}{4}$ yard wide?

28. If when flour is worth $9 per bbl., a penny loaf weighs 4oz., what will it weigh when flour is worth $6 per bbl.?

29. If 10 horses eat 45 bushels of oats in 3 weeks, how many bushels will 12 horses eat in the same time?

30. Three men can do a piece of work in 12 days; how many men must be added to the number to do the same in 4 days?

31. A ship's crew of 12 men has food for 24 days, how many men must be discharged that it may last 12 days longer?

32. Paid \$1.50 for 3lb. of tea; what should I pay for 9lb.?

33. If .25 of a ship cost \$3000, what cost .375 of her?

34. At \$24 per cwt., what is the cost of $62\frac{1}{2}$lb.?

35. If a steeple 180 feet high casts a shadow 240 feet, what is the length of the shadow cast by a staff 3 feet high, at the same time?

NOTE 2. Since *each* of the three terms in the above example is in *feet*, the learner may be uncertain which number to place as the *third term*; but he has only to notice that he is required to find the length of a *shadow*, ∴ the third term should be the number expressing the length of *shadow* in the given example, viz. 240ft.; thus,

$$180 : 3 :: 240 : \text{4th term} = 4\text{ft., Ans.}$$

36. If a staff 3 feet long casts a shadow 4 feet, what is the hight of a steeple which, at the same time, casts a shadow 240 feet? Ans. 180 ft.

37. If a staff 3 feet long casts a shadow 4 feet, how long is the shadow of a steeple which is 180 feet high, at the same time?

38. If a steeple 180 feet high casts a shadow 240 feet, what is the hight of a staff which, at the same time, casts a shadow 4 feet?

39. The interest of \$300 for 1yr. being \$18, what is the interest of \$850 for the same time?

40. The interest of \$800 for 6m. being \$24, what principal will gain \$45 in the same time?

41. If a man's salary amounts to \$2700 in 3 years, what will it amount to in 11 years?

42. If a man's salary amounts to \$9900 in 11 years, in how many years will it amount to \$2700?

43. If $12\frac{1}{2}$ yards of silk that is $\frac{3}{4}$ of a yard wide will make a dress, how many yards of muslin that is $1\frac{3}{8}$ yards wide will be required to line it?

44. If $\frac{3}{8}$ of an acre of land is worth \$36.40, what is the value of $15\frac{3}{13}$ acres, at the same price?

45. If 6 men can mow 12a. 3r. 16rd. of grass in 2 days, by working 6 hours per day, how many days will it take them to do the same if they work only 4 hours per day?

46. If 2bbl. of flour are worth as much as 3 cords of wood, how many barrels of flour will pay for 45 cords of wood?

47. A bankrupt, owing $25000, has property worth $15000; how much will he pay on a debt of $500?

48. A man, owning $\frac{3}{5}$ of a ship, sells $\frac{2}{3}$ of his share for $20000; what is the value of the ship?

49. A and B hired a pasture for $45.90, in which A pastured 11 oxen and B 19; what shall each pay?

50. If 13 men perform a piece of work in 45 days, how many men must be added to perform the same in $\frac{1}{3}$ of the time?

51. If the interest on $700 is $42 in one year, what will be the interest on the same sum for $3\frac{1}{2}$ years?

52. How many yards of paper 2 feet in width will paper a room that is $13\frac{1}{2}$ feet long, 12 feet wide, and 9 feet high?

53. If I pay $168 for 63 gallons of wine, how much water shall I add that I may sell it at $2 per gallon without loss?

54. A certain house was built by 30 workmen in 98 days, but, being burned, it is required to rebuild it in 60 days; how many men must be employed?

55. A garrison of 1500 men has provisions for 12 months, how long will the same provisions last if the garrison is reenforced by 300 men?

56. If a piece of land 20 rods long and 8 rods wide contains an acre, how long must it be to contain the same when it is but 2 rods wide.

57. If the earth revolves 366 times in 365 days, in what time does it revolve once? Ans. 23h. $56\frac{4}{61}$m.

58. A wall which was to be built 24 feet high was raised 8 feet by 6 men in 12 days; how many men must be employed to build the remainder of the wall in 12 days more?

59. A wall was completed by 12 men in 12 days; how many men would complete the same in 4 days?

60. If a man perform a journey in 6 days when the days are 12 hours long, in how many days of 8 hours each will he perform the same?

61. A cistern has a pipe that will fill it in 6 hours; how many pipes of the same size will fill it in 45 minutes?

62. A cistern has 3 pipes; the first will fill it in 3 hours, the second in 4 hours, and the third in 5 hours; in what time will they together fill the cistern?

63. Paid $3.50 for 7lb. of tea; what should I pay for 19lb.?

64. A can cut a field of grain in 8 days; A and B can cut it in 6 days. In what time can B do the same?

65. If 2 horses can draw a load of 16 tons upon a railway, how many horses will be required to draw 72 tons?

66. A farm was sold at $25.50 per acre, amounting to $1925.25; how many acres did the farm contain?

67. A garrison of 1000 men have 14oz. of bread each per day for 120 days; how long will the same bread last them if each man is allowed but 12oz. per day?

68. If $\frac{5}{11}$ of a ship cost $25000, what is $\frac{1}{18}$ of her worth?

69. At $27 per cwt., what is the cost of $37\frac{1}{2}$lb.?

70. The earth moves 19 miles per second in her orbit; how far does she go in 3m. 27sec.

COMPOUND PROPORTION.

332. Compound Proportion is an *equality* of two ratios, one of which is *compound* and the other *simple;* thus,

$$\left.\begin{array}{r} 3 : 12 \\ 16 : \ 2 \end{array}\right\} :: 18 : 9, \text{ is a compound proportion;}$$

and $48 : 24 :: 18 : 9$, is the same reduced to a simple form.

Note. The *compound ratio* may consist of any number of couplets.

333. Every compound proportion may be reduced to a simple form, and, moreover, every example in compound proportion may be solved by means of two or more simple proportions.

Ex. 1. If 6 men in 8 hours thresh 30 bushels of wheat, in how many hours will 2 men thresh 5 bushels?

BY SIMPLE PROPORTION.

$2 : 6 :: \ 8 : 24$, and

$30 : 5 :: 24 : \ 4$, Ans.

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332. What is Compound Proportion? **333.** May an example in compound proportion be solved by simple proportion? Analyze Ex. 1.

In solving this question by simple proportion, we, in the first place, disregard the *amount of labor*, and inquire how long it will take 2 men to do as much as 6 men in 8 hours. Having found 24 hours to be the answer to this question, we next disregard the *number of men*, and inquire how long it will take to thresh 5 bushels of wheat if 30 bushels are threshed in 24 hours, and thus obtain 4 hours, the true answer to the question.

In this operation, the given number of hours, 8, is first multiplied by 6 and the product divided by 2, then this quotient is multiplied by 5 and the product divided by 30; but it will answer the same purpose to multiply the 8 by the product of the two multipliers, 6 and 5, then divide the number so obtained by the product of the two divisors, 2 and 30; thus,

BY COMPOUND PROPORTION.

$$\left.\begin{array}{rcr} 2 & : & 6 \\ 30 & : & 5 \end{array}\right\} :: 8 : \text{4th term.}$$

$$\begin{array}{rrl} 60 & 30 & \\ & 8 & \\ \hline 60) & 240 & (4, \text{ Ans.} \\ & 240 & \\ \hline \end{array}$$

Here 2 is multiplied by 30 for a divisor, and the product of 6 and 5 is multiplied by 8 for a dividend.

It will be seen that, of the first two couplets, $\left\{\begin{array}{c} 2 : 6 \\ 30 : 5 \end{array}\right\}$, one ratio is *less than a unit* and the other *greater;* but there is no impropriety in this, for one condition of the question requires the answer to be greater than the 3d term, and the other condition requires it to be less. Hence,

334. To solve questions in Compound Proportion,

RULE. *Write that given number which is of the same kind as the required answer for the 3d term; take any two of the remaining terms* THAT ARE ALIKE, *and, considering the question as* DEPENDING ON THESE ALONE, *arrange them as in simple proportion; arrange each pair of* LIKE TERMS *by the same principles; and then multiply the continued product of the 2d terms by the 3d term, and divide this result by the continued product of the 1st terms; the quotient will be the term sought.*

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334. Rule for compound proportion?

NOTE. The work may often be much abridged by canceling factors in the 2d and 3d terms, with like factors in the 1st terms (144, Note).

Ex. 2. If 6 men in 15 days earn \$135, how many dollars will 9 men earn in 18 days?

$$\left.\begin{array}{l}6 \text{ men} : 9 \text{ men} \\ 15 \text{ days} : 18 \text{ days}\end{array}\right\} :: \$135 : \text{4th term.}$$

$9 \times 18 \times 135 = 21870 =$ continued product of 2d and 3d terms.
$6 \times 15 = 90 =$ continued product of 1st terms.
$21870 \div 90 = 243$, Ans.

THE SAME CANCELED.

$$\left.\begin{array}{l}\begin{array}{r|r}\cancel{6} & : 9 \\ \cancel{15} & : \cancel{18} \\ 5 & 3\end{array}\;\; :: \overset{27}{\cancel{135}} : \\ 9 \times 27 = 243, \text{ Ans.}\end{array}\right\} \text{or, } \left\{\frac{9 \times \overset{3}{\cancel{18}} \times \overset{27}{\cancel{135}}}{\cancel{6} \times \underset{5}{\cancel{15}}} = 243, \text{ Ans.}\right.$$

3. If 4 men, in 24 days of 9 hours each, build a wall 40ft. long, 9ft. high, and 4ft. thick, in how many days of 6 hours each can 8 men build a wall 60ft. long, 12ft. high, and 5ft. thick? Ans. 45.

$$\left.\begin{array}{rcl}8 \text{ men} & : & 4 \text{ men} \\ 6 \text{ hours} & : & 9 \text{ hours} \\ 40 \text{ ft. long} & : & 60 \text{ ft. long} \\ 9 \text{ ft. high} & : & 12 \text{ ft. high} \\ 4 \text{ ft. thick} & : & 5 \text{ ft. thick}\end{array}\right\} \quad :: 24 \text{ days} :$$

4. If a family of 6 persons spend \$600 in 8 months, how many dollars will be required for a family of 10 persons in 14 months? Ans. 1750.

5. If a family of 6 persons spend \$600 in 8 months, how many months will \$1750 sustain a family of 10 persons?

6. If a family of 6 persons spend \$600 in 8 months, how large a family may be sustained 14 months for \$1750?

7. If the transportation of 12 boxes of sugar, each weighing 4cwt., 40 miles, cost \$8, what must be paid for carrying 40 boxes, weighing $3\frac{1}{2}$cwt. each, 75 miles? Ans. \$43.75.

8. If 4 men dig a trench 84 feet long in $2\frac{1}{2}$ days, how many men can dig a trench 336 feet long in 4 days? Ans. 10.

9. If 4 men dig a trench 84ft. long and 5ft. wide in 3 days, how many men can dig a trench 420ft. long and 3ft. wide in 4 days? Ans. 9.

10. If 2 men dig a trench 50ft. long, 5ft. wide, and 3ft. deep in $3\frac{1}{2}$ days, how many men can dig a trench 300ft. long, $2\frac{1}{2}$ft. wide, and 4ft. deep in 7 days? Ans. 4.

11. If 6 men dig a trench of 4 degrees of hardness, 35ft. long, 6ft. wide, and 5ft. deep in 5 days, how many men can dig a trench of 6 degrees of hardness 105ft. long, 4ft. wide, and 3ft. deep in 2 days? Ans. 27.

12. If 5 men, in 4 days of 10 hours each, dig a trench of 10 degrees of hardness, 50ft. long, 3ft. wide, and $6\frac{1}{4}$ft. deep, how many men can dig a trench of 5 degrees of hardness, 75ft. long, $4\frac{1}{2}$ft. wide, and $4\frac{1}{4}$ft. deep, in 9 days of $8\frac{1}{2}$ hours each?

13. If $100 gain $6 in 1 year, what will $300 gain in 8m.?

14. If $300 gain $12 in 8 months, what will $100 gain in 1 year?

15. If $100 gain $6 in 1 year, in what time will $300 gain $12?

16. If $100 gain $6 in 1 year, what principal will gain $12 in 8 months?

17. If a 2-penny loaf weighs 9oz. when wheat is 6s. 6d. per bushel, how much bread may be bought for 3s 2d. when wheat is worth 4s. 9d. per bushel? Ans. 14lb. 10oz.

18. A wall, which was to be built 32 feet high, was raised 8 feet by 6 men in 12 days; how many men must be employed to build the remainder of the wall in 9 days? Ans. 24.

19. If 6bbl. of flour serve a family of 8 persons 12m., how many bbl. will serve a family of 12 persons 16 months?

20. If 16 horses eat 24 bushels of oats in 6 days, how many bushels will 23 horses eat in 20 days?

21. A garrison of 1600 men have bread enough to allow 24 ounces per day to each man for 25 days; but, the garrison being re-enforced by 400 men, how many ounces per day may each man have in order that they may hold out against the enemy 30 days?

22. If 3 compositors, in 2 days of 9 hours each, set type for 27 pages, each page consisting of 36 lines of 45 letters each, how may compositors will set 36 pages of 40 lines of 54 letters each, in 6 days of 8 hours each?

23. If a man, walking 12 hours a day for 8 days, travel 384 miles, in how many days of 10 hours each would he walk 240 miles, traveling at the same rate?

24. If a man travel 280 miles in 7 days, traveling 10 hours each day, how many miles will he go in 12 days, traveling at the same rate, only 9 hours each day?

25. If 12 horses or 10 oxen eat 2 tons of hay in 8 weeks, how much hay will 18 horses and 25 oxen eat in 6 weeks?

26. If it take 33 reams of paper to make 1500 copies of a book of 11 sheets, how many reams will be required to make 2500 copies of a book of 9 sheets?

27. If 600 tiles, each 12 inches square will pave a court, how many tiles that are 10 inches long and 8 inches wide will pave another court which is 3 times as long and half as wide?

28. How many bricks, each 8 inches long, 4 inches wide, and 2 inches thick, would occupy the same space as 600 stones, each 2 feet long, $1\frac{1}{2}$ feet wide, and 8 inches thick?

29. If 7 shares in a bank yield their owner $17.50 in 3 months, how much will 12 shares yield in 2 years?

30. If 3 men, in 16 days of 12 hours each, build a wall 30ft. long, 8ft. high, and 3ft. thick, how many men will be required to build a wall 45ft. long, 9ft. high, and 6ft. thick, in 24 days of 9 hours each?

31. If the transportation of 9hhd. of sugar, each weighing 12 cwt., 20 leagues, cost $50, what must be paid for the transportation of 50 tierces, each weighing $2\frac{1}{2}$ cwt., 300 miles?

32. If $300 gain $18 in 9 months, what is the rate per cent.?

33. If a bar of silver 2ft. 1in. long, 6in. wide, and 3in. thick, be worth $2725, what is the value of a bar of gold 1ft. $9\frac{13}{16}$in. long, 8in. wide, and 4in. thick, the specific gravity of silver to that of gold being as 10.47 to 19.26, and the value per oz. of silver being to that of gold as 2 to 33? Ans. $128293.

34. If 496 men, in 5 days of 12h. 6m. each, dig a trench of 9 degrees of hardness 465 feet long, $3\frac{2}{3}$ feet wide and $4\frac{2}{3}$ feet deep, how many men will be required to dig a trench of 2 degrees of hardness $168\frac{3}{4}$ feet long, $7\frac{1}{2}$ feet wide, and $2\frac{1}{3}$ feet deep, in 22 days of 9 hours each? Ans. 15.

ALLIGATION.

335. Alligation treats of mixing simple substances of different qualities, producing a compound of some intermediate quality. It is of two kinds, *Medial* and *Alternate*.

ALLIGATION MEDIAL.

336. Alligation Medial is the process by which we find the price of the mixture, when the quantities and prices of the simples are given.

Ex. 1. A merchant mixes 5 gallons of oil worth 4s. per gal. with 4 gal. at 5s., 2 gal. at 11s., and 3 gal. at 12s. What is the value of a gallon of the mixture?

5	gal. at 4s. per gal.		are worth		20s.
4	"	5s.	"	"	20s.
2	"	11s.	"	"	22s.
3	"	12s.	"	"	36s.
∴ 14	gal.		are worth		98s.

and 1 gal. is worth $\frac{1}{14}$ of 98s = 7s., Ans.

All examples of this nature are solved on this plan. Hence,

337. To find the price of a mixture when the number of articles mixed and their prices are given,

Rule. *Divide the total value of the articles mixed by the sum of the simples, and the quotient is the price of* one.

2. A miller mixes 20 bushels of corn worth 80c. per bush. with 10 bush. of rye at $1, 40 bush. of oats at 35c., and 30 bush. of barley at 90c.; what is the price per bushel of the mixture?

3. A grocer mixes 10 pounds of sugar worth 6c. per lb. with 12 lb. at 8c., 4 lb. at 12c., and 5 lb. at 15c.; what is a pound of the mixture worth?

ALLIGATION ALTERNATE.

338. Alligation Alternate is the process of mixing

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335. What does Alligation treat of? It is of how many kinds? What? 336. What is Alligation Medial? 337. Rule?

quantities of different prices so as to obtain a mixture of a required intermediate price. There are three problems.

339. PROBLEM 1. The prices of several kinds of goods being given, to ascertain how much of each kind may be taken to form a compound of a proposed medium price.

Ex. 1. A farmer wishes to mix oats worth 30c. per bush. with barley worth 45c., so as to make a mixture worth 42c.; how many bushels of each may he take?

ANALYSIS. It is evident that he must mix them in such proportions as to gain just as much on his oats as he loses on the barley. Now, he gains 12c. on 1 bush. of oats, and loses but 3c. on 1 bush. of barley; ∴, for each bushel of oats he must take $12 \div 3 = 4$ bushels of barley.

SECOND METHOD.

$$42 \begin{cases} 30 & 3 \qquad -12c. \times 3 = -36c., \text{ deficiency.} \\ 45 & 12 \qquad +\ 3c. \times 12 = +36c., \text{ surplus.} \end{cases}$$

Having written the prices of the oats and barley in a vertical column and the price of the mixture at the left, as above, we write the difference between the mean price (i. e. the price of the mixture) and the price of the oats against the price of the barley, and the difference between the mean price and that of the barley against the price of the oats, and the differences standing against the prices of the oats and barley, respectively, will represent the *proportionl quantities* of oats and barley to be taken; for it will be seen that the product of the deficiency in the value of a bushel of oats, multiplied by the number of bushels of oats ($-12c. \times 3 = -36c.$), is *necessarily* equal to the product of the surplus in the value of a bushel of barley multiplied by the number of bushels of barley ($+3c. \times 12 = +36c.$), *since the two products are composed of the* SAME FACTORS; and one representing a deficiency and the other a surplus, *they will balance each other.*

In the same manner, any number of *pairs* of simples may be

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338. What is Alligation Alternate? How many Problems? **339.** Object of Problem 1? Explain the analysis of Ex. 1. Explain the 2d method.

made to balance, as in Ex. 2, the price of one simple in each pair being *less* and that of the other *greater* than the mean price.

In performing the operation, the terms are connected by a line merely for convenience of reference in comparing them.

2. A merchant has 4 kinds of sugar, worth severally 6c., 8c., 13c., and 16c. per lb.; how may he mix them so as to make a mixture worth 10c. per lb.?

OPERATION.

$$10\left\{\begin{array}{r}6\\8\\13\\16\end{array}\right. \quad \begin{array}{r}6\\3\\2\\4\end{array} \quad \begin{array}{r}-4c.\times 6 = -24c.\\-2c.\times 3 = -\ \ 6c.\\+3c.\times 2 = +\ \ 6c.\\+6c.\times 4 = +24c.\end{array} \begin{array}{l}\left.\begin{array}{l}\\ \\\end{array}\right\}-30c.\\ \left.\begin{array}{l}\\ \\\end{array}\right\}+30c.\end{array}$$

Each *pair* of these products, viz. the 1st and 4th, and the 2d and 3d, will necessarily balance; *for they are composed of the* SAME FACTORS, and the one marked + represents a *surplus* and the one marked — represents a *deficiency*. By this method the quantities always balance in pairs, however many simples may be put in the mixture.

340. There evidently may be as many independent answers, all correct, as there are different ways of *pairing* the simples; and, by taking multiples of these, the results may be varied indefinitely, so that there may be an infinite number of answers to one question.

Among other methods, the 2d example may have the following solutions, and each may be *proved* correct by *Alligation Medial.*

$$10\left\{\begin{array}{r}6\\8\\13\\16\end{array}\right. \quad \begin{array}{l}3\text{ lb. at } 6c.\\6\text{ lb. `` } 8c.\\4\text{ lb. `` } 13c.\\2\text{ lb. `` } 16c.\end{array} \qquad 10\left\{\begin{array}{r}6\\8\\13\\16\end{array}\right. \quad \begin{array}{r}3+6=9\text{ lb. at } 6c.\\6\text{ lb. `` } 8c.\\4\text{ lb. `` } 13c.\\4+2=6\text{ lb. `` } 16c.\end{array}$$

From these illustrations:

RULE. *Write the prices of the several simples in a vertical column; on the left, separated by a line, write the proposed medium price; connect, by a line, each price that is less than the*

339. Explain Ex. 2. How are the prices connected? How do they balance?
340. How many answers may there be? How proved correct? Rule?

medium with one or more that is greater, and each that is greater with one or more that is less; write the difference between the medium price and the price of each simple against the number or numbers with which the simple is connected; these differences, or their sum if two or more stand against one price, will be the proportional parts of the several simples which may be taken to form the mixture.

341. Each of the foregoing methods is simple and correct, but, for the convenience of the merchant, there is a better mode, viz.: *Assume* the quantities of the simples, and then, by calculation, correct the assumption, as follows:

3. A merchant has 5 kinds of wine worth 5s., 6s., 8s., 13s., and 15s. per gal. What quantities of each may he take to make a mixture worth 9s. per gallon?

	s.	gal.		s.		s.	s.
9s.	5	7	×	−4	=	−28	
	6	6	×	−3	=	−18	
	8	6	×	−1	=	−6	−52, deficiency.
	13	2	×	+4	=	+8	
	15	4	×	+6	=	+24	+32, surplus.
							−20, deficiency.
		gal.		s.			
Add wine at 15s.,		4	×	+6	=		+24, surplus.
							+4, surplus.
Subtract wine at 13s.,		−1	×	+4	=		−4, deficiency.

Having *assumed* 7gal. at 5s., 6gal. at 6s., 6gal. at 8s., 2gal. at 13s., and 4gal. at 15s., we find the mixture is not worth so much as it should be by 20s. Now this may be remedied by putting in more of the higher priced wines or less of the cheaper. If we add 4gal. more of the 15s. wine, this will balance the deficiency and create a surplus of 4s., and this may be corrected by taking out 1gal. of the 13s. wine. There are now in the mixture 7gal. at 5s., 6gal. at 6s., 6gal. at 8s., 1gal. at 13s., and 8gal. at 15s.

Remark. The deficiencies are marked by the sign — and the excesses by + to aid the mind in making corrections.

---

341. Another method, explain it. What is the remark?

NOTE. This mode of correcting may be indefinitely varied, hence the merchant may take the simples in a ratio more nearly as he desires than by either of the other modes.

Let the pupil solve the following examples by each of the three modes, and prove them:

4. A grocer wishes to mix teas worth 25c., 33c., 48c., 56c., and 75c. so that the compound may be worth 45c. per pound. How many pounds of each may he take?

5. A farmer has cows worth \$16, \$20, \$28, \$40, and \$50 per head; what number of each may he sell at an average price of \$30 per head?

342. PROBLEM 2. The price of each of the simples, the price of the compound, and the quantity of one kind being given, to find how much of each of the other simples may be taken:

RULE. *Find the proportional parts as in the preceding Problem; then say, as the proportional part of that simple whose quantity is given is to the given quantity, so is each of the other proportional parts to the required quantity of each of the other simples, severally.*

Ex. 1. How many pounds of sugar at 4, 6, 9, and 10c. per lb. may be mixed with 12 lb. at 13c. to make a compound worth 8c. per lb.? Ans. 15, 9, 6, and 6 lb. at 4, 6, 9, and 10c.

	c.		lb.
	4		5
	6	2 + 1 =	3
8c.	9		2
	10		2
	13		4

If we connect the prices as in the margin, we obtain 5, 3, 2, 2, and 4 lb. for the proportional parts. Now if the 4 lb. at 13c. be increased in a 3 fold ratio, it will become 12 lb., *the given quantity*, and if each of the other proportional parts be increased in the same ratio, evidently the price per lb. of the mixture will remain unaltered; thus,

4 lb. at 13c. : 12 lb. at 13c. : : 5 lb. at 4c. : 15 lb. at 4c.
4 lb. at 13c. : 12 lb. at 13c. : : 3 lb. at 6c. : 9 lb. at 6c.
etc. etc.

---

341. What is the Note? **342.** Object of Problem 2? Rule? Explanation?

2. How many gallons of wine at 8, 10, and 15s. per gal. may be mixed with 15gal. of water of no exchangeable value, to make a mixture worth 12s. per gal.?

3. How many lb. of wool at 30, 40, and 50c. per lb. may be mixed with 24lb. at 45c. to make a mixture worth 42c. per lb.?

343. PROBLEM 3. The prices of the several simples, the price of the compound, and the entire quantity in the compound being given, to find how much of each simple may be taken:

RULE. *Find the proportional parts as in Problem* 1; *then say, as the sum of the proportional parts is to the whole compound, so is each of the proportional parts to the required quantity of each.*

Ex. 1. I have 4 kinds of coffee, worth 8, 11, 14, and 20c. per pound; how many pounds of each may I take to form a compound of 60lb. at 13c. per lb.?

Ans. 28, 4, 8, and 20lb. at 8, 11, 14, and 20c.

	c.	lb.
13c.	8	7
	11	1
	14	2
	20	5

15 : 60lb. :: 7lb. : 28lb. at 8c.
15 : 60lb. :: 1lb. : 4lb. at 11c.
etc. etc.

We find that the sum of the proportional parts, if linked as above, is 15lb., and if this be quadrupled, 60lb., the required compound, will be obtained; but the whole compound will be quadrupled by increasing each of the proportional parts in a four fold ratio.

2. How many ounces of gold, that is 16, 18, 20. and 24 carats fine, may be taken to form a mass of 72 ounces. 21 carats fine?

3. How many sheep worth 9, 12, 16, 18, and 24s. each, may be taken to form a flock of 125 sheep worth 15s. each?

---

343. Object of Problem 3? Rule? Explanation?

INVOLUTION.

344. A Power of a number is the product obtained by using the number *two* or *more times* as a *factor.*

Involution *is the process of raising a number to a power.*

The number involved is the 1*st power* of itself. It is also the *root* of the other powers (Art. 112, Notes 3 and 6).

345. The Index or Exponent of a power is a figure placed at the right and a little above the root *to show how many times it is used as a factor* (Art. 112, Note 4); thus,

$4 \times 4 = 16 = 4^2$, i.e. the 2d power or square of 4.
$4 \times 4 \times 4 = 64 = 4^3$, i.e. the 3d power or cube of 4.
$4 \times 4 \times 4 \times 4 = 256 = 4^4$, i.e. the 4th power of 4.
$4 \times 4 \times 4 \times 4 \times 4 = 1024 = 4^5$, i.e. the 5th power of 4.

Hence,

346. To involve a number to any required power,

Rule 1. *Write the index of the power over the root;* or,

Rule 2. *Multiply the number by itself, and (if a higher power than the second is required) multiply this product by the original number, and so on until the root has been taken as a factor as many times as there are units in the index of the required power.*

Ex. 1. What is the 3d power of 6?

Ans. $6^3 = 6 \times 6 \times 6 = 216$.

2. What is the 5th power of 3? Ans. 243.
3. What is the 4th power of 5?
4. What is the 8th power of 2?
5. What is the 2d power of 16? Ans. 256.
6. What is the 3 power of $\frac{4}{6}$?

$$\frac{4}{6} = \frac{2}{3};\ \left[\frac{2}{3}\right]^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2^3}{3^3} = \frac{8}{27}, \text{ Ans.}$$

---

344. What is a Power of a number? What is Involution? What is the number involved? **345.** What is the Index or Exponent of a power? **345.** Rule for involution? Second rule?

7. What is the 2d power of $\frac{6}{15}$?
8. What is the 3d power of .03? Ans. .000027.
9. What is the 4th power of .12?
10. What is the square of $2\frac{1}{4}$?

$$2\tfrac{1}{4}=\tfrac{9}{4};\ \text{and}\ (\tfrac{9}{4})^2=\tfrac{81}{16}=5\tfrac{1}{16},\ \text{Ans.}$$

11. What is the cube of $3\frac{1}{6}$?

NOTE 1. It will be observed that a mixed number is first reduced to an improper fraction, and a common fraction is reduced to its lowest terms, and then each term is involved separately. Also that the number of decimal places in the power of a decimal is equal to the number of decimal places in the root multiplied by the index of the power (Art. 171).

NOTE 2. The powers of a number greater than unity, are greater than the root, and the powers of a proper fraction are less than the root; thus, the cube of 2 is 8, which is greater than 2; and the square of $\frac{3}{2}$ is $\frac{9}{4}$, which is greater than $\frac{3}{2}$; but the square of $\frac{2}{3}$ is $\frac{4}{9}$, which is less than $\frac{2}{3}$.

347. To multiply different powers of the same number together:

RULE. *Add the indices of the factors together, and the sum will be the index of the product.*

12. Multiply the square of 3 by the cube of 3.

Ans. $3^2 \times 3^3 = 3^5$; i. e. $\overline{3 \times 3} \times \overline{3 \times 3 \times 3} = 3^5 = 243$.

13. Multiply 5^2 by 5^4. Ans. 5^6.

348. To involve a quantity that is already a power:

RULE. *Multiply the index of the given number by the index of the power to which it is to be raised.*

Thus, the 3d power of 2^2 is 2^6, for $2^2 = 2 \times 2$, and the 3d power of 2×2 is $\overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$.

14. What is the 4th power of 3^3? Ans. 3^{12}.
15. What is the 5th power of 2^4?

---

346. What is done with a mixed number? How is a common fraction involved? How many decimal places in the power of a decimal? If the root is greater than *one*, are its powers greater or less than the root? If the root is less than one? **347.** Rule for multiplying different powers of the same number together? **348.** Rule for involving a power?

349. To divide a power of a number by any other power of the same number:

RULE. *Subtract the exponent of the divisor from the exponent of the dividend.*

16. Divide 5^7 by 5^3. Ans. $5^7 \div 5^3 = 5^4$, for $5^7 \div 5^3 = \frac{5^7}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5 \times 5 \times 5 \times 5 = 5^4 = 625$.

17. Divide 8^9 by 8^7. 18. Divide 4^7 by 4^3.

EVOLUTION.

350. A ROOT of a number is one of the *equal* factors whose continued product is that number (Art. 112, Note 3).

EVOLUTION *or* EXTRACTING ROOTS *is the resolving of a quantity into as many equal factors as there are units in the index of the root.*

351. There are two methods of *indicating* a root, one by means of the *radical sign*, $\surd$, and the other by means of a *fractional index*.

The figure placed over the radical sign is the *index* of the root, and is always the same as the denominator of the fractional index; thus, the *cube root of* 8 is $\sqrt[3]{8}$ or $8^{\frac{1}{3}}$.

The *square root* of the *cube* of 4, or the *cube* of the *square root* of 4, is $\sqrt{4^3}$ or $4^{\frac{3}{2}}$.

If no number is over the radical sign, 2 is understood.

352. EVOLUTION is the reverse of INVOLUTION.

In Involution the root is given and the power required.

In Evolution the power is given and the root required.

---

349. Rule for dividing one power by another power of the same number? **350.** What is a Root of a number? What is Evolution? **351.** How many methods of indicating a root? What? What is the index of the root? What of the index 2?

353. *All* numbers can be *involved* to *any* required power, but comparatively *few* can be *evolved.*

Those numbers which can have their roots extracted are called *perfect* powers, and their roots are *rational* numbers. Numbers whose roots cannot be taken are called *imperfect* powers, and their roots are *irrational* or *surd* numbers.

A number may be a perfect power of one name or degree, and an imperfect power of another; thus, 16 is a perfect square, but an imperfect cube, whereas 27 is a perfect cube, but an imperfect square; again, 64 is a perfect square, cube, and sixth power.

354. Every power and every root of 1 is 1. There is no other number whose powers and roots are all alike.

The roots of a proper fraction are greater than the fraction, and the roots of any number greater than unity are less than the number; thus, $\sqrt{\frac{4}{9}} = \frac{2}{3}$, which is greater than $\frac{4}{9}$; $\sqrt[3]{\frac{27}{64}} = \frac{3}{4}$, which exceeds $\frac{27}{64}$; but $\sqrt{\frac{64}{36}} = \frac{8}{6}$, which is less than $\frac{64}{36}$; $\sqrt[3]{8} = 2$, which is less than 8.

EXTRACTION OF THE SQUARE ROOT.

355. TO EXTRACT THE SQUARE ROOT *of a number is to resolve it into two equal factors, i. e. to find a number which, multiplied into itself, will produce the given number.*

356. The *square* of a number consists of *twice* as many figures as the *root,* or of *one less* than twice as many; thus,

Roots,	1,	9,	10,	99,	100,	999.
Squares,	1,	81,	100,	9801,	10000,	998001.

Hence, to ascertain the number of figures in the square root of a given number,

---

353. Can all numbers be involved? Evolved? What are perfect powers? Rational roots? Imperfect powers? Irrational or surd roots? May a number be a perfect power of one degree and an imperfect power of another degree? A perfect power of several degrees? Illustrate. 154. What of 1? The roots of a proper fraction, are they greater or less than the fraction? The roots of a number greater than one? 355. To extract the square root of a number, what? 356. How many figures in the square of a number?

RULE. *Beginning at the right, point off the number into periods of two figures each, and there will be one figure in the root for each period of two figures in the square, and if there is an odd figure in the square there will be a figure in the root for that.*

Ex. 1. How large a square floor can be laid with 576 square feet of boards?

If we knew the length and breadth of a floor, we should find its area by multiplying the length by the breadth (Art. 101), or, in this example (since length and breadth are equal), by multiplying the length by itself. *But we are now to reverse this process, and, knowing the area, to find the length of one side.*

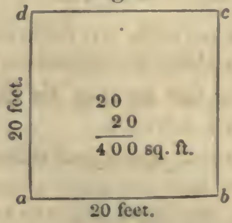
Since the number, 576, consists of *three* figures, its root will consist of *two* figures, *tens* and *units*, and the *square* of the tens must be found in the 5 (hundreds).

OPERATION.

$$\begin{array}{r} \dot{5}\,7\,\dot{6}\,(2\,4 \\ 4 \\ \hline 44)\,1\,7\,6 \\ 1\,7\,6 \\ \hline 0 \end{array}$$

Now the square of 2 (tens) is 4 (hundreds) and the square of 3 (tens) is 9 (hundreds); and, as 5 (hundreds) is less than 9 (hundreds) there can be but 2 (tens) in the root. Let us now construct a square, Fig. 1, each side of which shall be 2 tens (= 20 feet) in length. The area of this square is $20 \times 20 = 400$ square feet, which, deducted from 576 feet, will leave 176 square feet to be used in enlarging the floor. To preserve the square form, this addition must be made upon the 4 sides of the floor, or, more conveniently, equally upon 2 adjacent sides, as in Fig. 2. From the nature of the case, the 2 additions, *bm* and *cr*, are of a uniform breadth; and, if their *length* were known, we could determine their breadth by dividing their area, 176 feet, by their length (Art. 102). But we *do* know the length of *bh* + *cr*, viz. twice the tens of

Fig. 1.



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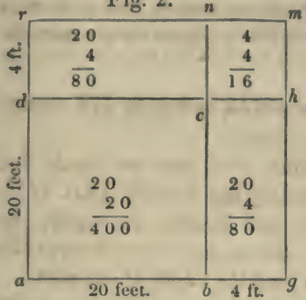
356. To ascertain the number of figures in a square root, Rule? Explain Ex. 1.

the root = 4 (tens or 40 ft.), and this is sufficiently near to the whole length of the additions to serve as a *trial divisor.* Now $176 \div 40$, or, what is the same in effect, $17 \div 4$, gives 4 feet for the breadth of the addition, and this added to the *trial divisor*, 40, or annexed to the 4 (tens) will give 44, the whole length of $bm + cr$, the *true divisor.* And $44 \times 4 = 176$; i. e. the length of the addition multiplied by its breadth gives its area.

Fig. 2.



It will be seen that every foot of board is used, and the floor is a square, each side of which is $20 + 4 = 24$ft. long, Ans.

357. The same species of reasoning applies, however many figures there may be in the root. Hence,

To Extract the Square Root of a number,

RULE. 1. *Separate the given number into periods of two figures each, by placing a dot over units, hundreds, etc.*

2. *Find the greatest square in the left-hand period and set its root at the right, in the place of a quotient in long division.*

3. *Subtract the square of this root figure from the left-hand period, and to the remainder annex the next period for a dividend.*

4. *Double the root already found for a* TRIAL DIVISOR, *and, omitting the right-hand figure of the dividend, divide and set the quotient as the next figure of the root. Also set it at the right of the trial divisor, and so form the* TRUE DIVISOR.

5. *Multiply the true divisor by this new root figure, and subtract the product from the dividend.*

6. *To the remainder annex the next period for a new dividend. double the part of the root already found for a trial divisor, and proceed as before until all the periods have been employed.*

---

357. Rule for extracting the square root of a number?

NOTE 1. The left-hand period *may* consist of but one figure.

NOTE 2. The trial divisor being smaller than the true divisor, the quotient is frequently too large, *and a smaller number must be set in the root.* This usually occurs when the addition to the square, *a c*, is wide, and, consequently, the square, *h n*, large; or, in other words, when the trial divisor is much less than the true divisor.

358. PROOF. *Square the root;* thus, in Ex. 1, $24^2 = 576$.

2. What is the square root of 401956?

OPERATION.

```
        4 0̇ 1 9̇ 5 6̇(6 3 4, Ans.
        3 6
 1 2 3) 4 1 9
          3 6 9
 1 2 6 4) 5 0 5 6
            5 0 5 6
                  0
```

3. What is the square root of 191844? Ans. 438.
4. What is the square root of 677329?
5. What is the square root of 67081?

OPERATION.

```
      6̇ 7 0̇ 8 1̇ (2 5 9, Ans.
      4
 4 5) 2 7 0
      2 2 5
 5 0 9) 4 5 8 1
        4 5 8 1
              0
```

In this example, the left-hand period consists of but *one* figure. So, also, the trial divisor, 4, is contained in 27 six times; and the 2d remainder, 45, equals the divisor; still, the true root figure is but 5.

6. What is the square root of 9765625? Ans. 3125.
7. What is the square root of 136161?
8. What is the square root of 42016324?
9. What is the square root of 43046721?
10. What is the square root of 22014864?
11. What is the square root of 1522756?
12. What is the square root of 18671041?
13. What is the square root of 6091024?

---

357. What is Note 1? Note 2? **358.** Proof? Explain Ex. 5?

14. What is the square root of 16777216?

OPERATION.

```
 1 6 7 7 7 2 1 6 ( 4 0 9 6, Ans.
 1 6
809 ) 7 7 7 2
      7 2 8 1
8186 ) 4 9 1 1 6
       4 9 1 1 6
               0
```

When a root figure is 0, as in this example, we simply annex 0 to the trial divisor, and bring down the next period to complete the new dividend.

15. What is the square root of 5764801?

16. What is the square root of 1048576?

17. What is the square root of 282475249?

NOTE 3. In extracting the root of a *decimal*, put the first point over *hundredths* and point toward the *right*, and if the last period is not full, annex 0.

18. What is the square root of .4096? Ans. .64.

19. What is the square root of .0625?

20. What is the square root of 39.0625? Ans. 6.25.

21. What is the square root of 6046.6176?

22. What is the square root of 5.6? Ans. 2.36+.

OPERATION.

```
    5.6 0 ( 2.3 6 +
    4
 4 3 ) 1 6 0
       1 2 9
 4 6 6 ) 3 1 0 0
         2 7 9 6
  4 7 2 ) 3 0 4 0 0.
```

If there is a remainder after employing all the periods in the given example, the operation may be continued at pleasure by annexing successive periods of ciphers, decimally; there will, however, in such examples, *always be a remainder;* for the right-hand figure of the dividend is *a cipher*, whereas the right-hand figure of the subtrahend is, *necessarily*, the right-hand figure of the square of some one of the nine significant figures, *the right-hand figure of the root and of the divisor being always alike.* Now, no one of these nine figures, squared, will give a number ending with a cipher; ∴, the last figure of the dividend and of the subtrahend being unlike, *there must be a remainder.*

23. What is the square root of 2? Ans. 1.41421+.

---

358. Explain Ex. 14. What is Note 3? Explain Ex. 22.

24. What is the square root of 20? Ans. 4.472+.
25. What is the square root of 316?
26. What is the square root of 31.6?

359. To extract the root of a common fraction, or of a mixed number:

RULE. *Reduce the fraction or mixed number to its simplest form, and then take the root of the numerator and denominator separately; or, if either term of the fraction, when reduced, is an imperfect square, reduce the fraction to a decimal* (Art. 173), *and then proceed as in the foregoing examples.*

27. What is the square root of $\frac{27}{48}$?

$$\sqrt{\tfrac{27}{48}} = \sqrt{\tfrac{9}{16}} = \tfrac{3}{4}, \text{ Ans.}$$

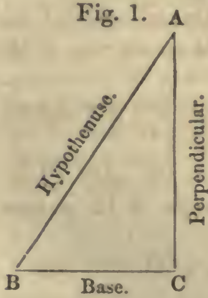
28. What is the square root of $\frac{75}{147}$? Ans. $\frac{5}{7}$.
29. What is the square root of $\frac{1008}{4375}$?
30. What is the square root of $20\frac{1}{4}$?

$$\sqrt{20\tfrac{1}{4}} = \sqrt{\tfrac{81}{4}} = \tfrac{9}{2} = 4\tfrac{1}{2}, \text{ Ans.}$$

31. What is the square root of $10\frac{6}{25}$?
32. What is the square root of $\frac{2}{3}$ of $\frac{32}{27}$?
33. What is the square root of $\frac{3}{7}$? Ans. .654+.
34. What is the square root of $\dfrac{2\frac{3}{4}}{7\frac{1}{3}}$?

APPLICATION OF THE SQUARE ROOT.

Fig. 1.



360. A TRIANGLE is a figure bounded by three straight lines.

A *right-angled triangle* has *one* of its angles a *right angle*, as at C.

The side opposite the right angle is called the *hypothenuse;* the other two sides are the *base* and *perpendicular.*

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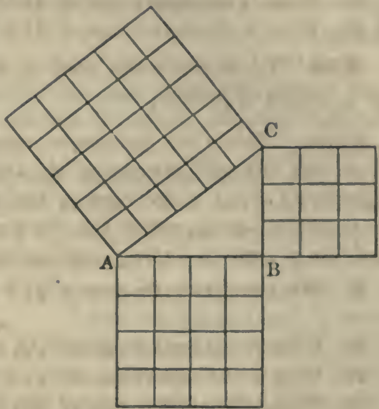
359. Rule for extracting the root of a common fraction or mixed number?
360. What is a Triangle? A right-angled triangle? Hypothenuse? Base?

Fig. 2.



The square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides. Also the square of either of the two sides which form the right angle is equal to the square of the hypothenuse diminished by the square of the other side. This will be seen by counting the small squares in the square of the hypothenuse and those in the squares of the other two sides. Hence,

1st. To find the hypothenuse when the base and perpendicular are given,

RULE. *Add the square of the base to the square of the perpendicular, and extract the square root of the sum.*

2d. To find either side about the right angle when the hypothenuse and the other side are given,

RULE. *From the square of the hypothenuse, subtract the square of the other given side, and extract the square root of the remainder.*

Ex. 1. The base of a right-angled triangle is 6 feet and the perpendicular is 8 feet; what is the hypothenuse?

$6^2 = 36$, $8^2 = 64$; $36 + 64 = 100$; $\sqrt{100} = 10$. Ans. 10 ft.

2. The hypothenuse of a right-angled triangle is 15 and the base is 12; what is the perpendicular?

$15^2 = 225$, $12^2 = 144$; $225 - 144 = 81$; $\sqrt{81} = 9$, Ans.

---

360. The square of the hypothenuse equals what? The square of one of the other sides? How may this appear? Rule for finding the hypothenuse? Base or Perpendicular? Explain Ex. 1.

3. A ladder resting upon the ground 21 feet from a house, just reaches a window which is 28 feet high; how long is the ladder?

4. A tree that was 64 feet high is broken off 24 feet high, the part broken off turning upon the stub as upon a hinge; at what distance from the bottom of the tree does the top strike the ground?

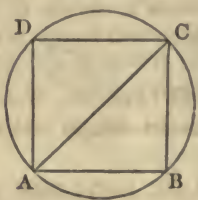
5. Two vessels sail from the same port, one due east 40 miles and the other due south 9 miles; how far apart are they?

6. A general has 9801 men; if he places them in a square, how many will there be in rank and file?

7. How many rods of fence will be required to inclose 640 acres of land in a square form?

8. A farmer sets out an orchard of 600 trees so that the number of rows is to the number of trees in a row as 2 to 3. The trees are 25 feet apart and no tree is within $12\frac{1}{2}$ feet of the fence; how many square feet of land in the field?

Fig. 3.



361. In figure 3 we have combined a *circle* (Art. 109), a *square* (Art. 101, Note), and two equal *right-angled triangles.* The line AC is the *diameter* of the circle, the *diagonal* of the square and the *hypothenuse* of each of the triangles. The square is said to be *inscribed* in the circle and the circle is *circumscribed* about the square.

The diameter of any circle is to its circumference in the ratio of 1 to 3.141592, nearly; hence the diameter multiplied by 3.141592 will give the circumference, and the circumference divided by 3.141592 will give the diameter.

The area of a circle may be found by multiplying the square of its diameter by .785398, nearly, and if the area is divided by .785398, the quotient will be the square of the diameter.

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361. What does Fig. 3 represent? What is the line AC? What is said of the square? Of the circle? Ratio of diameter to circumference? How is circumference found when diameter is given? Diameter when circumference is given? Area of a circle, how found? Diameter, when area is given?

362. *Similar figures* are figures that are of precisely the same form, whether large or small.

The *areas* of all similar figures are to each other as the *squares* of their corresponding lines.

9. What is the diameter of a circular pond which shall contain 25 times as much area as one 8 rods in diameter? Ans. 40rd.

10. The area of a triangle is 24 square inches and one side of it is 8 inches; what is the corresponding side of a similar triangle containing 96 square inches?

11. What is the side of a square that shall contain 36 times as much area as one whose side is 5 feet?

12. What is the side of a square equal in area to a circle 100 feet in diameter? Ans. 88.622sq. ft.

13. A circular field contains 10 acres; what is the length of its diameter?

14. What is the difference in the expense of fencing a circular 10-acre lot and one of the same area in a square form, the fence costing 75c. per rod?

15. If a pipe 3 inches in diameter will empty a cistern in 8 minutes, what is the diameter of the pipe which will empty it in 18 minutes?

16. The area of a rectangular piece of land (Art. 101, Note) is 50 acres, and the length of the piece is to its breadth as 5 to 1; what are the length and breadth?

17. A room is 16ft. long, 12ft. wide, and 9ft. high; what is the distance from one lower corner to the opposite upper corner? Ans. 21.931+ft.

18. The diameter of a circle is 10 inches; how many inches in the side of the inscribed square? Ans. $\sqrt{50} = 7.071+$.

SOLUTION. By figure 3 it is seen that the diameter of the circle is the hypothenuse of a right-angled triangle whose other sides are equal to each other; ∴ *the square of either side of the inscribed square is one half of the square of the diameter.*

19. What is the side of the greatest square stick of timber that can be hewn from a log 18 inches in diameter?

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362. What are similar figures? The ratio of the areas of similar figures?

EXTRACTION OF THE CUBE ROOT.

363. To EXTRACT THE CUBE ROOT *of a number is to resolve it into 3 equal factors; i. e. to find a number which, multiplied into its square, will produce the given number.*

364. The *cube* of a number consists of *three times* as many figures as the *root*, or of *one* or *two less* than three times as many; thus,

Roots,	1,	9,	10,	99,	100,	999.
Cubes,	1,	729,	1000,	970299,	1000000,	997002999.

Hence, to ascertain the number of figures in the cube root of a given number,

RULE. *Beginning at the right, point off the number into periods of three figures each, and there will be one figure in the root for each period of three figures in the cube, and if there are one or two figures besides full periods in the cube, there will be a figure in the root for this part of a period.*

Ex. 1. Suppose we have 74088 blocks of wood, each a cubic inch in size and form, how large a cubical pile can be formed by packing these blocks together?

OPERATION.

		74088	(42 Root.
Trial divisor,	4800	64	
	240	10088	Dividend.
	4	10088	
True divisor,	5044	0	

As there are *two* periods, the root must consist of *two* figures, tens and units; and we seek the cube of the tens in the left-hand period; the greatest cube in 74 is 64, whose root is 4. We place the root, 4, at the right of the number, and, having subtracted the cube, 64, from the left-hand period, we annex the next period to the remainder, 10, making 10088 for a dividend.

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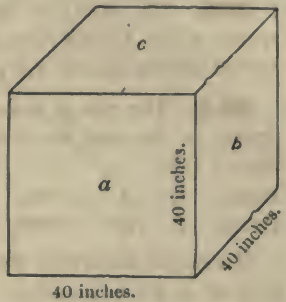
363. To extract the cube root of a number, what? **364.** How many figures in the cube of a number? To ascertain the number of figures in a cube root, Rule? Explain Ex. 1.

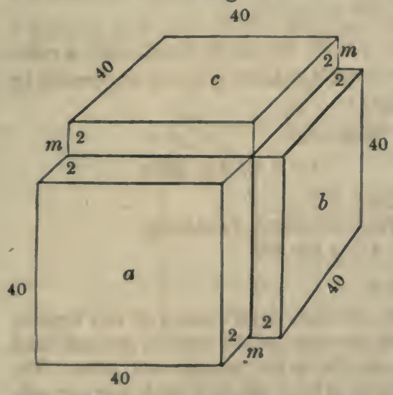
Thus, by using 64000 of the blocks, a cube is formed (Fig. 1) whose edge is 40 inches and whose contents are 64000 solid inches, and there are 10088 blocks remaining, with which to enlarge the cubic pile already formed.

Fig. 1.



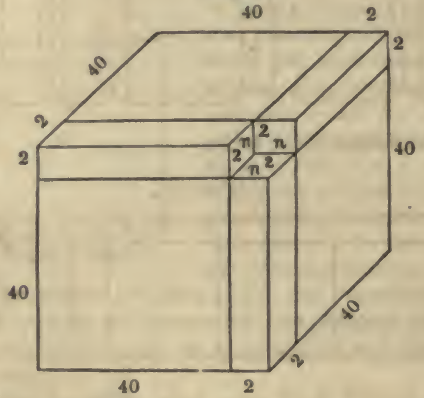
In enlarging this pile and preserving the cubic form, the additions must be made upon each of the 6 faces, or, more conveniently, equally upon any 3 adjacent faces, e. g. *a*, *b*, and *c*, as in Fig. 2. What may be the thickness of the addition? By dividing the contents of a rectangular solid by the area of one face, we obtain the thickness (Art. 105); now, the remaining 10088 solid inches are the contents, and the sum of the areas of the 3 square faces, *a*, *b*, and *c*, is sufficiently near the area to be covered by the additions to form a *trial divisor;* for the 3 additions, *a*, *b*, and *c* (Fig. 2), are the same as one solid 40 inches wide, 3 times 40 inches long, and of the thickness determined by trial. The area of these 3 faces is the square of 4 (tens), which is 16 (hundreds), multiplied by 3, which gives 4800; i. e., to obtain a trial divisor, we square the root figure and annex 00 (because the root figure is *tens*) for the area of one face, and then multiply this area by 3. Dividing 10088 by 4800, we obtain the quotient 2, *for the thickness of the additions*, i. e. for the *unit* figure of the root. Having made these additions, as in Fig. 2, we see that the pile does not retain the cubic form, three corners, *m*, *m*, and *m*, being vacant. Each of these corners is 40 inches long, 2 inches wide and 2 inches thick; i. e. the

Fig. 2.



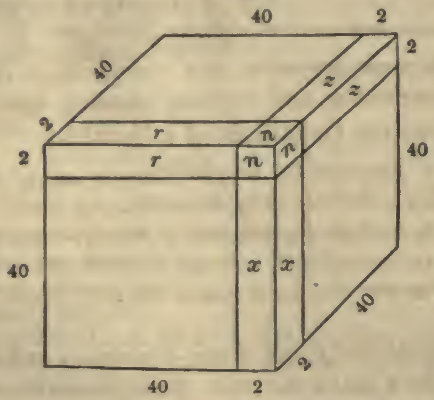
area covered to the depth of two inches by filling the vacant corners in Fig. 2, as seen in Fig. 3, is $2 \times 40 \times 3 = 240$ square inches; and still there is a vacant corner *n*, *n*, *n*, as seen in

Fig. 3,



which is a cube of 2 inches on each edge; i. e. it is a solid 2

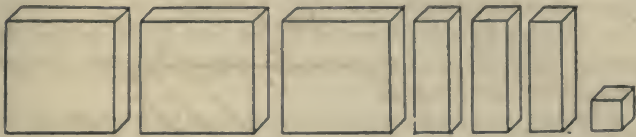
Fig. 4.



inches thick, (the *common* thickness of all the additions), covering $2 \times 2 = 4$ square inches, as seen in Fig. 4.

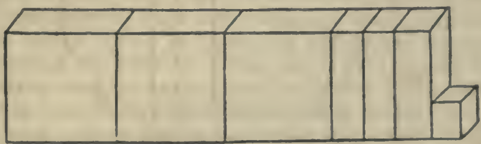
Now, if the several additions made in Figs. 2, 3, and 4, be spread out upon a plane, as in

Fig. 5,



or, in a consolidated form, as in

Fig. 6,



it will be readily seen that their collective solidity will be obtained by multiplying the entire area which they cover, $(40 \times 40 \times 3 + 40 \times 2 \times 3 + 2 \times 2 = 5044$ square inches), by their *common* thickness, 2, which will give 10088 solid inches; $\therefore$ a cube is formed (Fig. 4) whose edge is $40 + 2 = 42$ inches, and no blocks remain.

365. If there are more than two figures in the root, the same relations subsist, and the same reasoning applies. Hence,

To extract the Cube Root of a Number,

RULE. 1. *Separate the number into periods of three figures each by setting a dot over units, thousands, etc.*

2. *Find by trial the greatest cube in the left-hand period, place its root as in square root, subtract the cube from the left-hand period and to the remainder annex the next period for a dividend.*

3. *Square the root figure, annex two ciphers and multiply this result by 3 for a* TRIAL DIVISOR; *divide the dividend by the trial divisor and set the quotient as the next figure of the root.*

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365. Rule for extracting the cube root of a number?

4. *Multiply this root figure by the part of the root previously obtained, annex one cipher and multiply this result by three ; add the last product and the square of the last root figure to the trial divisor, and the* SUM *will be the* TRUE DIVISOR.

5. *Multiply the true divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

6. *Find a new trial divisor, and proceed as before, until all the periods have been employed.*

NOTE 1. The notes in Art. 357, with slight modifications, are equally applicable here.

NOTE 2. If the root consists of *three* figures it is plain that the cube, as completed in Fig. 4, must be enlarged just as Fig. 1 has already been enlarged. Hence, the new trial divisor will consist of 3 faces of Fig. 4; but the *true divisor already found* is the sum of the significant figures in these 3 faces, except *one* face each of *rr*, *xr*, and *zz*, and *two* faces of the little corner cube, *nnn* ; moreover, the number directly above the true divisor (in the operation) represents *one* face of *nnn*, and the number above *that* represents the sum of *one face each* of the 3 long corner blocks, *rr*, *xr*, and *zz* ; hence, *to find the next trial divisor*, we have only to *add the true divisor already found to* TWICE *the number above it, and* ONCE *the number above* THAT, *and to the sum annex two ciphers.* When there are many root figures this process is shorter than to square so much of the root as has been found, annex two ciphers, and multiply by 3, as directed in the 3d paragraph of the rule.

Ex. 2. What is the cube root of 21024576?

		$2\dot{1}02\dot{4}57\dot{6}$(276, Ans.	
1st Trial Divisor $= 20^2 \times 3 =$	1200	8	
$20 \times 7 \times 3 =$	420		
$7^2 =$	49		
1st True Divisor $=$	1669	13024	1st Dividend.
2d Trial Divisor $= 270^2 \times 3 =$	218700	11683	
$270 \times 6 \times 3 =$	4860		
$6^2 =$	36		
2d True Divisor $=$	223596	1341576	2d Dividend.
		1341576	

The 1st trial divisor is contained 10 times in the dividend, yet the root figure is only 7. The true root figure can never exceed 9, *and must in all cases be found by trial.*

Squaring 20 gives the same result as squaring 2 and annexing 00, as directed in the rule, 3d paragraph.

3. What is the cube root of 67917312?

	$\dot{6}79\dot{1}73\dot{1}2$ (408, Ans.
480000	64
9600	—
64	
489664	3917312
	3917312

In this example, the 1st trial divisor, 4800, is larger than the 1st dividend, 3917; ∴ we annex 0 to the root, 00 to the 1st trial divisor for the 2d trial divisor, and bring down the next period to complete a new dividend. The rule, followed literally, will give the same result.

NOTE 3. Prepare fractions and mixed numbers as directed in square root (Art. 359).

What is the value of the following expressions:

4. $\sqrt[3]{2803221}$? Ans. 141.
5. $\sqrt[3]{3176523}$?
6. $\sqrt[3]{382657176}$?
7. $\sqrt[3]{8024024008}$?
8. $\sqrt[3]{387420489}$?
9. $\sqrt[3]{134217728}$?
10. $\sqrt[3]{5}$? Ans. 1.709 +.
11. $\sqrt[3]{36.926037}$? Ans. 3.33.
12. $\sqrt[3]{10077.696}$?
13. $\sqrt[3]{40.353607}$?
14. $\sqrt[3]{166\frac{3}{8}}$? Ans. $5\frac{1}{2}$.
15. $\sqrt[3]{561\frac{33}{64}}$?
16. $\sqrt[3]{4\frac{1}{2}}$? Ans. 1.65 +.
17. $\sqrt[3]{43\frac{5}{7}}$?

APPLICATION OF THE CUBE ROOT.

366. Bodies which are of precisely the same form are *similar* to each other, and *the solid contents of similar bodies are to each other as the cubes of their corresponding lines*, and conversely, *the corresponding lines are to each other as the cube roots of the contents.*

Ex. 1. If an iron ball 5 inches in diameter weighs 16 pounds, what is the weight of a ball 30 inches in diameter?

$5^3 : 30^3 :: 16 :$ Ans., or $1^3 : 6^3 :: 16 :$ Ans.; i. e. $1 : 216 ::$ 16lb. : 3456lb., Ans.

---

365. What is Note 1? Note 2? Explain Ex. 2. Ex. 3. What is Note 3? **366.** What are similar bodies? The ratio of the contents of similar bodies?

2. If a ball 6 inches in diameter weighs 27 pounds, what is the diameter of a ball that weighs 64 pounds?

$\sqrt[3]{27} : \sqrt[3]{64} :: 6$in. : Ans.; i. e. $3 : 4 :: 6$in. : 8in., Ans.

3. How many bullets $\frac{1}{4}$ of an inch in diameter will be required to make a ball 1 inch in diameter?

4. If a globe of gold 1 inch in diameter is worth $100, what is the diameter of a globe worth $6400?

5. Suppose the diameter of the earth is 7912 miles, and that it takes 1404928 bodies like the earth to make one as large as the sun, what is the diameter of the sun?

6. A bin is 8 feet long, 4 feet wide, and 2 feet deep; what is the edge of a cubical box that will hold the same quantity of grain?

7. If a stack of hay 24 feet high weighs 27 tons, what is the hight of a similar stack which weighs 8 tons? Ans. 16ft.

8. If a bell 4 inches high, 3 inches in diameter, and $\frac{1}{4}$ of an inch thick weighs 1 lb., what are the dimensions of a similar bell that weighs 27 lb.?

9. If a loaf of sugar 10 inches high weighs 8 lb., what is the hight of a similar loaf weighing 1 lb.?

ARITHMETICAL PROGRESSION.

367. Any series of numbers *increasing* or *decreasing* by *a common difference* is in ARITHMETICAL PROGRESSION;

thus, 2, 5, 8, 11, 14, 17, etc. is an ascending series,

and 35, 30, 25, 20, 15, 10, etc. is a descending series.

The several numbers forming a series are called TERMS; the first and last terms, EXTREMES; the others, MEANS. The difference between two successive terms is the COMMON DIFFERENCE.

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367. When is a series of numbers in Arithmetical Progression? How many kinds of series? What? What are the Terms of a series?

In an arithmetical series 5 particulars claim special attention, viz. the first term, last term, common difference, number of terms, and sum of all the terms; and these are so related to each other that if any *three* of them are given the other *two* can be found.

368. In an ascending series, let 6 be the first term and 5 the common difference;

Then $6 = 1\text{st term.}$

$6 + 5 = 11 = 2\text{d term.}$

$6 + 5 + 5 = 6 + 2 \times 5 = 16 = 3\text{d term.}$

$6 + 5 + 5 + 5 = 6 + 3 \times 5 = 21 = 4\text{th term.}$

Thus we see that, in an ascending series, the *second* term is found by adding the common difference *once* to the *first* term; the *third* term, by adding the common difference *twice* to the *first* term, etc.

A similar explanation may be given when the series is descending. Hence,

369. PROBLEM 1. To find the last term, the first term, common difference, and number of terms being given:

RULE. *Multiply the common difference by the number of terms less* 1*; add the product to the first term if the series is ascending, or subtract the product from the first term if the series is descending, and the sum or difference will be the term sought.*

Ex. 1. If the first term of an ascending series is 5, the common difference 4, and the number of terms 7, what is the last term? $5 + 6 \times 4 = 29$, Ans.

2. The first term of a descending series is 47 and the common difference 8; what is the 6th term?

$47 - 5 \times 8 = 7$, Ans.

3. What is the amount of $100, at 6 per cent., simple interest, for 25 years?

---

367. What are the Extremes of a series? Means? Common Difference? How many particulars claim special attention? What are they? How many of them must be given? **368.** How is an ascending series formed? How a descending series? **369.** Object of Problem 1? Rule?

370. Problem 2. To find the common difference, the extremes and number of terms being given.

By inspecting the formation of the series in Art. 368, it will be seen that *the difference between the extremes is equal to the common difference multiplied by* 1 *less than the number of terms;* e. g. the difference between the 1st and 4th terms ($21 - 6 = 15$), is the sum of 3 *equal* additions; $\therefore$ this difference, divided by 3 ($15 \div 3 = 5$), gives *one* of these additions, i. e. the common difference. Hence,

Rule. *Divide the difference of the extremes by the number of terms less one, and the quotient will be the common difference.*

Ex. 1. The extremes of an arithmetical series are 3 and 38, and the number of terms is 8; what is the common difference?

$$\frac{38-3}{8-1} = \frac{35}{7} = 5, \text{ Ans.}$$

2. A man has 6 sons whose ages form an arithmetical series; the youngest is 2 years old and the oldest 22; what is the difference of their ages? Ans. 4 yr.

3. The amount of $100 at simple interest for 10 years is $160; what is the rate per cent.?

371. Problem 3. To find the number of terms, the extremes and common difference being given.

By Art. 368 it is evident that the difference of the extremes is the common difference multiplied by one less than the number of terms. Hence, conversely,

Rule. *Divide the difference of the extremes by the common difference, and the quotient, increased by* 1, *is the number of terms.*

Ex. 1. The extremes of an arithmetical series are 3 and 31 and the common difference is 4; what is the number of terms?

$$\frac{31-3}{4} + 1 = \frac{28}{4} + 1 = 7 + 1 = 8, \text{ Ans.}$$

2. The common difference in the ages of the children in a family is 2 years; the youngest is 1 year old and the oldest 19; how many children in the family?

372. Problem 4. To find the sum of a series, the extremes and number of terms being given.

The sum of the extremes is equal to the sum of any two terms that are equally distant from the extremes; thus, in the series, 3, 5, 7, 9, 11, 13, we have

$$\begin{array}{l} 1\text{st} + 6\text{th} = 2\text{d} + 5\text{th} = 3\text{d} + 4\text{th}. \\ \;\;3 + 13 \;= \;5 + 11 \;= 7 + 9 = 16; \end{array}$$

and ∴ the sum of all terms is $16 \times 3 = 48$. Hence,

Rule. *Multiply the sum of the extremes by half the number of terms, and the product is the sum of the series.*

Ex. 1. The extremes of a series are 3 and 39 and the number of terms is 10; what is the sum of the series?

$$3 + 39 = 42;\ 10 \div 2 = 5;\ 42 \times 5 = 210, \text{ Ans.}$$

2. How many strokes does a clock strike in 12 hours?

GEOMETRICAL PROGRESSION.

373. Any series of numbers *increasing* or *decreasing* by *a common ratio* is in Geometrical Progression;

thus, 2, 6, 18, 54, 162, etc. is an ascending series,
and 64, 32, 16, 8, 4, etc. is a descending series.

In the above, 3 is the ratio in the 1st series and ½ in the 2d.

The first term, last term, ratio, number of terms, and sum of all the terms are so related to each other that if any *three* of them are given the other *two* can be found.

374. In a series, let 2 be the first term, and 4 the ratio;

$$\begin{array}{lrl} \text{Then} & 2 &= 1\text{st term.} \\ & 2 \times 4 = 8 &= 2\text{d term.} \\ & 2 \times 4 \times 4 = 2 \times 4^2 = 32 &= 3\text{d term.} \\ & 2 \times 4 \times 4 \times 4 = 2 \times 4^3 = 128 &= 4\text{th term.} \end{array}$$

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370. Object of Problem 2? Rule? **371.** Object of Problem 3? Rule. **372.** Object of Problem 4? Rule? **373.** What constitutes a series in Geometrical Progression? How many kinds of series? What? How many particulars claim attention? What? How many of them must be given?

In forming the foregoing series we see that the *second* term is found by multiplying the *first* term by the *ratio;* the *third* term, by multiplying the *first* by the *square* of the ratio; the *fourth*, by multiplying the *first* by the *cube* of the ratio, *the index of the power of the ratio always being one less than the number of the term sought.* A similar explanation may be given when the series is descending. Hence,

375. PROBLEM 1. To find the last term, the first term, ratio, and number of terms being given,

RULE. *Multiply the first term by that power of the ratio whose index is equal to the number of terms preceding the required term, and the product will be the term sought.*

Ex. 1. The first term of a geometrical series is 4, the ratio 3, and the number of terms 6; what is the last term?

$6-1=5$; $3^5=243$; and $243\times4=972$, Ans.

2. The 1st term is 3, and the ratio $\frac{1}{2}$; what is the 5th term?

$5-1=4$; $(\frac{1}{2})^4=\frac{1}{16}$; and $\frac{1}{16}\times3=\frac{3}{16}$, Ans.

3. The 1st term is 5, the ratio 1.06; what is the 4th term?

Ans. 5.95508.

4. What is the amount of \$10 at compound interest for 4 years at 5 per cent. per annum?

5. Supposing money at compound interest to double once in 12 years, to what will \$100 amount in 72 years?

Ans. \$6400.

376. Since the last term is obtained (Art. 374) by multiplying the first term by that power of the ratio whose index is equal to the number of terms less one, so, conversely,

PROBLEM 2. To find the ratio, the extremes and number of terms being given:

RULE. *Divide the last term by the first, and the quotient will be that power of the ratio whose index is one less than the number of terms; the corresponding root of the quotient will therefore be the ratio.*

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374. How is an ascending series formed? A descending series? **375.** Object of Problem 1? Rule? **376.** Object of Problem 2? Rule?

Ex. 1. The first term in a geometrical series is 2, the last term 250, and the number of terms 4; what is the ratio?

$250 \div 2 = 125$; $4 - 1 = 3$; and $\sqrt[3]{125} = 5$, Ans.

2. The Extremes are 3 and 48, and the number of terms 3; what is the ratio? Ans. 4 or $\frac{1}{4}$.

3. The extremes are 3 and 243, and the number of terms 5; what is the ratio?

377. PROBLEM 3. To find the sum of a series, the extremes and ratio being given.

Having a series given, e. g. 2, 10, 50, 250, 1250, 6250, *multiply* each term *except the last* by the *ratio*, 5; thus,

Given series,	2,	10,	50,	250,	1250,	[6250],
Product by 5,		10,	50,	250,	1250,	6250;

and we shall evidently form a *new* series like the *old*, except the first term of the *old* is not found in the *new*. Now, if the *old* except *the last term* be *subtracted* from the *new*, the *remainder will be the difference of the extremes in the old series* the other terms in the two series canceling each other; the remainder will also be 4 times the *sum* of all the terms except the last in the old series; for *once* a series from 5 *times* a series *must* leave 4 *times* the series; $\therefore \frac{1}{4}$ of this remainder *plus the last term must be the sum of all the terms in the old series;* but 4 is the ratio less 1.

A similar explanation is always applicable. Hence,

RULE. *Divide the difference of the extremes by the ratio less one, and to the quotient add the greater extreme.*

Ex. 1. The extremes are 2 and 486, and the ratio 3; what is the sum of the series?

$486 - 2 = 484$; $3 - 1 = 2$; $484 \div 2 = 242$; and $242 + 486 = 728$, Ans.

2. The extremes are 4 and 5184, and the ratio 6; what is the sum of the series?

3. What debt will be discharged by 12 monthly payments, the 1st payment being \$1, the 2d \$2, and so on in a geometrical series?

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377. Object of Problem 3? Rule?

ANNUITIES.

378. AN ANNUITY is a sum of money payable annually, or at any regular period, either for a limited time or forever.

An annuity is in *arrears* when the installments remain unpaid after they are due.

The AMOUNT of an annuity in arrears is the interest of the unpaid installments added to their sum.

379. PROBLEM 1. To find the amount of an annuity in arrears, at simple interest.

Ex. 1. An annuity of $100 per annum has remained unpaid 4 years; what is its amount? Ans. $436.

The 4th payment is due to-day and is worth just $100; the 3d payment due 1 year ago is worth $106; the 2d payment due 2 years ago is worth $112; and the 1st payment due 3 years ago is worth $118. But these numbers, $100, $106, $112, and $118, are in arithmetical progression. Hence,

RULE. *Find the last term of the series by Art.* 369, *and the sum of the series by Art.* 372.

2. Purchased a farm for $5000, agreeing to pay for it in 5 equal annual installments; the 5 years having elapsed without any payment being made, what is now due, allowing simple interest? Ans. $5600.

3. A salary of $600 per annum is in arrears for 8 years; to what does it amount, allowing simple interest at 7 per cent.?

380. PROBLEM 2. To find the amount of an annuity in arrears at compound interest.

Ex. 1. What is the amount of $1 annuity, per annum, in arrears for 3 years, at 6 per cent. compound interest?

The 3d instalment becoming due to-day, is worth just $1; the 2d having been due 1 year, is worth $1.06; and the 1st having

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378. What is an Annuity? When is an annuity in arrears? What is the Amount of an annuity? **379.** Object of Problem 1? Rule? **380.** Problem 2?

been due 2 years, is worth $1.1236; $\therefore$ \$1 + \$1.06 + \$1.1236 = \$3.1836, the sum sought. But these numbers are in geometrical progression. Hence,

RULE 1. *Find the last term of the series by Art. 375, and the sum of the series by Art. 377;* or,

RULE 2. *Multiply the amount of* \$1, *found in the following table, by the annuity, and the product will be the required amount.*

TABLE,

Showing the amount of the annuity of \$1, £1, etc., at 4, 5, 6, and 7 per cent. compound interest, from 1 to 20 years.

Years.	4 per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	Years.
1	1.000000	1.000000	1.000000	1.000000	1
2	2.040000	2 050000	2.060000	2.070000	2
3	3.121600	3.152500	3.183600	3.214900	3
4	4.246464	4.310125	4.374616	4.439943	4
5	5.416323	5.525631	5.637093	5.750739	5
6	6.632975	6.801913	6.975319	7.153291	6
7	7.898294	8.142008	8.393838	8.654021	7
8	9 214226	9.549109	9.897468	10.259803	8
9	10.582795	11.026564	11.491316	11.977989	9
10	12.006107	12.577893	13.180795	13.816448	10
11	13.486351	14.206787	14.971643	15.783599	11
12	15.025805	15.917127	16 869941	17.888451	12
13	16.626838	17.712983	18.882138	20.140643	13
14	18.291911	19.598632	21.015066	22.550488	14
15	20.023588	21.578564	23.275970	25.129022	15
16	21.824531	23.657492	25.672528	27.888054	16
17	23.697512	25.840366	28.212880	30.840217	17
18	25.645413	28.13[illegible]85	30.905653	33.999032	18
19	27.671229	30.53[illegible]4	33.759992	37 378965	19
20	29.778079	33.065[illegible]4	36.785591	40.995492	20

2. What is the amount of an annual salary of \$1000, in arrears for 5 years, at 6 per cent.? Ans. \$5637.093.

3. What is the amount of an annual rent of 100£, in arrears for 15 years, at 5 per cent.?

Ans. 2157.8564£ = 2157£ 17s. 1d. 2qr.

4. What is the amount of an annual pension of \$500, in arrears for 12 years, at 6 per cent.?

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380. 1st Rule? 2d Rule?

PERMUTATIONS.

381. Permutation is the arranging of a given number of things in every possible order of succession.

382. Problem. To find the number of permutations of a given number of things.

The single letter, a, can have but 1 position, i. e. it cannot stand either before or after itself; the 2 letters, a and b, furnish the 2 permutations,

$\left\{\begin{matrix} a\ b \\ b\ a \end{matrix}\right\}$, the number of which is expressed by the product of $1 \times 2 = 2$; and if a 3d letter, c, be introduced, we have $\left\{\begin{matrix} c\ a\ b, & c\ b\ a \\ a\ c\ b, & b\ c\ a \\ a\ b\ c, & b\ a\ c \end{matrix}\right\}$; i. e. the new letter, c, may stand 1st, 2d, or 3d in each of the 2 permutations of a and b; hence the number of permutations of 3 things is expressed by the product, $1 \times 2 \times 3 = 6$. If a 4th letter, d, be taken, it may stand as 1st, 2d, 3d, or 4th, in each of the 6 permutations of a, b, and c, and, of course, furnish 4 times $6 = 1 \times 2 \times 3 \times 4 = 24$ permutations.

By the above, it is evident that the number of permutations

$$\begin{array}{lrr} \text{Of 1 thing} & = & 1 \\ \text{Of 2 things} = & 1 \times 2 = & 2 \\ \text{Of 3 things} = & 1 \times 2 \times 3 = & 6 \\ \text{Of 4 things} = & 1 \times 2 \times 3 \times 4 = & 24 \end{array}$$

and so on to any extent. Hence,

Rule. *Form the series of numbers,* 1, 2, 3, 4, *etc., up to the number of things to be permuted, and their continued product will be the number of permutations.*

Ex. 1. How many different integral numbers may be expressed by writing the 9 significant digits in succession, each figure to be taken once, and but once, in each number?

Ans. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880$.

2. In how many different orders may a family of 10 persons seat themselves around the tea table?

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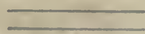
381. What is Permutation? **382.** Object of the Problem? Rule?

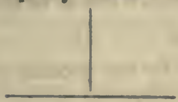
MENSURATION.

383. MENSURATION is the art of measuring lines, surfaces, and solids.

The principles are all Geometrical, and are very numerous. A few only of the more simple are here presented.

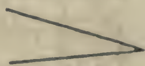
384. Two *parallel* lines are everywhere equally distant from each other.





When two lines meet so as to form *equal* angles, the lines are *perpendicular* to each other and the angles are *right angles.* A right angle contains 90°.

An *acute angle* is an angle of less than 90°.



An *obtuse angle* is an angle of more than 90°.

Two lines are *oblique* to each other when they meet so as to form *acute* or *obtuse angles*, and the angles are *oblique angles.*

385. A TRIANGLE is a plane figure which is bounded by three lines.

The *base* of a triangle (or any other figure) is the side on which it is supposed to stand. The *altitude* of a triangle is the perpendicular distance from the angle opposite the base to the base, or to the base extended.

386. PROBLEM 1. To find the area of a triangle:

RULE. *Multiply the base by half the altitude.*

Ex. 1. The base of a triangle is 7 inches and the altitude 8 inches; what is its area? Ans. 28sq. in.

2. The base is 8ft. and the hight 11ft.; what is the area?

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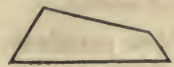
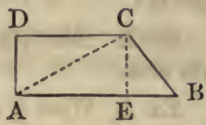
383. What is Mensuration? **384.** What of two parallel lines? What is a right angle? An acute angle? Obtuse angle? What are oblique lines? Oblique angles? **385.** What is a Triangle? Its base? Its altitude? **386.** Rule for finding its area?

387. A QUADRILATERAL or QUADRANGLE is a plane figure, having four sides and four angles.

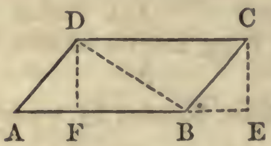
There are three kinds of quadrilaterals, viz.:



1st. *Trapeziums, none* of whose sides are *parallel ;*



2d. *Trapezoids,* as A B C D, only *one* pair of whose sides are *parallel ;* and,



3d. *Parallelograms, each* pair of whose opposite sides are *parallel,* as A B C D, or F E C D.

The *diagonal* of a figure is a line which joins two opposite angles, as A C in the above *trapezoid,* and B D in the *parallelogram.* The *altitude* of a trapezoid or parallelogram is the perpendicular between two parallel sides.

388. PROBLEM 2. To find the area of a trapezium :

RULE. *Draw a diagonal dividing the trapezium into two triangles, and find the area of each triangle by Problem* 1. *The sum of these triangles will be the area of the trapezium.*

Ex. What is the area of a trapezium, one of whose diagonals is 20 inches, and the length of the perpendiculars let fall upon it, from the other angles of the trapezium, 6 and 8 inches?

Ans. 140sq. in.

389. PROBLEM 3. To find the area of a trapezoid :

RULE. *Multiply the half sum of the parallel sides by the altitude, and the product will be the area.*

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387. What is a Quadrilateral? How many kinds? What is a trapezium? Trapezoid? Parallelogram? What is the diagonal of a figure? Altitude of a trapezoid? Of a parallelogram? **388.** Rule for finding the area of a trapezium? **389.** Rule for finding the area of a trapezoid?

Ex. 1. The parallel sides of a trapezoid are 10 and 12 feet, and its altitude is 6 feet; what is its area? Ans. 66sq. ft.

2. What is the area of a board, whose length is 10ft., the wider end being 2ft. and the narrower 18 inches in width?

390. PROBLEM 4. To find the area of a parallelogram:

RULE. *Multiply the base by the altitude, and the product is the area.*

Ex. 1. What is the area of a rectangular field, whose length is 40 rods, and altitude or width 8 rods? Ans. 2 acres.

2. The base of a parallelogram is 6 feet, and the altitude 4 feet; what is its area?

391. A POLYGON is a plain figure bounded by *straight lines.*

NOTE 1. Three straight lines, at least, are required to bound a polygon.

The lines which bound a polygon, taken together, are called the *perimeter* of the polygon.

A polygon of 5 sides is called a *pentagon;* of 6, a *hexagon;* 7, a *heptagon;* 8, an *octagon;* 9, a *nonagon;* 10, a *decagon;* 11, an *undecagon;* 12, a *dodecagon;* etc.

NOTE 2. A polygon may be divided into triangles by drawing diagonals, and then its area may be found by Problem 1.

392. PROBLEM 5. To find the area of a circle when the radius and circumference are given (Art. 109 and 361):

RULE 1. *Multiply the circumference by half the radius;* or,

RULE 2. *Multiply the square of the radius by* 3.141592, *and the product is the area.*

Ex. 1. What is the area of a circle, whose radius is 6 and circumference 37.699104? Ans. 113.097312.

2. What is the area of a circle whose radius is 10?

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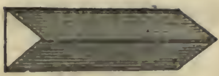
390. Rule for finding the area of a parallelogram? 391. What is a Polygon? Note 1? Perimeter of a polygon? Name the different polygons? 392. Rule for finding the area of a circle? Second Rule?



393. A PRISM is a solid that has two similar, equal, parallel faces, called *bases*, and all its other faces parallelograms.

NOTE. A prism is triangular, quadrangular, pentagonal, etc., according as its bases are triangles, quadrangles, pentagons, etc.

A CYLINDER is a round body whose diameter is the same throughout its entire length, and whose ends or bases are equal, parallel circles.

394. PROBLEM 6. To find the surface of a prism or cylinder:

RULE. *Multiply the perimeter or circumference of the base by the length of the solid, and to the product add the area of the two ends.*

Ex. 1. What is the surface of a prism, whose length is 10 inches and base 4 inches square? Ans. 192sq. in.

2. What is the surface of a cylinder, whose length is 20 feet and diameter 4 feet?

395. PROBLEM 7. To find the solid contents of a prism or cylinder:

RULE. *Multiply the area of the base by the altitude.*

Ex. 1. What are the contents of a cylinder, whose length is 20 inches and whose diameter is 10 inches?

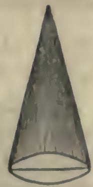
Ans. 1570.796c. in.

2. What are the contents of a quadrangular prism, whose length is 25 feet and whose base is 3 feet square?

396. A PYRAMID is a solid, having a polygonal face, called the *base*, and all its other faces are triangles which meet at a common point, called the *vertex* of the pyramid. The *slant hight* is the distance from the vertex to the middle of one side of the base.

393. What is a Prism? A Cylinder? 394. Rule for finding the surface of a prism or cylinder? 395. Rule for finding the contents of a prism or cylinder? 396. What is a Pyramid? Its vertex? Slant hight?

NOTE. A pyramid is triangular, quadrangular, etc., according as its base is a triangle, quadrangle, etc.



A CONE is a solid, like a pyramid, except that its *base* is a *circle.* The *altitude* of the pyramid or cone is its *perpendicular hight.*

397. PROBLEM 8. To find the contents of a pyramid or of a cone:

RULE. *Multiply the area of the base by one third of the altitude.*

Ex. 1. What are the contents of a cone, whose base is 10 feet in diameter and whose altitude is 24 feet?

Ans. 628.3184 cu. ft.

2. What are the contents of a pyramid, whose altitude is 12 inches and whose base is a triangle, having its base 6 inches and its altitude 8 inches?

398. The FRUSTUM *of a pyramid* or *cone* is the part remaining after a portion next the vertex has been cut off by a plane parallel to the base. The two ends are called the *upper and lower bases.*

399. PROBLEM 9. To find the contents of the frustum of a pyramid or cone:

RULE. *Multiply the sum of the two bases, added to the mean proportional between the two bases, by one third of the altitude of the frustum.*

Ex. 1. What are the contents of the frustum of a quadrangu-

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396. What is a Cone? Altitude of a pyramid or cone? 397. Rule for finding the solid contents? 398. What is the Frustum of a pyramid or cone? 399. Contents of a frustum, how found?

lar pyramid, whose altitude is 21 feet and whose bases are 5 feet and 3 feet square? Ans. 343cu. ft.

2. What are the contents of the frustum of a cone, whose hight is 12 feet and whose bases are 6 feet and 4 feet in diameter?

400. A SPHERE or GLOBE is a solid bounded by a curved surface, all parts of the surface being equally distant from a point within, called the *center.*

A *diameter* of the sphere is a line passing through the center, and limited in both directions by the surface.

401. PROBLEM 10. To find the surface of a sphere:

RULE. *Multiply the circumference by the diameter.*

Ex. 1. What is the surface of a sphere, whose diameter is 100 inches? Ans. 31415.92sq. in.

2. What is the surface of the earth, supposing it to be a sphere 8000 miles in diameter?

3. What is the surface of the sun, supposing it a sphere whose diameter is 885680 miles?

402. PROBLEM 11. To find the contents of a sphere:

RULE 1. *Multiply the surface of the sphere by one third of the radius.*

RULE 2. *Multiply the cube of the diameter by the decimal* .523599; *i. e. by* $\frac{1}{6}$ of 3.141592.

Ex. 1. What are the contents of a sphere, whose diameter is 100 inches? Ans. $523598\frac{2}{3}$c. in.

2. What is the volume or solidity of the earth, supposing it a sphere whose diameter is 8000 miles?

3. What is the volume or solidity of the sun, supposing it a sphere whose diameter is 885680 miles?

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400. What is a sphere? Its diameter? **401.** Rule for finding the surface of a sphere? **402.** Rule for finding the volume or solid contents of a sphere? Second rule?

MISCELLANEOUS EXAMPLES.

1. What number increased by ⅓ of itself gives 20?

2. What number diminished by 4⅔ gives 21?

3. The sum of two numbers is 54 and one of the numbers is 8½ times the other; what are the numbers?

4. Three roods and ten rods are what part of an acre?

5. The difference between two numbers is 37½ and the smaller number is 12½; what is the larger?

6. What number multiplied by 33⅓ gives 1000?

7. What number divided by 37½ gives 64?

8. What is the greatest common divisor of 84 and 144?

9. What is the least common multiple of 72 and 364?

10. What is the interest of $756.64 for 8m. 17d.?

11. The difference between two numbers is 25, and the smaller number is 10; what is the larger? What the sum of the two numbers?

12. The difference of two numbers is 563492, and the larger number is 3642538; what is the smaller? What the sum of the two numbers? 1st Ans. 3079046.

13. How many bricks 8 inches long, 4 inches wide, and 2 inches thick, will be required to build a wall 20 feet long, 16 feet high, and 2½ feet thick?

14. How many bricks whose dimensions are 8′, 4′, and 2′, will it take to build the walls of a house 40ft. long, 28ft. wide, and 22ft. high, the walls to be 1ft. 6′ thick, and no allowance made for doors and windows?

15. The salary of the President of the United States is $25000 per annum; what sum may he expend daily, and yet save $41560 in one term of office, viz. 4 years? Ans. $40.

16. What number, multiplied by ½ of itself, will produce 12½?

17. What number, multiplied by ¾ of itself, will produce 27?

18. How many square feet of boards will it take to lay a floor 20ft. long and 16ft. wide?

19. How large a square floor can be laid with 676 square feet of boards?

20. The fore wheel of a carriage is 9 feet, and the hind wheel $10\frac{1}{2}$ feet in circumference; how many times will each turn round in running from Boston to Andover, $20\frac{1}{2}$ miles?

21. A rectangular piece of land, containing 60 acres, has its length to its breadth as 3 to 2, what are its length and breadth?

22. Bought a cask of molasses, containing 84 gallons, for \$28; but 9 gallons having leaked out, at what price per gallon must I sell the remainder to gain \$1.25? Ans. 43 cents.

23. If a pipe 6 inches in diameter will discharge a certain quantity of water in 4 hours, in what time will a 4-inch pipe discharge the same quantity? Ans. 9 hours.

24. In 12gal. 3qt. 1pt. 2gi., how many gills?

25. In 1846542 seconds how many weeks, days, etc.?

26. Resolve 25740 into its prime factors.

Ans. 2, 2, 3, 3, 5, 11, 13.

27. Reduce $\frac{3}{12}$, $\frac{16}{128}$, $\frac{7}{16}$, and $\frac{8}{27}$ to equivalent fractions having the least common denominator.

28. Reduce 3s. 4d. 2qr. to the fraction of a pound.

29. Reduce $\frac{7}{12}$ of a pound to shillings and pence.

30. Add $\frac{3}{8}$ lb. $\frac{2}{3}$oz. $\frac{1}{8}$dwt. $\frac{3}{8}$gr. together.

31. From $\frac{3}{7}$ lb take $\frac{1}{3}$℥.

32. A colonel, arranging his men in a square battalion, found that he had 31 men remaining; but, increasing the rank and file by 1 soldier, he wanted 20 men to make up the square. Of how many men did his regiment consist? Ans. 656.

33. How shall I mark gloves that cost me 80c. per pair so that I may discount $33\frac{1}{3}$ per cent. from the marked price and yet gain 25 per cent. on the cost? Ans. \$1.50.

34. Suppose that in a shower the water falls to the depth of 2 inches, how many gallons will fall upon a township that is 6 miles square, each gallon containing 231 cubic inches?

35. How many bricks 8′ long, 4′ wide, and 2′ thick, will be required to build a house 32ft. long, $24\frac{2}{3}$ft. wide, and 20ft. high, the walls being 1ft. 4′ thick, the house having 2 doors, each 4ft. wide and 8ft. high, and 21 windows, each 3ft. wide and 6ft. high, no allowance being made for the space occupied by the mortar?

36. What is the square root of the square root of 16 times 81?

37. If a horse travels $6\frac{1}{4}$ miles per hour, how many hours will it take him to travel as far as a rail car will run in 6 hours, the car running $22\frac{1}{2}$ miles per hour?

38. Light moves about 192000 miles per second and sound about 1142 feet per second; what is the ratio of the velocity of light to that of sound? Ans. $887705\frac{445}{571}$.

39. What is the square root of 4 times the square of 8?

40. What is the cube of the square root of 25?

41. What is the cube root of the square of 8?

42. What is the square of the cube root of 8?

43. Two ships sail from the same port, one due north and the other due west, one at the rate of 6 miles and the other 8 miles per hour. Suppose the surface of the ocean to be plane, how far apart are the ships in 10 hours?

44. An army consists of 59049 men; how many shall be placed in rank and file to form them into a square?

45. What is the diameter of a circular pond which shall contain 36 times as much area as one 20 rods in diameter?

46. What is the mean proportional between 16 and 64?

47. What is the third proportional to 3 and 30?

48. A ladder 41 feet long, will reach a window 40 feet high on one side of a street, and, without moving the foot, it will reach a window 9 feet high on the other side; how wide is the street?

Ans. 49ft.

49. What is the difference in the expense of fencing a circular 40-acre lot and one of the same area in a square form, the fence costing 50c. per rod?

50. Sold to J. P. F. goods as follows:

Jan. 18, 1862,	on 6m.,	75yd. of cloth,	at $4,	$300.
Mar. 12, "	" 3m.,	600gal. of molasses,	" $33\frac{1}{3}$c.,	200.
June 15, "	" 4m.,	50bbl. of flour,	" $8.	400.

Also bought of him:

Feb. 18, 1862,	on 4m.,	30c. of wood,	at $ 6,	$180.
May 24, "	" 6m.,	10t. of hay,	" 12,	120.
July 6, "	" 5m.,	10 cows,	" 30,	300.
" 24, "	" 4m.,	1 horse,	"	100.

When shall he pay me the balance of the debt?

51. What is the side of a square equivalent in area to a rectangular field, which is 81 rods long and 49 rods wide?

52. Sent an invoice of goods to my agent in Liverpool which he sold for \$25000; what sum can he invest for me, his commission for selling being 2 per cent. and for investing 1 per cent.?

53. A house worth \$8000 is insured for $\frac{3}{4}$ its value; what is the premium at $\frac{3}{8}$ of 1 per cent.?

54. What is the amount of \$325, at 6 per cent., compound interest, for 3yr. 8m. 12d.?

55. \$1200. *Boston, May* 12, 1860.

For value received of A. B. I promise to pay him, or his order, one thousand two hundred dollars, on demand, with interest. CHARLES DANE.

INDORSEMENTS: Aug. 18, 1860, \$300; Dec. 18, 1860, \$10; May 6, 1861, \$16.50; June 24, 1861, \$400; Dec. 24, 1861, \$100; what was due Apr. 12, 1862?

56. A bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep; what are the dimensions of a similar measure that holds half a peck? Ans. $9\frac{1}{4}$in. diameter; 4in. deep.

57. Sold a lot of goods for \$100 and thereby gained 25 per cent.; what per cent. should I have gained, had I sold them for \$120?

58. A garden whose breadth is 5 rods, and whose length is $1\frac{3}{8}$ times its breadth, has a wall $3\frac{1}{2}$ feet thick and 4 feet high, around it, outside of the line; what was the cost of this wall at $3\frac{1}{4}$c. per cubic foot?

59. What will be the cost of digging a ditch around the above-mentioned garden, within and adjacent to the wall $3\frac{1}{2}$ feet wide and $2\frac{3}{4}$ feet deep, at $\frac{5}{8}$ of a cent per cubic foot?

60. What would be the cost of walling the above-mentioned garden, the central line of the wall to be on the bounding line, the wall to be $3\frac{1}{2}$ feet thick and $3\frac{3}{4}$ feet high and to cost $6\frac{1}{2}$c. per cubic foot?

61. A hare has 45 rods the start of a hound, but the hound runs 12 rods while the hare runs 9; how many rods will the hare run before the hound overtakes him?

62. A hare has 32 rods the start of a hound, but the hare runs

only 16 rods while the hound runs 20; how far will the hound run before he overtakes the hare?

63. What is the interest of $72.50 from Aug. 8, 1861, to July 20, 1862?

64. A, B, and C engage to do a piece of work; A can do it in 20 days, B in 24, and C in 30. In what time can the three together do the work?

65. A gentleman left his son an estate, $\frac{1}{4}$ of which he spent in 1 year and $\frac{5}{12}$ of the remainder in 6 months more, when he had only $1400 remaining; what was the value of the estate?

66. The commander of a besieged fortress has 2lb. of bread per day for each soldier for 45 days, but wishes to prolong the seige to 60 days; what must be the allowance per day?

67. A man sold a watch for $60, which was $\frac{5}{7}$ of its cost; what was lost by the transaction?

68. If a bar of silver 1ft. 6in. long, 4in. wide, and 2in. thick, is worth $1240, what is the value of a bar of gold 1ft. 3in. long, 8in. wide, and 1in. thick, the weight of a cubic inch of silver being to the weight of a cubic inch of gold as 10 to 19, and the value per ounce of silver being to that of gold as 2 to 33?

69. Jan. 1, 1861, A, B, and C form a partnership for 1 year, and each furnishes $2000. May 1, A furnishes $1000 more; June 1, B furnishes $1500 and C withdraws $500; Oct. 1, A withdraws $500, and B and C furnish $1000 each. Having gained $3000, at the close of the year the partnership is dissolved. What is each partner's share of the gain?

70. How many gallons of wine at 6, 10, 15, and 20s. per gal. may be taken to form a mixture of 95 gallons worth 12s. per gallon?

71. Find the difference in time due to a difference of 17° 20′ 40″ in longitude.

72. The difference in the time of two places is 3h. 18m. 15sec.; what is the difference in longitude?

73. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$yd., at the rate of $7 for 5yd., and sold them out at the rate of $11 for 7yd., and gained $200 by the bargains; how many bales were there? Ans. 9.

74. The trans-Atlantic telegraph laid in 1857 from St. John's, Newfoundland, to Valentia, Ireland, 1640 miles in a straight line, consisted of 7 copper wires, twisted together, imbedded in gutta percha, and surrounded by 18 bundles of iron wire. Each bundle of iron wire consisted of 7 wires which were twisted together, and the bundles ran spirally round the cable. Now, to allow for deviations from a straight course, inequalities of the sea-bottom, etc., suppose the cable was $1\frac{4}{8}\frac{3}{2}$ times as long as would be required for a straight course, and that it was necessary to increase the wire 1 mile in every 20 in consequence of twisting the wires, and 1 mile in every 24 because of the bundles running spirally, what length of wire was required for the cable?

Ans. $362906\frac{1}{4}$ miles.

75. By the census of 1860, the number of inhabitants of Alabama was 964296; of Arkansas, 435427; of California, 380016; of Connecticut, 460151; of Delaware, 112218; of Florida, 140439; of Georgia, 1057329; of Illinois, 1711753; of Indiana, 1350941; of Iowa, 674948; of Kansas, 107110; of Kentucky, 1155713; of Louisiana, 709290; of Maine, 628276; of Maryland, 687034; of Massachusetts, 1231065; of Michigan, 749112; of Minnesota, 172022; of Mississippi, 791396; of Missouri, 1182317; of New Hampshire, 326072; of New Jersey, 672031; of New York, 3880735; of North Carolina, 992667; of Ohio, 2339599; of Oregon, 52464; of Pennsylvania, 2906370; of Rhode Island, 174621; of South Carolina, 703812; of Tennessee, 1109847; of Texas, 602432; of Vermont, 315116; of Virginia, 1596079; of Wisconsin, 775873; of the District of Columbia, 75076; and of the Territories, 220143; what was the population of the United States in 1860?

Ans. 31443790.